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# Polaron Rashba effect in an asymmetric quantum dot

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We study the influence of polaron Rashba effect in an asymmetric quantum dot. Using variational method, we derive the expression of the polaron ground state energy. We also discuss the dependencies of the ground state energy on the wave vector and the transverse (longitudinal) effective confinement length. It is found that the ground state energy splits into two branches due to the Rashba effect. The spin splitting energy is an increasing function of the wave vector and the Rashba SO parameter.

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72.20.Pa Thermoelectric and thermomagnetic effects.

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## 1. Introduction

In recent years, with the development of nanofabrication technology, the semiconductor spintronics in low-dimensional materials has drawn greater physicists' attention in the world. It becomes possible to fabricate the electron device with bran-new physical properties by using electron spin and even realize quantum memory and quantum computation, and then improve rapidly the efficiency of electron device. The spintronics is an intense research topic in condensed matter physics as it has a potential impact on information technology. The Rashba effect is a major branch of the spintronics which plays an important role in the field. Recently, there are many reports on the Rashba effect in low-dimensional systems, especially in quantum dot systems. For example, Y.S. Liu *et al.* [1] proposed a pure thermoelectric spin generator based on a Rashba quantum dot molecular junction by using the temperature difference instead of the usual voltage bias difference. $Z_S T$ of the molecular junction are calculated in terms of the Green's function formulism and the equation of motion technique. Johnson Lee *et al.* [2] investigated the effect of the Rashba spin splitting and a magnetic field on the energy levels of electrons in parabolic quantum dots. Electronic transport through a double quantum dot interferometer with Rashba spin-orbit interaction was studied by X.J. Liu *et al.* [3] with means of the slave-boson mean-field approximation and Green function technique. Using the *k.p* method and valence force field model, X.W. Zhang *et al.* [4] discussed the Rashba spin splitting of the minibands of coupled InAs/GaAs pyramid quantum dots. K.W. Chen *et al.* [5] reported the quantum interference and spin accumulation on double quantum dots with Rashba spin-orbit coupling and electron interaction based on Keldysh non-equilibrium Green's function formalism. M. Governale [6] presented results on the effect of spin-orbit coupling on the electronic structure of few-electron interating quantum dots. The ground state properties as a function of the number of electrons in the dot N are calculated by means of spin-density functional theory. So much works are studied about the influence of the electron Rashba effect in quantum dot system. However, the study of the polaron Rashba effect in the field is quite few so far. Considering the influence of the Rashba SO interaction on the condition of the electron–LO phonon strong coupling in a parabolic quantum dot, J.W. Yin *et al.* [7] calculated the bound polaron ground state energy by the variational method of Pekar. The condition of electric-LO phonon strong coupling in a parabolic quantum dot was studied in detail by W.P. Li *et al.* [8] and the polaron ground state energy was derived by the variational method of Pekar, considering the influence of the Rashba SO interaction.

In the present paper, we study the polaron Rashba effect in an asymmetric quantum dot. First, we derive the expression of the ground state energy of the polaron by using the variational method of Pekar. Then, our numerical results are presented and discussed. Finally, a brief conclusion is drawn in our investigation.

## 2. Theory and model

We consider a polar crystal system in which an electron with heavy hole characteristics is moving and interacting with bulk LO phonons. Due to the phonon field and the polar crystal boundary effect, the moving of the electron in every direction is quantized. On the basis of effective mass approximation, the Hamiltonian of electron–phonon system in an asymmetry quantum dot can be written as

$$H = \frac{P^2}{2m} + V(\mathbf{r}) + \sum_{\mathbf{k}} \hbar\omega_{LO} a_{\mathbf{k}}^{+} a_{\mathbf{k}} + \sum_{\mathbf{k}} (V_{\mathbf{k}} a_{\mathbf{k}} \, e^{i\mathbf{k}\cdot\mathbf{r}} + \text{h.c}) + i\frac{\alpha_R}{2\hbar^3}(P_-^3 \hat{\sigma}_+ - P_+^3 \hat{\sigma}_-) . \quad (1)$$

The first term denotes the kinetic energy of the electron and the second term represents the three-dimensional anisotropic harmonic potential in the $x$–$y$ plane and $z$-direction and it can be expressed as

$$V(\mathbf{r}) = \frac{1}{2} m\omega_1^2 \rho^2 + \frac{1}{2} m\omega_2^2 z . \quad (2)$$

The third and the fourth terms in Eq. (1) describe the LO-phonon field and the interaction energy of the electron–LO phonon, respectively. The last term is the contribution of the Rashba spin-orbit interaction. Where $a_{\mathbf{k}}^{+} (a_{\mathbf{k}})$ is the creation (annihilation) operator of the LO phonons with wave vector $\mathbf{k}$ and the frequency $\omega_{LO}$, $\mathbf{r} = (\rho, z)$ refers to the position operator of an electron and $\omega_1 (\omega_2)$ is the measure of the transverse (longitudinal) confinement strength of quantum dot. Using the notation $P_\pm = P_x \pm iP_y$, $\hat{\sigma}_\pm = \hat{\sigma}_x \pm i\hat{\sigma}_y$, where $\hat{P}$, $\hat{\sigma}$ denote the electron momentum operator and Pauli matrices, respectively. In semiconductor structure, Rashba parameter $a_R$ is determined by many factors.

The Fourier coefficient for the interaction is described by

$$V_k = i\left(\frac{\hbar\omega_{LO}}{k}\right)\left(\frac{\hbar}{2m\omega_{LO}}\right)^{1/4}\left(\frac{4\pi\alpha}{V}\right)^{1/2} . \quad (3)$$

Here $V$ is the volume of the crystal and the electron–LO phonon coupling constant is represented by $\alpha$.

We then carry out the unitary transformation to Eq. (1)

$$U = \exp\left(\sum_{\mathbf{k}} a_{\mathbf{k}}^{+} f_k - a_{\mathbf{k}} f_k^*\right), \quad (4)$$

where $f_k (f_k^*)$ is the variational function to be determined by minimizing the energy.

For the ground state of the system, we choose the following variational trial wave function

$$|\psi\rangle = (\lambda^2/\pi)^{1/2} \exp(-\lambda^2\rho^2/2)(\mu^2/\pi)^{1/4} \times \exp(-\mu^2 z^2/2)\left(a x_{1/2} + b x_{-1/2}\right)\left|0_{\text{ph}}\right\rangle , \quad (5)$$

where

$$\chi_{1/2} = \begin{pmatrix}1\\0\end{pmatrix} \text{ and } \chi_{-1/2} = \begin{pmatrix}0\\1\end{pmatrix}$$

refer to the up and down states of the spin, $a$ and $b$ are coefficients and $|0\rangle_{\text{ph}}$ is the unperturbed zero-phonon state which satisfies $a_{\mathbf{k}} |0\rangle_{\text{ph}} = 0$. The $\lambda$ and $\mu$ are variational parameters which can be determined by minimizing the total energy of the system.

The expectation value of $\langle\psi|U^{-1}HU|\psi\rangle$ for the ground state is

$$F(\lambda, \mu, f_k, f_k^*) = \langle\psi|U^{-1}HU|\psi\rangle = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2}{2m\lambda^2 l_1^4} + \frac{\hbar^2}{4m\mu^2 l_2^4} + \sum_k \hbar\omega_{LO}|f_k|^2 + \sum_k \left(V_k f_k + \text{h.c}\right)\exp\left(-\frac{k_\parallel^2}{4\lambda^2} - \frac{k_z^2}{4\mu^2}\right) \pm \alpha_R k^3 . \quad (6)$$

Using the variational method, we get

$$f_k = -\frac{V_k^* \exp\left(-\frac{k_\parallel^2}{4\lambda^2} - \frac{k_z^2}{4\mu^2}\right)}{\hbar\omega_{LO}}$$

and

$$f_k^* = -\frac{V_k \exp\left(-\frac{k_\parallel^2}{4\lambda^2} - \frac{k_z^2}{4\mu^2}\right)}{\hbar\omega_{LO}} . \quad (7)$$

Substituting Eq. (7) into Eq. (6) and replacing the summation with integration, we have

$$E_{\uparrow\downarrow} = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2}{2m\lambda^2 l_1^4} + \frac{\hbar^2}{4m\mu^2 l_2^4} - \sqrt{\frac{2}{\pi}}\alpha\hbar\omega_{LO}\left(\frac{\hbar}{2m\omega_{LO}}\right)^{1/2} \times \mu\left(1 - \frac{\mu^2}{\lambda^2}\right)^{-1/2} \arcsin\left(1 - \frac{\mu^2}{\lambda^2}\right)^{1/2} \pm \alpha_R k^3 , \quad (8)$$

where $E_\uparrow$, $E_\downarrow$ are the ground state energy of the spin-up and spin-down components with the spin-splitting energy $E_{SO} = a_R k^3$ and $l_1 = \sqrt{\hbar/m\omega_1}$, $l_2 = \sqrt{\hbar/m\omega_2}$ are the transverse and longitudinal effective confinement lengths, respectively.

From Eq. (8) we can see that the spin-splitting energy $E_{SO}$ due to the Rashba effect is proportional to $k^3$ rather than $k$. The reason is that the electron in conduction band owns heavy hole characteristic, so that the spin-splitting energy $E_{SO}$ is proportional to $k^3$.

### 3. Numerical results and discussion

In this section, to show more obviously the influence of the Rashba effect on the properties of the polaron in an asymmetric quantum dot, we perform a numerical calculation. For the sake of simple, we choose the usual polaron units ($\hbar = 2m = \omega_{LO} = 1$). The results are presented in Figs. 1–5.

Figure 1 presents the polaron ground state energy $E_0$ as a function of the wave vector $k$ for fixed $l_1 = 0.2$, $l_2 = 0.4$, $\alpha = 5$ and $a_R = 0.05$. The solid line, the dash dotted line and the dotted line correspond to the cases of ground state energy $E_0$, the spin-up splitting energy $E_\uparrow$ and the spin-down splitting energy $E_\downarrow$, respectively. One can see that the ground state energy will enhance with increasing wave vector. This is because the first term in Eq. (8) is the contribution of the wave vector to the ground state energy with a positive value, which makes the ground state energy increase with the increase of the wave vector. We can also see from it that the ground state energy $E_0$ splits into two branches because of the Rashba effect and the energy spacing between the spin-up and the spin-down becomes larger with the increase of the wave vector. The origin of the increase of the energy spacing is attributed to the increase of wave vector $k$.

In Fig. 2 we plot the dependencies of the ground state energy $E_0$, the spin-up splitting energy $E_\uparrow$ and the spin-down splitting energy $E_\downarrow$ on the transverse effective confinement length $l_1$ with $k = 3$, $\alpha = 5$, $l_2 = 0.6$ and $a_R = 0.05$. The ground state energy $E_0$, the spin-up splitting energy $E_\uparrow$ and the spin-down splitting energy $E_\downarrow$ as functions the longitudinal effective confinement length $l_2$ are shown in Fig. 3 for $k = 3$, $\alpha = 5$, $l_2 = 0{,}2$ and $a_R = 0.05$. In the two figures, the solid line, the dash dotted line and the dotted line correspond to the cases of ground state energy $E_0$, the spin-up splitting energy $E_\uparrow$ and the spin-down splitting energy $E_\downarrow$, respectively. From Figs. 2 and 3, we can see that the polaron energies decrease when the transverse and longitudinal effective confinement lengths increase. These can be attributed to the quantum size effects. We can also see from the two figures that the energy spacing between the spin-up and the spin down becomes bigger with decreasing transverse and longitudinal effective confinement lengths. Comparing Fig. 2 and Fig. 3, we see that the influence of the Rashba effect on the transverse effective confinement length is smaller than the longitudinal effective confinement length.

Figure 4 illustrates the connection between the ground state energy $E_0$, the spin-up splitting energy $E_\uparrow$ and the spin-down splitting energy $E_\downarrow$ with the electron–LO phonon-coupling constant $\alpha$ for fixed $k = 3$, $l_1 = 0.4$, $l_2 = 0.8$ and $\alpha_R = 0.05$. The solid line, the dash-dotted line and the dotted line correspond to the cases of ground state energy $E_0$, the spin-up splitting energy $E_\uparrow$ and the spin-down

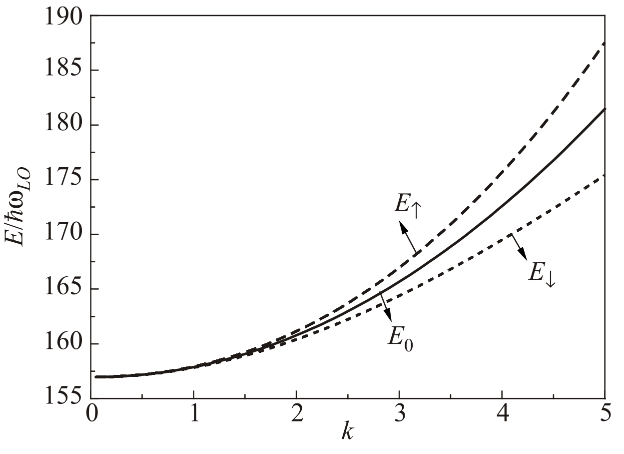


*Fig. 1.* The dependencies of polaron ground state energy $E_0$ and spin-up (spin-down) splitting energy $E_\uparrow(E_\downarrow)$ on the wave vector $k$.

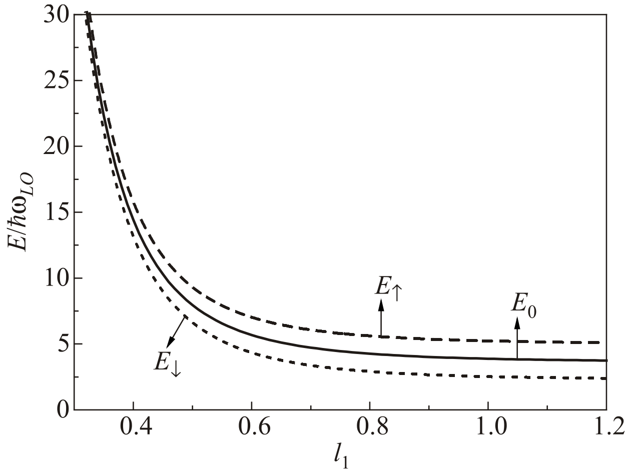


*Fig. 2.* The dependencies of polaron ground state energy $E_0$ and spin-up (spin-down) splitting energy $E_\uparrow(E_\downarrow)$ on the transverse effective confinement length $l_1$.

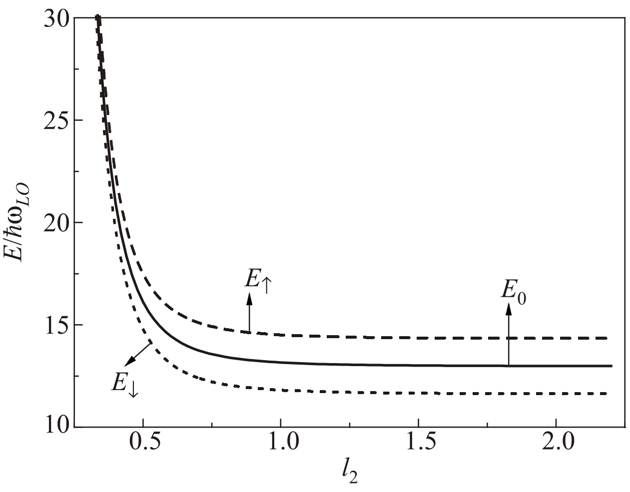


*Fig. 3.* The dependencies of polaron ground state energy $E_0$ and spin-up (spin-down) splitting energy $E_\uparrow(E_\downarrow)$ on the longitudinal effective confinement length $l_2$.

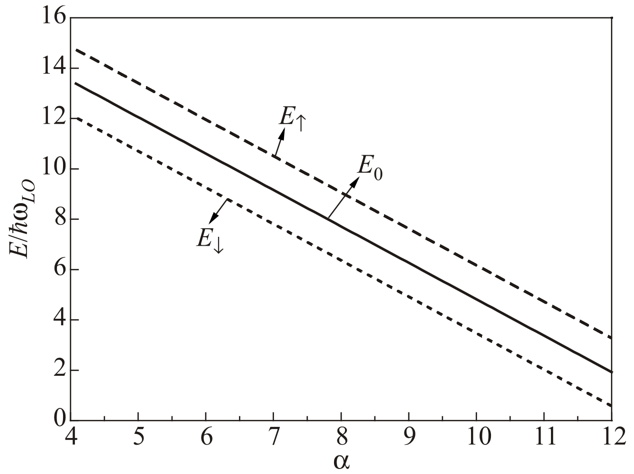


*Fig. 4.* The connection of polaron ground state energy $E_0$ and spin-up (spin-down) splitting energy $E_{\uparrow}(E_{\downarrow})$ with the electron–phonon coupling strength $\alpha$.

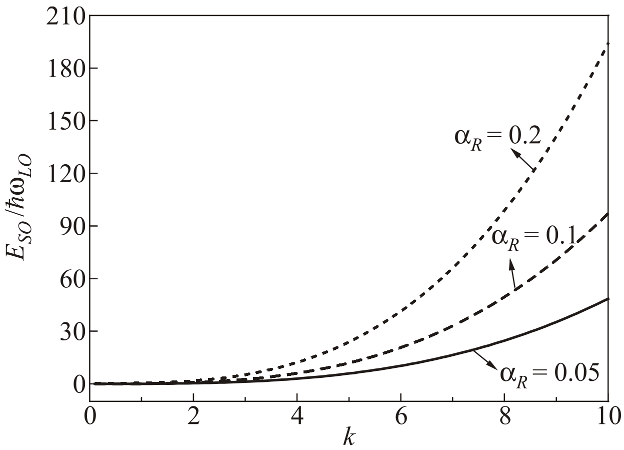


*Fig. 5.* The change of polaron spin splitting energy $E_{SO}$ with the wave vector $k$ at different Rashba SO parameter $\alpha_R$.

splitting energy $E_{\downarrow}$, respectively. It can be seen that the polaron ground state energy is a decreasing function of the electron–LO phonon coupling constant. Since the larger the electron-phonon coupling strength is, the stronger the electron–phonon interaction is. So it makes the electron interact with more phonons. However, the fourth term in Eq. (8) is the contribution of the electron–phonon coupling strength to the polaron energy which is a negative value. As a result the polaron energy will decrease with the increase of the electron–phonon coupling strength. One can also see from the figure that the change of the energy spacing between spin-up and spin-down is zero with increasing electron–phonon coupling strength. That is the coupling strength has no influence on the Rashba effect. The fourth term in Eq. (8) includes the electron–phonon interaction energy. In calculating, we take a constant value to $\lambda$ and $\mu$. So electron–phonon coupling strength is proportional to the interaction energy. Therefore, we can get a conclusion that the influence of the interaction between the electron and LO-phonon on the Rashba effect also can be neglected.

Figure 5 indicates the spin-splitting energy $E_{SO}$ change with the wave vector $k$ at different Rashba SO parameter $a_R$. The solid line, the dash dotted line and the dotted line correspond to the cases of Rashba parameter $a_R = 0.05$, $a_R = 0.1$ and $a_R = 0.2$, respectively. We can see from the figure that the spin-splitting energy will parabolic enhance with the increase of the wave vector. Because the spin-splitting energy $E_{SO}$ is proportional to $k^3$. Hence, the spin-splitting energy is an increasing function of the wave vector. From Fig. 1 and Fig. 4, it can be seen an interesting phenomena that the spin splitting is zero near $k = 0$.

## 4. Conclusion

In this paper we investigate the polaron Rashba effect in an asymmetric quantum dot by using variational method. We discuss the relations of the ground state energy varying with the wave vector, the transverse effective confinement length and the longitudinal effective confinement length. It is found that the ground state energy splits into two branches because of the Rashba effect. The results indicate that the ground state energy and the spin-up (spin-down) splitting energy increase with increasing wave vector, but they decrease with the increase of the transverse (longitudinal) effective confinement length. The spin splitting energy is an increase function of the wave vector and the Rashba SO parameter.

1. Y.S. Liu, F. Chi, X.F. Yang, and J.F. Feng, *J. Appl. Phys.* **109**, 053712 (2011).
2. J. Lee, H.N. Spector, W.C. Chou, and C.S. Chu, *J. Appl. Phys*. **99**, 113708 (2006).
3. X.J. Liu, H.T. Yin, W.L. Wan, H. Li, X.Z. Wang, and C.B. Yao, *Phys. Status Solidi B* **248**, 1735 (2011).
4. X.W. Zhang, Q. Xu, W.J. Fan, and J.W. Luo, S.S. Li, and J.B. Xia, *Appl. Phys. Lett*. **92**, 143113 (2008).
5. K.W. Chen and C.R. Chang, *Phys. Rev. B* **78**, 235319 (2008).
6. M. Governale, *Phys. Rev. Lett*. **89**, 206802 (2002).
7. J.W. Yin, W.P. Li, Y.F. Yu, and J.L. Xiao, *J. Low Temp. Phys*. **163**, 53 (2011).
8. W.P. Li, J.W. Yin, Y.F. Yu, and J.L. Xiao, *J. Low Temp. Phys*. **160**, 195 (2010).