Fizika Nizkikh Temperatur, 2004, v. 30, Nos. 7/8, p. 834–840

Spin and current variations in Josephson junctions

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Received February 2, 2004

We study the dynamics of a single spin embedded in the tunneling barrier between two superconductors. As a consequence of pair correlations in the superconducting state, the spin displays rich and unusual dynamics. To properly describe the time evolution of the spin we derive the effective Keldysh action for the spin. The superconducting correlations lead to an effective spin action, which is nonlocal in time, leading to unconventional precession. We further illustrate how the current is modulated by this novel spin dynamics.

PACS: **74.50.+r**, 75.20.Hr, 73.40.Gk

Introduction

The analysis of spins embedded in Josephson junctions has had a long and rich history. Early on, Kulik [1] argued that spin flip processes in tunnel barriers reduce the critical Josephson current as compared to the Ambegaokar–Baratoff limit [2]. More than a decade later, Bulaevskii et al. [3] conjectured that π-junctions may be formed if spin flip processes dominate. The competition between the Kondo effect and the superconductivity was elucidated in [4]. Transport properties formed the central core of these and many other pioneering works, while spin dynamics was relegated to a relatively trivial secondary role. In the present article, we report on new nonstationary spin dynamics and illustrate that the spin is affected by the Josephson current. As a consequence of the Josephson current, spins exhibit novel nonplanar precessions while subject to the external magnetic field. A spin in a magnetic field exhibits circular Larmor precession about the direction of the field. As we report here, when the spin is further embedded between two superconducting leads, new out-of-plane longitudinal motion, much alike that displayed by a mechanical top, will arise. We term this new effect the Josephson nutation. We further outline how transport is, in turn, modulated by this rather unusual spin dynamics. Our predictions are within experimental reach, and we propose a detection scheme.

The system

The system under consideration is illustrated in Fig. 1. It consists of two identical ideal *s*-wave superconducting leads coupled each to a single spin; the entire system is further subject to a weak external magnetic field. In Fig. 1, $\mu_{L,R}$ denote the chemical potentials of the left and right leads, **B** is a weak external magnetic field along the z axis, and $\mathbf{S} = (S_x, S_y, S_z)$ is the operator of the localized spin.

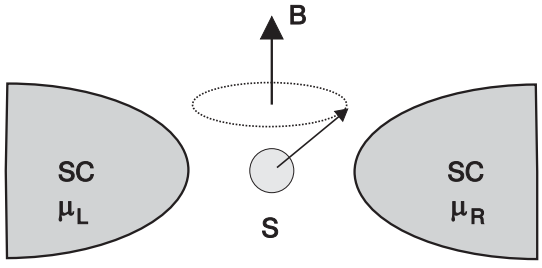

Fig. 1. Magnetic spin coupled to two superconducting leads.

The Hamiltonian of the system reads

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_T, \qquad \mathcal{H}_0 = \mathcal{H}_L + \mathcal{H}_R - \mu B_z S_z, \quad (1)$$

$$\mathcal{H}_T = \sum_{\mathbf{k},\mathbf{p},\alpha,\alpha'} e^{i\varphi/2} c^{\dagger}_{R\mathbf{k}\alpha} [T_0 \delta_{\alpha\alpha'} + T_1 \boldsymbol{\sigma}_{\alpha\alpha'} \cdot \mathbf{S}] c_{L\mathbf{p}\alpha'} + \text{h.c.}, \quad (2)$$

where $\mathcal{H}_L$ and $\mathcal{H}_R$ are the Hamiltonians in the left and right superconducting leads, while $c^{\dagger}_{ik\alpha}$ ($c_{ik\alpha}$) creates (annihilates) an electron in the lead in the state $\mathbf{k}$ with spin α in the right (left) lead for $i = L(R)$. The vector $\boldsymbol{\sigma}$ represents the three Pauli matrices and μ is the magnetic moment of the spin. When a spin is embedded in the tunneling barrier, the conduction electron tunneling matrix becomes, not too surprisingly, spin-dependent: $\hat{T} = [T_0 \hat{1} + T_1 \mathbf{S} \cdot \hat{\boldsymbol{\sigma}}]$ [5,6]. Here T_0 is a spin-independent tunneling matrix element and T_1 is a spin-dependent matrix element originating from the direct exchange coupling J of the conduction electron to the localized spin $\mathbf{S}$. We take both tunneling matrix elements to be momentum independent. This is not a crucial assumption and is merely introduced to simplify notations. Typically, from the expansion of the work function for tunneling, $T_1/T_0 \sim J/U$, where U is the height of a spin-independent tunneling barrier [7]. A weak external magnetic field $B_z \sim 100$ G does not influence the superconductors and we may ignore its effect on the leads. In what follows, we abbreviate μB_z by B. The operator $e^{i\varphi/2}$ is the single electron number operator. When the junction is linked to an external environment, the coupling between the junction and the environment induces fluctuation of the superconducting phase φ.

The effective action

Josephson junctions are necessarily embedded into external electrical circuits. This implies that the dynamics will explicitly depend on the superconducting phase φ. The evolution operator is given by the real-time path integral

$$Z = \int D\varphi D\mathbf{S} \exp[i\mathcal{S}_{\text{circuit}}(\varphi) + i\mathcal{S}_{\text{spin}}(\mathbf{S}) + i\mathcal{S}_{\text{tunnel}}(\varphi, \mathbf{S})]. \quad (3)$$

The effective action $\mathcal{S}_{\text{tunnel}}$ describes the junction itself. We generalize the formerly known effective tunneling action for a spinless junction [8–10] to the spin-dependent arena to obtain

$$\mathcal{S}_{\text{tunnel}} = -2\oint_K dt \oint_K dt' \alpha(t,t')[T_0^2 + T_1^2 \mathbf{S}(t) \cdot \mathbf{S}(t')] \times \cos\frac{\varphi(t) - \varphi(t')}{2} -$$

$$-2\oint_K dt \oint_K dt' \beta(t,t')[T_0^2 - T_1^2 \mathbf{S}(t) \cdot \mathbf{S}(t')] \cos\frac{\varphi(t) + \varphi(t')}{2}, \quad (4)$$

where

$i\alpha(t,t') \equiv G(t,t')G(t',t)$, $i\beta(t,t') \equiv F(t,t')F^{\dagger}(t,t')$

and the Green functions are

$$G(t,t') \equiv -i\sum_{\mathbf{k}} \left\langle T_K c_{\mathbf{k}\sigma}(t) c^{\dagger}_{\mathbf{k}\sigma}(t') \right\rangle, \quad (5)$$

$$F(t,t') \equiv -i\sum_{\mathbf{k}} \left\langle T_K c_{\mathbf{k}\uparrow}(t) c_{-\mathbf{k}\downarrow}(t') \right\rangle, \quad (6)$$

$$F^{\dagger}(t,t') \equiv -i\sum_{\mathbf{k}} \left\langle T_K c^{\dagger}_{\mathbf{k}\uparrow}(t) c^{\dagger}_{-\mathbf{k}\downarrow}(t') \right\rangle. \quad (7)$$

In Eq. (4) $\oint_K$ denotes integration along the Keldysh contour. We now express the spin action on Keldysh contour in the basis of coherent states

$$\mathcal{S}_{\text{spin}} = \oint_K dt\, \mathbf{B} \cdot \mathbf{S} + S_{WZNW}. \quad (8)$$

Here, S denotes the magnitude of the spin $\mathbf{S}$. The second, Wess–Zumino–Novikov–Witten (WZNW), term in Eq. (8) depicts the Berry phase accumulated by the spin as a result of motion of the spin on a sphere of radius S [11,12]. Explicitly,

$$\mathcal{S}_{WZNW} = \frac{1}{S^2} \int_0^1 d\tau \oint_K dt [\mathbf{S}(t,\tau) \cdot (\partial_\tau \mathbf{S}(t,\tau) \times \partial_t \mathbf{S}(t,\tau))]. \quad (9)$$

The additional integral over τ allows us to express the action in a local form. At $\tau = 0$ the spin is set along the z direction at all times, $\mathbf{S}(t,0) = \text{const}$; at $\tau = 1$ the spin field corresponds to the physical configurations, $\mathbf{S}(t,1) = \mathbf{S}(t)$.

Dynamics

We now perform the Keldysh rotation, defining the values of the spin and the phase variables on the forward/backward branches of the Keldysh contour (see Fig. 2, $\mathbf{S}^{u,l}$ for the upper and lower branch) and rewriting all the expressions in terms of their average (classical component $\mathbf{S}$) and difference (quantum component $\mathbf{l}$):

$$\mathbf{S} \equiv (\mathbf{S}^u + \mathbf{S}^l)/2, \quad \mathbf{l} \equiv \mathbf{S}^u - \mathbf{S}^l, \quad \mathbf{S} \cdot \mathbf{l} = 0. \quad (10)$$

After the Keldysh rotation we obtain [13,14]

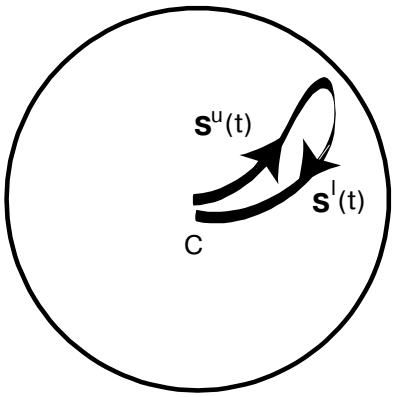

Fig. 2. The sphere of radius S for the vectors $\mathbf{S}^{u,l}(t)$ is shown. The path C describes the evolution of the spin along the upper (u) and lower (l) branches of the Keldysh contour. To properly describe the spin dynamics on this closed contour, we analyze the WZNW action, see Eq. (9). For clarity, we draw a small piece of the closed trajectories.

$$\mathcal{S}_{WZNW} = \frac{1}{S^2}\int_0^1 d\tau \int dt [\mathbf{S}^u(t,\tau)\cdot(\partial_\tau \mathbf{S}^u(t,\tau)\times(\partial_t \mathbf{S}^u(t,\tau)) - (u\to l)]. \tag{11}$$

The relative minus sign stems from the backward time ordering on the return part of C. The individual WZNW phases for the upper (u) and lower (l) branches are given by the areas spanned by the trajectories $\mathbf{S}^{u,l}(t)$ on the sphere of radius S divided by the spin magnitude (S). The WZNW term contains odd powers of $\mathbf{l}$. Insofar as the WZNW term of Eq. (11) is concerned, the standard Keldysh transformation to the two classical and quantum fields, $\mathbf{S}$ and $\mathbf{l}$, mirrors the decomposition of the spin in an antiferromagnet (AF) to the two orthogonal slow and fast fields.* The difference between the two individual WZNW terms in Eq. (11) is the area spanned between the forward and backward trajectories. For close forward and backward trajectories the WZNW action on the Keldysh loop may be expressed as

$$\mathcal{S}_{WZNW} = \frac{1}{S^2}\int dt\, \mathbf{l}\cdot(\mathbf{S}\times\partial_t\mathbf{S}). \tag{12}$$

For the spin part of the (semiclassical) action we, then, obtain

$$\mathcal{S}_{\text{spin}} = \int dt\, \mathbf{B}\cdot\mathbf{l} + \frac{1}{S^2}\int dt\, \mathbf{l}\cdot(\mathbf{S}\times\partial_t\mathbf{S}). \tag{13}$$

Next, we perform the Keldysh rotation to the classical and quantum components with respect to both the phase and spin variables in the tunneling part of the effective action. Towards this end, we introduce (with notations following Refs. 8, 10)

$$\varphi = (\varphi^u + \varphi^d)/2, \qquad \chi = \varphi^u - \varphi^d. \tag{14}$$

With these definitions in hand, the tunneling part of the action reads

$$\mathcal{S}_{\text{tunnel}} = \mathcal{S}_\alpha + \mathcal{S}_\beta, \tag{15}$$

where the normal (quasi-particle) tunneling part $\mathcal{S}_\alpha$ is expressed via the Green functions

$$\alpha^R \equiv \theta(t-t')(\alpha^> - \alpha^<); \quad \alpha^K(\omega) \equiv \alpha^> + \alpha^<,$$

where

$$i\alpha^>(t,t') \equiv G^>(t,t')G^<(t',t);$$
$$i\alpha^<(t,t') \equiv G^<(t,t')G^>(t',t).$$

Similarly the Josephson-tunneling part $\mathcal{S}_\beta$ is expressed via the off-diagonal Green's functions

$$\beta^R \equiv \theta(t-t')(\beta^> - \beta^<); \quad \beta^K(\omega) \equiv \beta^> + \beta^<,$$

where

$$i\beta^>(t,t') \equiv F^>(t,t')F^{\dagger>}(t,t');$$
$$i\beta^<(t,t') \equiv F^<(t,t')F^{\dagger<}(t,t').$$

In this paper we are interested in the interaction between the supercurrent and the spin. Thus we provide the expression for the Josephson part:

$$\mathcal{S}_\beta = \int dt \int dt' 4\beta^R(t,t')\left[\left\{[2T_0^2 - 2T_1^2\mathbf{S}(t)\cdot\mathbf{S}(t')]\sin\frac{\chi(t)}{4}\cos\frac{\chi(t')}{4} - \frac{1}{2}T_1^2\mathbf{l}(t)\cdot\mathbf{l}(t')\cos\frac{\chi(t)}{4}\sin\frac{\chi(t')}{4}\right\}\times\right.$$

* These two orthogonal AF fields represent (i) the slowly varying staggered spin field (the antiferromagnetic staggered moment $\mathbf{m}$ taking on the role of $\mathbf{S}$ and (ii) the rapidly oscillating uniform spin field $\mathbf{l}$ (paralleling our $\mathbf{l}$). In the antiferromagnetic correspondence, the two forward time spin trajectories at two nearest neighbor AF sites become the two forward (u) and backward (l) single spin trajectories of the nonequilibrium problem. This staggered doubling correspondence is general.

$$\times \sin\frac{\varphi(t)+\varphi(t')}{2}+\left\{T_1^2\mathbf{l}(t)\cdot\mathbf{S}(t')\cos\frac{\chi(t)}{4}\cos\frac{\chi(t')}{4}-T_1^2\mathbf{S}(t)\cdot\mathbf{l}(t')\sin\frac{\chi(t)}{4}\sin\frac{\chi(t')}{4}\right\}\cos\frac{\varphi(t)+\varphi(t')}{2}\Bigg]+$$

$$+\int dt\int dt\beta^K(t,t')\Bigg[\left\{[4T_0^2-4T_1^2\mathbf{S}(t)\cdot\mathbf{S}(t')]\sin\frac{\chi(t)}{4}\sin\frac{\chi(t')}{4}+T_1^2\mathbf{l}(t)\cdot\mathbf{l}(t')\cos\frac{\chi(t)}{4}\cos\frac{\chi(t')}{4}\right\}\cos\frac{\varphi(t)+\varphi(t')}{2}-$$

$$-\left\{2T_1^2\mathbf{l}(t)\cdot\mathbf{S}(t')\cos\frac{\chi(t)}{4}\sin\frac{\chi(t')}{4}+2T_1^2\mathbf{S}(t)\cdot\mathbf{l}(t')\sin\frac{\chi(t)}{4}\cos\frac{\chi(t')}{4}\right\}\sin\frac{\varphi(t)+\varphi(t')}{2}\Bigg]. \qquad (16)$$

The normal-tunneling part $\mathcal{S}_\alpha$ is obtained from $\mathcal{S}_\beta$ by the following substitution: $\beta^{R/K}(t,t')\to\alpha^{R/K}(t,t')$, $\varphi(t')\to-\varphi(t')$, and $\chi(t')\to-\chi(t')$. The Keldysh terms (those including β^K and α^K), which normally give rise to random Langevin terms (see, e.g., Ref. 10) are, in our case, suppressed at temperatures much lower than the superconducting gap ($T \ll \Delta$), due to the exponential suppression of the correlators $\beta^K(\omega)$ and $\alpha^K(\omega)$ at $\omega < \Delta$.

To obtain β^R, we start from the Gorkov Green functions

$$F^{>}(t,t') = -i\sum_k \frac{\Delta}{2E_k}\,\mathrm{e}^{-iE_k(t-t')},$$
$$F^{>\dagger}(t,t') = i\sum_k \frac{\Delta}{2E_k}\,\mathrm{e}^{-iE_k(t-t')}, \qquad (17)$$

where the quasiparticle energy $E_k \equiv \sqrt{\Delta^2+\epsilon_k^2}$, ϵ_k being the free-conduction-electron dispersion in the leads. Putting all of the pieces together, we find that

$$\beta^R(t-t') = \theta(t-t')\sum_{k,p}\frac{\Delta^2}{2E_kE_p}\sin[(E_k+E_p)(t-t')]. \qquad (18)$$

The kernel $\beta^R(t-t')$ decays on short time scales of order $O(\hbar/\Delta)$. Varying the full action with respect to the quantum components $\mathbf{l}$ and χ and setting these to zero, we obtain coupled equations of motion for both the spin and phase:

$$\frac{d\mathbf{S}(t)}{dt} = \mathbf{S}(t)\times\mathbf{B} + T_1^2\int dt' 4\beta^R(t-t')\mathbf{S}(t)\times\mathbf{S}(t')\times$$
$$\times\cos\frac{\varphi(t)+\varphi(t')}{2}, \qquad (19)$$

$$\frac{\delta\mathcal{S}_{\text{circuit}}}{\delta\chi(t)}\Big|_{\chi\to0} = -\int dt' 2\beta^R(t-t')(T_0^2-T_1^2\mathbf{S}(t)\cdot\mathbf{S}(t'))\times$$
$$\times\sin\frac{\varphi(t)+\varphi(t')}{2}. \qquad (20)$$

Note, that, if the rest of the circuit contains dissipative elements, e.g., resistors, then $\mathcal{S}_{\text{circuit}}$ will contain the nonvanishing Keldysh components, and one should include the corresponding Langevin terms into Eq. (20). The rather complicated equations of motion (19) and (20) are very general. To make headway, we now adopt a perturbative strategy. In Eq. (19), we first assume an ideal voltage bias, i.e., an imposed phase $\varphi(t) = \omega_J t$, where the «Josephson frequency» $\omega_J = 2eV/\hbar$. To this lowest order, we neglect the influence of the spin on the phase. Next, we use the separation of characteristic time scales to our advantage. To this end, we note that the spin dynamics is much slower as compared to electronic processes, i.e., $\omega_J, B \ll \Delta$. This separation of scales allows us to set $\mathbf{S}(t') \simeq \mathbf{S}(t) + (t-t')d\mathbf{S}/dt$ in the integrand of Eq. (19), wherein we obtain

$$\frac{d\mathbf{S}}{dt} = \lambda\mathbf{S}\times\frac{d\mathbf{S}}{dt}\sin\omega_J t + \mathbf{S}\times\mathbf{B}. \qquad (21)$$

Here

$$\lambda = -4T_1^2\int_0^\infty dt\, t\beta^R(t)\sin\frac{\omega_J t}{2} \approx -2T_1^2\omega_J\int_0^\infty dt\, t^2\beta^R(t) =$$
$$= 2\omega_J\sum_{k,p}\frac{|\Delta|^2|T_1|^2}{E_kE_p(E_k+E_p)^3} =$$
$$= 2T_1^2\rho^2\frac{\omega_J}{\Delta}\int\int\frac{dz_1dz_2}{(\cosh z_1+\cosh z_2)^3} =$$
$$= \frac{\pi^2}{8}T_1^2\rho^2\frac{\omega_J}{\Delta} = \frac{g_1}{32}\frac{\omega_J}{\Delta} \qquad (22)$$

with $g_1 \equiv (2\pi T_1\rho)^2$ being the spin channel conductance. In Eq. (22) we employ the separation of time scales ($\omega_J \ll \Delta$) again. When expressed in the spherical coordinates (in the semiclassical limit) $\mathbf{S} = S(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$, Eq. (21) transforms into two simple first order differential equations

$$\frac{d\varphi}{dt} = -\frac{B}{1+S^2\lambda^2\sin^2(\omega_J t)}, \qquad (23)$$

$$\frac{d\theta}{dt} = -S\lambda\frac{d\varphi}{dt}\sin\theta\sin\omega_J t. \qquad (24)$$

These equations can be solved exactly. For a spin oriented at time $t = 0$ at an angle θ_0 relative to $\mathbf{B}$,

$$\varphi(t) = -\frac{B}{\omega_J\sqrt{1+S^2\lambda^2}}\tan^{-1}[\sqrt{1+S^2\lambda^2}\tan(\omega_J t)],$$

$$\theta(t) = 2\tan^{-1}\left(\left[\frac{(1-c\cos(\omega_J t))(1+c)}{(1+c\cos(\omega_J t))(1-c)}\right]^{\gamma}\tan\frac{\theta_0}{2}\right),$$

with $c = S\lambda/\sqrt{1+S^2\lambda^2}$ and $\gamma = -S\lambda B/2\omega_J c$. For $S\lambda << 1$, $\varphi \simeq -Bt$ and $\theta \simeq \theta_0 - S\lambda(B/\omega_J)\sin\theta_0 \times \times\cos\omega_J t$. Typically, whenever a spin is subjected to a uniform magnetic field, the spin azimuthally precesses with the Larmor frequency $\omega_L = B$. In a Josephson junction, however, the spin exhibits additional polar (θ) displacements. The resulting dynamics may be likened to that of a rotating rigid top. The Josephson current leads to a nonplanar gyroscopic motion (Josephson nutations) of the spin much like that generated by torques applied to a mechanical top. For small λ, we find nutations (see Fig. 3) of amplitude

$$\theta_1 - \theta_2 \propto S\lambda\frac{B}{\omega_J}\sin\theta \propto Sg_1\frac{B}{\Delta}\sin\theta.$$

The origin of the first term on the right-hand side of Eq. (21) can be understood as follows (this origin can be also traced in the calculations): the spin is subject to the electron-induced fluctuating field $\mathbf{h} = T_1\Sigma\, e^{i\varphi/2}c^{\dagger}\boldsymbol{\sigma}c + \text{h.c}$. The same coupling may be thought of as an influence of the spin on the leads, which results in a nonzero low-frequency contribution $\delta\mathbf{h}$ to $\mathbf{h}$. Since the response function of the electron liquid is isotropic but retarded, $\delta\mathbf{h}(t)$ is not aligned with $\mathbf{S}(t)$ but contains information about the values of $\mathbf{S}(t')$ at earlier times. The response function decays on a time scale $\sim \hbar/\Delta$, much shorter than the period of the spin precession, $\sim 1/B$. As a consequence, in addition to a contribution $\propto \mathbf{S}$ the field $\mathbf{h}$ acquires a component $\propto \dot{\mathbf{S}}/\Delta$, which leads to the first term on the right-hand side of Eq. (21).

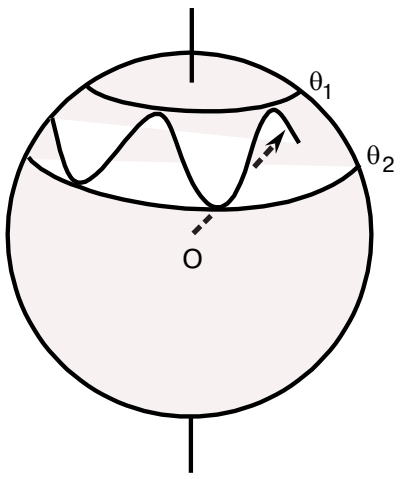

Fig. 3. The resulting spin motion on the unit sphere in the general case. As in the motion of classical spinning top, the spin exhibits undulations along the polar direction.

The right-hand side of the second equation of motion (20) clearly corresponds to the Josephson current. Indeed, in the Keldysh formalism one has $I = (2\pi/\Phi_0)\partial\mathcal{S}/\partial\chi$ (instead of $I = (2\pi/\Phi_0)\partial\mathcal{S}/\partial\varphi$). Thus we obtain for the Josephson current

$$I_J(t) = \frac{2\pi}{\Phi_0}\int dt' 2\beta^R(t-t')(T_0^2 - T_1^2\mathbf{S}(t)\cdot\mathbf{S}(t'))\times$$

$$\times\sin\frac{\varphi(t)+\varphi(t')}{2}. \qquad (25)$$

We start from the lowest order (local in time) adiabatic approximation, i.e., we set $\mathbf{S}(t) = \mathbf{S}(t')$ and $\varphi(t) = \varphi(t')$. This yields

$$I_J(t) = \frac{2\pi}{\Phi_0}E_{J,0}\left(1 - \frac{T_1^2}{T_0^2}\mathbf{S}^2\right)\sin\varphi(t), \qquad (26)$$

where $E_{J,0} \equiv 2T_0^2\int dt\beta^R(t) = \pi^2\rho^2T_0^2\Delta = (1/4)g_0\Delta$ is the spin-independent Josephson energy [2] (g_0 being the conductance of the spin-independent channel). The second term of Eq. (26) gives the spin-related reduction of the Josephson critical current studied in Ref. 1. We now evaluate the lowest-order correction to this equation due to deviations from locality in time and spin precessions. Expanding $\mathbf{S}(t')$ in Eq. (25) in $(t'-t)$ and using the fact that for the Larmor precession we have $\mathbf{S}\cdot\dot{\mathbf{S}} = 0$ and $\mathbf{S}\cdot\ddot{\mathbf{S}} = B^2(S_z^2 - \mathbf{S}^2)$, we find a correction to the Josephson current which depends on S_z^2:

$$I_J(t) = \frac{2\pi}{\Phi_0}\left[E_{J,0}\left(1 - \frac{T_1^2}{T_0^2}\mathbf{S}^2\right) + \delta E_J(S_z^2 - \mathbf{S})\right]\sin\varphi(t), \qquad (27)$$

where

$$\delta E_J \equiv -T_1^2B^2\int dt\beta^R(t)t^2 = \frac{\pi^2}{16}T_1^2\rho^2(B^2/\Delta).$$

Here we clearly elucidated the manner in which the spin dynamics alters the Josephson current.

For $S = 1/2$ the semiclassical approximation is insufficient. In this case it is easier to perform a calculation with spin operators [13], rather than a path integral. One, then, obtains [13] an expression for the Josephson current identical to Eq. (25) with $\mathbf{S}(t)$ being, however, the spin operator in the interaction representation. Using the commutation relations of the spin operators one obtains an extra contribution to the Josephson current proportional to S_z. This allows

reading out of the spin state via the Josephson current. This extra contribution scales as S while the spin-dependent contributions in Eq. (27) scale as S^2.

Detection

We now briefly discuss a detection scheme for the Josephson nutations for $S >> 1$, e.g., in the semiclassical limit. In principle the nutations should affect the Josephson current. The level of approximation employed in this paper was, however, insufficient to describe this effect. Indeed, one has to substitute $\mathbf{S}(t)$ containing the nutations into Eq. (25). As the amplitude of the nutations is of the order g_1, the correction to the current will be of the order g_1^2. We will study this correction elsewhere.

Here we discuss a more direct detection strategy. The spin motion generates a time-dependent magnetic field,

$$\delta\mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi r^5}[3\mathbf{r}(\mathbf{r}\cdot\mathbf{m}(t)) - r^2\mathbf{m}(t)],$$

superimposed on the constant external field $\mathbf{B}$. Here $\mathbf{r}$ is the position relative to the spin, with magnetic moment $\mathbf{m}(t) = \mu\mathbf{S}(t)$. A ferromagnetic cluster of spin $S = 100$ generates a detectable field $\delta B \sim 10^{-10}$ T which appears a micron away from the spin. For a SQUID loop of micron dimensions located at that position, the corresponding flux variation $\delta\Phi \sim 10^{-7}\Phi_0$ (with Φ_0 a flux quantum) are within reach of modern SQUID's. For such a setup with $T_1/T_0 \sim 0.1$, the typical critical Josephson current is $J_S^{(0)} \sim 10\ \mu A$, $|\Delta| = 1$ meV, and $eV \sim 10^{-3}|\Delta|$. We find that $\lambda S \sim 0.1$. Since $S_x = S\sin\theta\cos\varphi$, $S_y = S\sin\theta\sin\varphi$, the spin components orthogonal to $\mathbf{B}$ vary, to first order in (λS), with Fourier components at frequencies $|\omega_L \pm \omega_J|$ $(\omega_L = B)$, leading to a discernible signal in the magnetic field $\mathbf{B} + \delta\mathbf{B}$. For a field $B \sim 200$ G, $\omega_L \sim 560$ MHz, and a new side band will appear at $|\omega_L - \omega_J|$, whose magnitude may be tuned to 10–100 MHz. This measurable frequency is markedly different from the Larmor frequency ω_L.

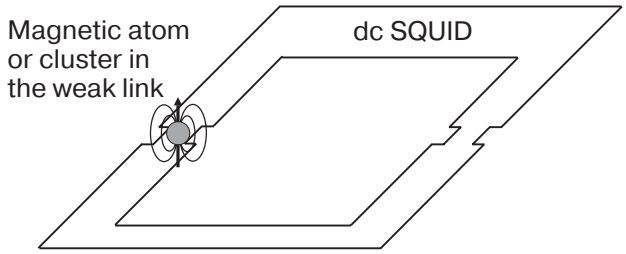

Fig. 4. A SQUID-based detection scheme. The SQUID monitors the magnetic field produced by the magnetic cluster in one of the junctions.

The efficiency of the detector may be further improved by embedding the spin in one of the Josephson junctions of the SQUID itself. The setup is sketched in Fig. 4. The Josephson junction containing the spin is used both for driving the nutations and, together with the second junction of the SQUID, for detecting them.

Conclusion

In this article, we illustrated that the dynamics of a spin embedded in a Josephson junction is richer than appreciated hitherto. We reported unusual nonplanar spin motion (in a static field), which might be probed directly and which was further shown to influence the current in the Josephson junction. Using a path-integral formalism, we described this nonplanar spin dynamics and the ensuing current variations that it triggers. To describe the time evolution we derived the effective action for a spin of arbitrary amplitude S on the Keldysh contour. In passing, we noted a similarity between the resultant effective action and that encountered in quantum antiferromagnetic spin chains. Our central results are encapsulated in the effective action (16).

In the semiclassical limit of large S, relevant to ferromagnetic spin clusters [14], we obtained two coupled equations of motion (Eqs. (19) and (20)). These equations may be solved perturbatively, as outlined above, or numerically. We presented an exact limiting-case solution and illustrated how the new spin dynamics may be experimentally probed.

The formalism developed can also be applied to the minimal $S = 1/2$ system. In this case, however, it is simpler to perform a calculation with spin operators [13], rather than a path integral.

This work was supported by the US Department of Energy under LDRD X1WX, the CFN of the DFG, and the S. Kovalevskaya award (YM). We thank L.N. Bulaevskii and G. Schön for discussions.

1. I.O. Kulik, *Sov. Phys. JETP* **22**, 841 (1966).
2. V. Ambegaokar and A. Baratoff, *Phys. Rev. Lett.* **10**, 486 (1963).
3. L.N. Bulaevskii, V.V. Kuzii, and A.A. Sobyanin, *JETP Lett.* **25**, 290 (1977).
4. L.I. Glazman and K.A. Matveev, *JETP Lett.* **49**, 659 (1989).
5. A.V. Balatsky and I. Martin, *Quan. Inform. Process.* **1**, 53 (2002).
6. A.V. Balatsky, Y. Manassen, and R. Salem, *Phys. Rev.* **B66**, 195416 (2002).

7. J.X. Zhu and A.V. Balatsky, *Phys. Rev.* **B67**, 174505 (2003).
8. V. Ambegaokar, U. Eckern, and G. Schön, *Phys. Rev. Lett.* **48**, 1745 (1982).
9. A.I. Larkin and Y.N. Ovchinnikov, *Phys. Rev.* **B28**, 6281 (1983).
10. U. Eckern, G. Schön, and V. Ambegaokar, *Phys. Rev.* **B30**, 64191 (1984).
11. E. Fradkin, *Field Theories of Condenced Matter Systems*, Addison-Wesley, Redwood City (1991).
12. S. Sachdev, *Quantum Phase transmissions*, Cambridge University Press, London (1999).
13. L.N. Bulaevskii, M. Hruška, A. Shnirman, D. Smith, and Y. Makhlin, *Phys. Rev. Lett.* **92**, 177001 (2004).
14. J.X. Zhu, Z. Nussinov, A. Shnirman, and A.V. Balatsky, *Phys. Rev. Lett.* **92**, 107001 (2004).