

Mechanical-magneto coupled model of polymer-bonded magnetostrictive composites

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In order to accurately predict the magneto-elastic property of polymer-bonded magnetostrictive materials, a new mechanical-magneto coupled nonlinear model is proposed in this paper. In the proposed model, the total strain of the composites is expressed by the matrix strain, magnetostriction, eyestrains and strain concentration factors. Firstly, the interaction between the matrix and inclusion phase of the magnetostrictive composites is analyzed and the strain concentration factors are calculated with the Mori-Tanaka mean field method. Then, the magnetization process of the composites is analyzed in detail according to the demagnetizing field theory. Finally, the proposed model of magnetostrictive composites is obtained. Furthermore, in order to verify the proposed model, the performance of the magnetostrictive composites is tested, and the theoretical calculations are also compared with the experimental data. The results show that the given model can greatly simulate the stress concentration and precisely predict the magnetostrictive coefficient, saturation magnetostrictive coefficient as well as the magnetization of the composites.

Keywords: Magnetostrictive composites, Mechanical-magneto coupled model, demagnetization field, Mori-Tanaka method

Предлагается новая магнито-механическая в сочетании с нелинейной модель для прогнозирования магнитоупругих свойств полимерных магнитострикционных материалов. В предложенной модели полная деформация композитов выражается матрицей деформации, магнитострикции и концентрацией деформаций. Проанализировано взаимодействие между матрицей и фазой включения магнитострикционных композитов, с помощью метода Мори-Танакы вычислены коэффициенты концентрации деформаций. Согласно теории размагничивающего поля подробно анализируется процесс намагничивания композитов. С целью проверки предложенной модели, тестируется производительность магнитострикционных композитов, сравниваются теоретические расчеты с экспериментальными данными. Результаты показывают, что данная модель может в значительной степени имитировать концентрацию напряжений и точно предсказывать магнитострикционный коэффициент, величину коэффициента магнитострикции, а также намагниченность композитов.

Механіко-магнітна модель полімерних магнітострікційних композитів. *Ран Чжу, Бовен Ван, Яфан Лі, Шуїн Цао, Цзюань Ян*

Пропонується нова магнито-механічна у поєднанні з нелінійною модель для прогнозування магнітопружних властивостей полімерних магнітострікційних матеріалів. У запропонованій моделі повна деформація композитів виражається матрицею деформації,

магнітострикції і концентрацією деформацій. Проаналізована взаємодія між матрицею і фазою включення магнітострикційних композитів, за допомогою методу Морі-танака обчислені коефіцієнти концентрації деформацій. Згідно теорії розмагнічуючого поля детально аналізується процес намагнічення композитів. З метою перевірки запропонованої моделі, тестується продуктивність магнітострикційних композитів, порівнюються теоретичні розрахунки з експериментальними даними. Результати показують, що дана модель може в значній мірі імітувати концентрацію напруги і точно передбачати магнітострикційний коефіцієнт, величину коефіцієнта магнітострикції, а також намагніченість композитів.

1. Introduction

In recent years, extensive researches have been conducted on Terfenol-D, a new kind of driving element, due to its giant magnetostriction, large electro-mechanical coupling factor and high energy density. However, the processing of Terfenol-D is difficult to be realized owing to the brittleness caused by the rare earth element. As a result, its application to the sensors is limited. In addition, the low resistivity of the material leads to the strong eddy current loss at high frequency, making the Terfenol-D still unable to replace the traditional piezoelectric material at high frequency.

For purpose of improving the mechanical performance and the working frequency of magnetostrictive materials, A. E. Clark proposes the idea for the first time that the magnetostrictive composites should be prepared by using the resin bonded Terfenol-D particles, which can be seen as the start of research on bonded magnetostrictive materials. Through experiment, it is found that in the preparation process, different contents of polymer matrix, particle size and particle shape of inclusion particles as well as the intensity of the oriented magnetic field during solidification can all influence the magnetostrictive properties of composite [1-8].

To predict the performance of the magnetostrictive composites, related theoretical models have been studied by numerous scholars. A magnetic-elastic model of the magnetostrictive composites with chain structure is established by Yin Green et al. [9] with Green Function, while this model has poor prediction effect at the nearly saturated magnetic field. In addition, another model is established by Guan with the Mori-Tanaka method [10], and the demagnetizing factor is considered. Nevertheless, this model is only applicable to the case without external stress. With the application of the mean field method, T.Wang solves the equivalent elastic modulus and equivalent magnetostriction of magnetostrictive multiphase alloy with the glass matrix [11]. Furthermore, J. Aboudi and X.J. Zheng establish the constitutive equations to predict the average strain and average magnetic flux density of magnetostrictive composites under mechanical-magneto-thermal coupling conditions [12], whereas the interac-

tion between matrix and particles is not taken into consideration in this model. Therefore, none of above models has comprehensively considered the effect of mechanical-magneto coupling and the interaction between matrix and particles, which will influence the accuracy and applicability of the model.

In order to solve these problems, a novel nonlinear mechanical-magneto coupled model of polymer-bonded magnetostrictive materials is proposed in this paper. Firstly, the total strain of the composites is decomposed into matrix elastic strain and magnetostrictive strain, and the strain concentration factors are obtained by the Mori-Tanaka method. Then, according to the demagnetization field theory, the magnetization process of the magnetostrictive composites is analyzed. Finally, the proposed model is obtained by utilizing the nonlinear constitutive model of magnetostrictive materials. In addition, in order to verify the given model, the composite sample is tested, and the details are discussed in following contents.

2. Composites Model

2.1 Homogenization Assumption

The ideal structure of the magnetostrictive composites is shown in Fig.1a. It is assumed that there is no particle aggregation, voids or impurities, and the magnetostrictive particles are distributed uniformly in the matrix. During the analysis of the composites, an element D- Ω shown in Fig.1b is employed, where D is the polymer matrix and Ω the magnetostrictive particles.

According to the experiments, a larger magnetostriction coefficient can be obtained by using the needle-like and directional crystalline magnetostrictive particles when preparing the magnetostrictive composites. Besides, the rotation ellipsoid is the most similar shape to the actual shape of the elongated magnetostrictive particles, and thus the rotation ellipsoid in Fig.1c is considered (the long axis is l , while the short axes are d_1 and d_2 respectively). To find the solution to the magneto-elastic problem of the ellipsoid, the elastic Eshelby tensor and magnetic Eshelby tensor are proposed in the lecture [13], whereas the calculation is complex. For the sake of simplifying the model, it

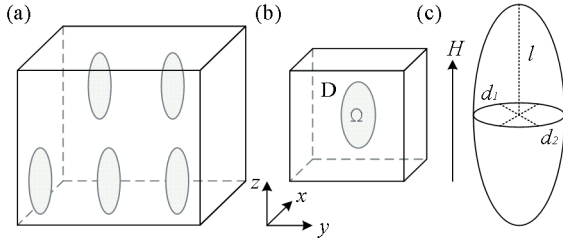


Fig. 1. (a) Structure of magnetostrictive composites (b) Element of the composites (c) Ellipsoidal inclusion and magneto-elastic normalization model.

is assumed that the easy magnetizing axis of Terfenol-D crystal coincides with the long axis of the ellipsoid. According to the ellipsoid magneto-elastic normalization hypothesis, only the shape of particles in the scale factor needs to be considered during the calculation of the magnetic problems. Moreover, to get larger magnetostriction, an oriented magnetic field [3, 5] is applied to the magnetostrictive composites during the preparation process. When the orientation magnetic field strength exceeds the saturated magnetic field intensity, it can be considered that the longitudinal axis of the magnetostrictive particle parallels to the applied magnetic field, as shown in Fig.1c.

2.2 Composites Strain

In order to obtain the magneto-elastic property of magnetostrictive composites, the total strain is decomposed, and the eigenstrain of matrix and magnetostrictive materials is used to express the total strain. The expression is shown in Eq.(1), in which ϵ_0 denotes the matrix strain and ϵ_λ is the eigenstrain of magnetostrictive materials, including $\epsilon_{\lambda,\sigma}$ as well as the magnetostrictive strain $\epsilon_{\lambda,H}$. In addition, A and B are strain concentration factors, which reflect the interaction between the matrix and particles as well as particles and particles in the composite.

$$\epsilon = A\epsilon_0 + B\epsilon_\lambda = A\epsilon_0 + B\epsilon_{\lambda,\sigma} + B\epsilon_{\lambda,H} \quad (1)$$

To achieve the solving of the strain concentration factors A and B , the element shown in Fig.1b is analyzed. According to above assumption of homogenization, the average stress $\langle\sigma\rangle$ and internal stress $\langle\sigma\rangle$ of the element are numerically equal to the total strain of the composite. Moreover, the mean field method as shown in Eq.(2) is employed [14], where ϵ_m is the matrix strain after considering the interaction between matrix and particles, ϵ_f refers to the eigenstrain of magnetostrictive particles after considering the interaction between matrix and particles, σ_m and σ_f denote the stress of

matrix and particles respectively, and V_f is the volume ratio of magnetostrictive particles.

$$\begin{cases} \langle\epsilon\rangle = (1 - V_f)\epsilon_m + V_f\epsilon_f \\ \langle\sigma\rangle = (1 - V_f)\sigma_m + V_f\sigma_f \end{cases} \quad (2)$$

According to Mori-Tanaka method[10, 14, 15], the matrix strain ϵ_m and magnetostrictive particle strain ϵ_f can be expressed as,

$$\epsilon_m = \epsilon_0 + \epsilon_p, \epsilon_f = \epsilon_0 + \epsilon_p + \epsilon' - \epsilon_\lambda \quad (3)$$

Where, ϵ_p is the disturbance strain occurred by the interaction effect between the matrix and particle, among the particles, and the effect of mismatch, ϵ' is strain difference between matrix and particles. According to Hooke's law, the stress on matrix and particles can be expressed by Eq.(4), where C_m is matrix stiffness tensor, C_f is particle stiffness tensor.

$$\sigma_m = C_m(\epsilon_0 + \epsilon_p), \sigma_f = C_f(\epsilon_0 + \epsilon_p + \epsilon' - \epsilon_\lambda) \quad (4)$$

Eshelby equivalent inclusion theory [16] considered that the perturbation strain induced by the eigenstrain of the composites is homogeneous. And which can be replaced by transformation strain generated by the equivalent inclusion with same shape. Relational expression Eq.(5) and Eq.(6) are given. Where S is Eshelby tensor, a function of Poisson's ratio and particle aspect ratio.

$$\epsilon' = S\epsilon^* \quad (5)$$

$$C_m(\epsilon_0 + \epsilon_p + \epsilon' - \epsilon^*) = C_f(\epsilon_0 + \epsilon_p + \epsilon' - \epsilon_\lambda) \quad (6)$$

According to Eq.(2), Eq.(4), Eq.(5) and Eq.(6), σ_f can be written as,

$$\begin{aligned} \sigma_f = [I + (1 - V_f)(S - I)L^{-1}(C_f - C_m)]\langle\sigma\rangle - \\ -(1 - V_f)(S - I)L^{-1}C_m C_f \epsilon_\lambda \end{aligned} \quad (7)$$

L is given by,

$$L = [V_f(S - I) - S]C_f + (1 - V_f)(S - I)C_m \quad (8)$$

According to Eq.(2)-Eq.(6) we can get the average strain, and the matrix strain is $\epsilon_0 = \sigma/C_m$.

$$\begin{aligned} \langle\epsilon\rangle = \frac{1}{V} \int_{1-V_f} \epsilon_m dv + \frac{1}{V} \int_{V_f} \epsilon_f dv = \\ = [I + V_f L^{-1}(C_f - C_m)]\epsilon_0 - V_f L^{-1} C_f \epsilon_\lambda \end{aligned} \quad (9)$$

Comparing Eq.(1) and Eq.(9), strain concentration factor can be obtained,

$$A = I + V_f L^{-1}(C_f - C_m), B = V_f L^{-1} C_f \quad (10)$$

2.3 Composites Magnetization

Due to the matrix is non-magnetic material, the magnetic field distribution in the magnetostrictive composites is different from applied magnetic field H . And the magnetic intensity of the composites is determined by the particle's shape. Considering the element $D-\Omega$ again, according to the demagnetizing field theory[19], seen from Eq.(11), where N is demagnetizing shape factor, H_i is internal magnetic field.

$$H_i = H - NM \quad (11)$$

The relationship between internal magnetic field and magnetization is $M=\chi H_i$ (where χ is the magnetic susceptibility), substituted into Eq.(11) we can get,

$$M = \frac{\chi}{1 + \chi N} H \quad (12)$$

Considering a 1- dimension model, demagnetization shape factor N_d along the magnetic field is given in Eq.(13) [10, 19], where $p=l/d$.

$$N_d = \frac{1}{p^2 - 1} \left[\frac{p}{\sqrt{p^2 - 1}} \ln(p + \sqrt{p^2 - 1}) - 1 \right] \quad (13)$$

Eq.(12) can be rewritten as,

$$H = \frac{1}{\chi} M + N_d M \quad (14)$$

Type reveals the influence of particle shape on the magnetization process, when the demagnetization effect, the magnetostrictive composites saturated magnetic field strength will increases.

2.4 Composites Constitutive Equations

According to Eq.(1), Eq.(10) and Eq.(14), employing the constitutive equation of magnetostrictive materials [17, 18] (appear in Appendix), we can get the mechanical-magnetic coupled equations of the magnetostrictive composites.

$$\varepsilon = A\varepsilon_0 - B \left(\frac{\sigma_f}{E_s} - \lambda_0(\sigma_f) \right) - B \frac{\lambda_s - \lambda_0(\sigma_f)}{M_s^2} M^2 \quad (17)$$

$$H = \frac{1}{k} f^{-1} \left(\frac{M}{M_s} \right) - N_d M - \frac{2(\lambda_s \sigma_f - \Lambda_0(\sigma))}{\mu_0 M_s^2} M \quad (18)$$

where:

$$A = I + V_f L^{-1}(C_f - C_m),$$

$$B = V_f L^{-1} C_f,$$

$$L = [V_f(S - I) - S]C_f + (1 - V_f)(S - I)C_m,$$

$$\lambda_0(\sigma_f) = \begin{cases} \lambda_s \tanh\left(\frac{\sigma_f}{\sigma_s}\right), & \frac{\sigma_f}{\sigma_s} \geq 0 \\ \frac{\lambda_s}{2} \tanh\left(\frac{2\sigma_f}{\sigma_s}\right), & \frac{\sigma_f}{\sigma_s} < 0 \end{cases},$$

$$\Lambda_0(\sigma_f) = \begin{cases} \ln\left[\cosh\left(\frac{\sigma_f}{\sigma_s}\right)\right] & \frac{\sigma_f}{\sigma_s} \geq 0 \\ \ln\left[\cosh\left(\frac{2\sigma_f}{\sigma_s}\right)\right]/4 & \frac{\sigma_f}{\sigma_s} < 0 \end{cases}.$$

By utilizing these equations, the performance of magnetostrictive composite can be predicted. First of all, need to simultaneous Eq.(7) and Eq.(17) of magnetostrictive particles by σ_f was used to solve the problem, then you can calculate the magneto-elastic performance. The parameters including: magnetostrictive material yield stress σ_s , saturation Young's modulus E_s , Poisson's ratio, saturation magnetostriction coefficient λ_s , saturation magnetization M_s , relaxation factor k ($k=3\chi_m/M_s$), susceptibility extreme χ_m , permeability of vacuum μ_0 , Young's modulus and Poisson's ratio of matrix, magnetostrictive particle shape demagnetization factor N_d , volume ratio of magnetostrictive particles V_f . These parameters are all based on physical properties of the material and the preparation process (which determined the shape factor and volume ratio). It means the additional experiments to was not necessary, on the contrary, some other model (i.e. Guan's model) had to do more experiments of determine the parameters.

3. Experiments and Discuss

In order to testify the model, the bonded magnetostrictive material samples as shown in Fig.2 of the samples were prepared. In the sample, the directional crystalline Terfenol-D rod was chosen as raw material, the physical parameters are shown in Tab.1. The Terfenol-D rod was smushed, and 110-150 μ m particles were selected. and observe its external morphology and size determination the particle aspect ratio using $p=2$. Resin as polymer matrix composite materials, parameters are shown in tab.2 the magnetostrictive particles and resin according to the volume ratio of 9:1 were mixed.

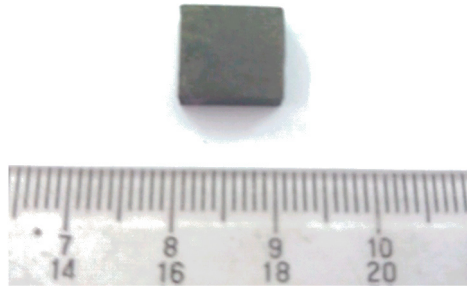


Fig. 2. Magnetostrictive composites sample

Solidification in a magnetic field under the 400kA/m condition, and exerts a certain pressure stress, to reduce air bubbles and cracks inside the material. The material forming the long axis direction the internal magnetostrictive particles along the magnetic field direction. Using the magnetostrictive measurement system, the performance of the sample can be tested. The magnetic field adjustment range is -500kA/m-500kA/m, and the range of stress adjustment is -15-0MPa.

Table.1 Parameters of Terfenol-D Rod

name	Physical meaning	Value
E_0	Initial Young's modulus	60 GPa
E_s	Saturation Young's modulus	110 GPa
ν	Poisson ratio	0.3
λ_s	Saturation magnetostriction	826 ppm
M_s	Saturation magnetostrictive	765 kA/m
σ_s	Maximum elastic stress	200 MPa
χ_m	Maximum magnetic susceptibility	55

Table.2 Parameters of Epoxy Resin

Young's modulus	Poisson ratio
3 GPa	0.35

The parameters are available to the constitutive equations to calculating the magnetostriction and magnetization. For the calculation, the solution of distribution stress σ_f is required. As seen form Fig.3a, when stress σ is applied, the stiffness coefficient of magnetostrictive particles is larger than the substrate, and which resulting composites inner stress concentration, magnetostrictive particle stress σ_f is a little larger than external stress. As shown in Fig3.b, -10MPa stress (press) under the condition of F with sigma magnetization curve, when the magnetic field increases. F will increase. As shown in fig.c, when subjected to tensile stress ($\sigma=5\text{MPa}$), a f with the magnetic field increases.

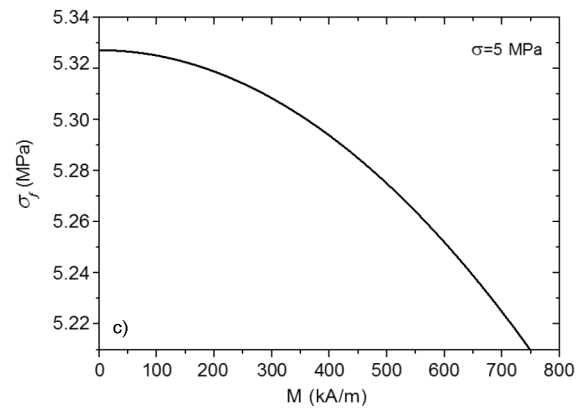
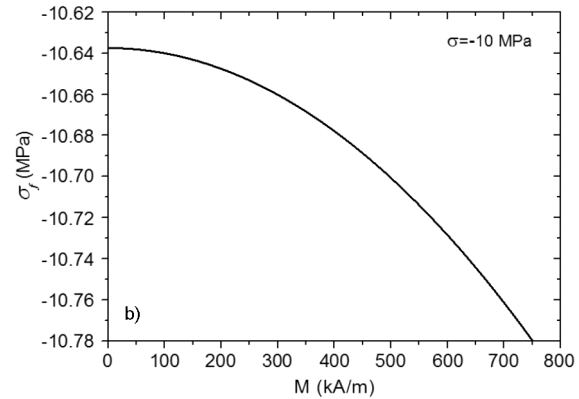
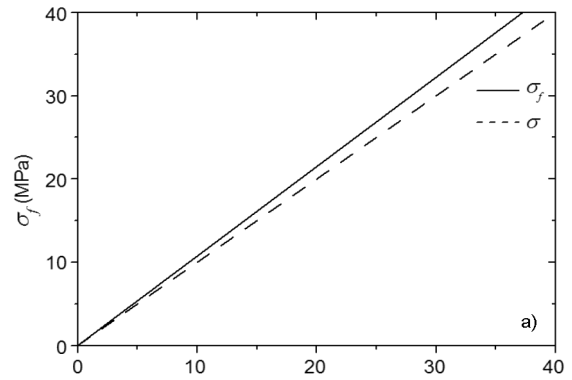


Fig. 3. Stress concentration phenomenon of magnetostrictive composites (a) of depends on external stress when $M = 0$ (b) of depends on magnetization when $\sigma = -10\text{MPa}$ (c) σ_f depends on magnetization when $\sigma = 5\text{MPa}$.

The composites sample was tested without external stress, and the experimental data, the proposed model and Guan's model [10] were compared. As shown in Fig.4, the theoretical value of two models were obvious larger than experimental value. With because in the preparation process, there is structural defects of magnetostrictive particles produced [9, 20], and the magnetostrictive particles were exposed in the air and oxidized, which made the magnetostrictive properties of particles became worse. By comparison, in the middle and low magnetic field, the proposed model is more ac-

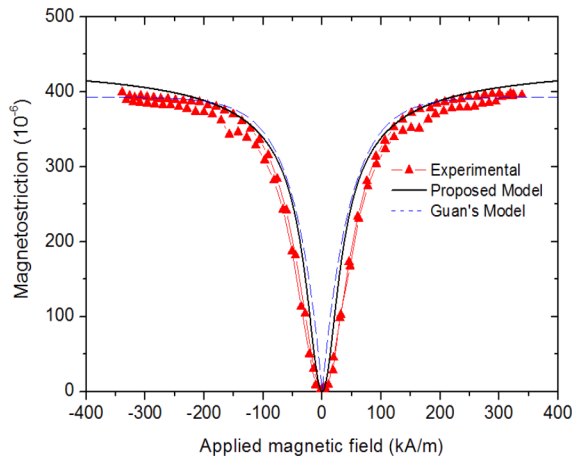


Fig. 4. Magnetostriction curve depends on applied magnetic field when $\sigma=0$

curate than Guan's model, which was due to the given model Employing Langevin function to describe the magnetization process. Which had a clear physical meaning, and could calculate the magnetization of magnetostrictive material accurately. In nearly saturation field, the prediction accuracy is slightly worse compared to Guan's Model.

Fig.5 shows the experimental and theoretical values of teh magnetostriction under different stress conditions. When the stress is very small (such as -2.2MPa), it had certain error of the calculation value in the middle magnetic field and near saturated magnetic field. And with increasing of the compression stress (-8.6MPa, -15MPa), the error was gradually decreasing. This tendency illustrated that the model takes into account the effect of stress interaction and effect of magnetic field. The stress conditions of composite magnetostrictive properties prediction effect is good. According to Eq.(15), the theoretical calculation of the magnetization of the material strength can be obtained.

As shown in Fig.6, **when external stress is -2.2MPa, -8.6MPa and -15MPa, respectively. The magnetization changes along the direction of arrow. This description with the stress increased composites at low magnetic field magnetization performance will become worse, which means the composites needs larger magnetic field to reach saturation. .**

Fig.7 shows the prediction effect of the model for the saturation of the composite under different compression stress conditions. As can be seen from the diagram, the predicted value of the proposed model is larger than the experimental data. This is because in the preparation of composite materials, due to the uniformity of magnetostrictive particles and distribution is difficult to achieve the ideal conditions and in resin matrix and materials under no stress pre-

dicted accuracy is slightly lower than that in Guan's model. When applying the stress, with

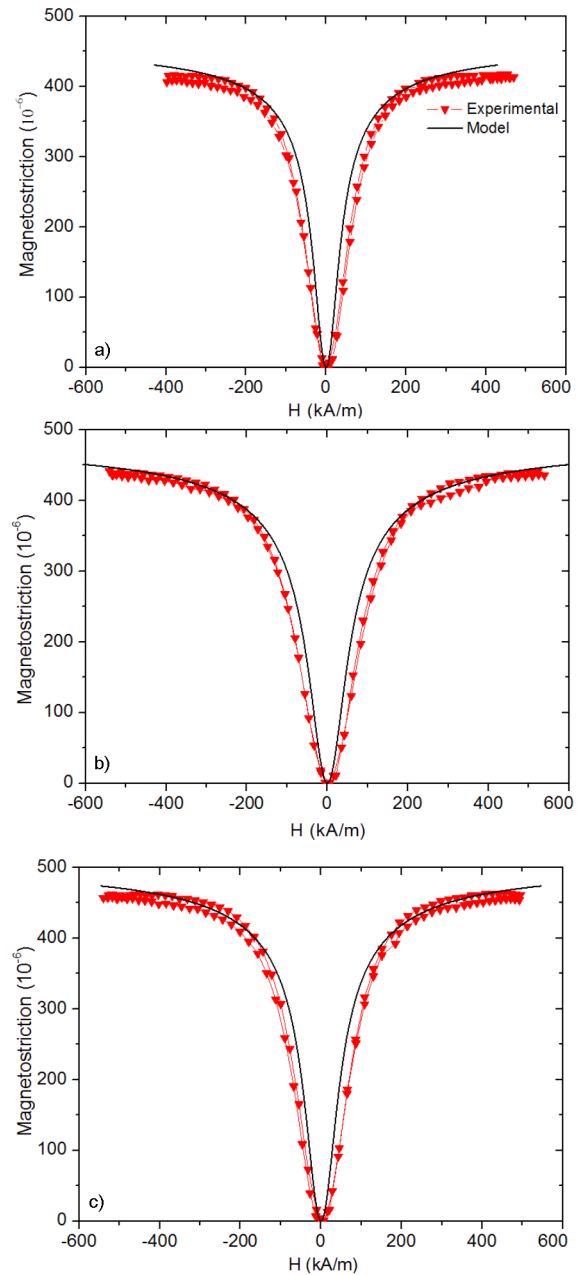


Fig. 5. Magnetostriction depends on applied magnetic field under different stress (a) $\sigma=-2.2$ MPa (b) $\sigma=-8.6$ MPa (c) $\sigma=-15$ MPa

the stress increases gradually with the theory calculation value decreases gradually with the gap between the experimental data and prediction error is less than 5%.

4. Conclusion

The proposed model can accurately predict the magnetization, magnetostriction and saturation magnetostriction of polymer-bonded mag-

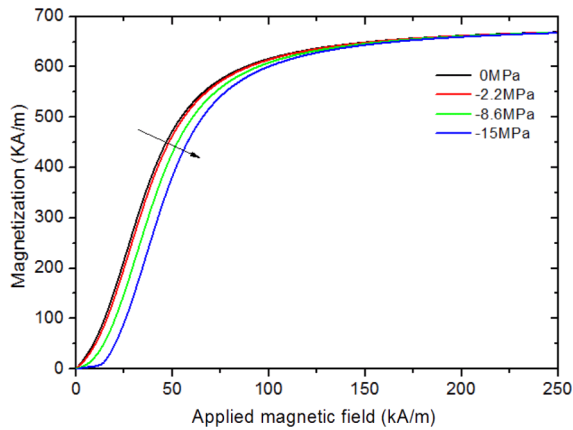


Fig. 6. Magnetization depends on applied magnetic field under variable stress

netostrictive composites. And the equations of the model involves few parameters, which can reduce the measurement work when the parameters are determined. The given model can be used to simulating the performance of the composites in the preparation stage, which is beneficial to control the properties and improve the stability of the preparation of the magnetostrictive composites.

The model can be used for not only polymer matrix but also glass matrix and other non magnetic matrix of magnetostrictive composites. Which means good applicability.

The model of composite materials should be carried out for the stress distribution σ_f which makes the calculation became complex. And the effect of magnetic hysteresis and temperature of the composite were not considered. Further research which will be in the future .

Acknowledgements

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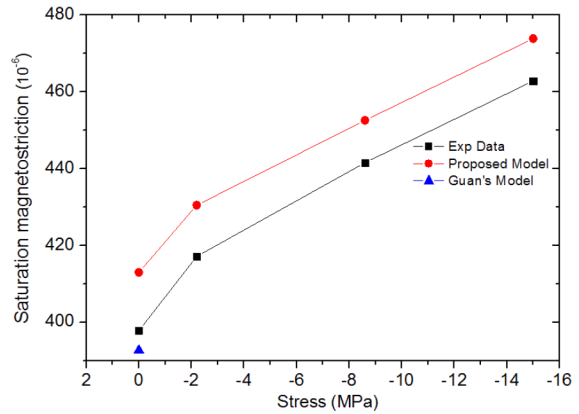


Fig. 7. Saturation magnetostrictive under variable stress

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