TO THE PHASE DYNAMICS OF RELATIVISTIC ELECTRON BUNCHES AT PLASMA WAKEFIELD EXCITATION

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Theoretical investigation of influence of radial component of plasma wakefield on phase dynamics of the bunches has been performed. The wakefield is excited in plasma by long sequence of the relativistic electron bunches. It has been shown that at certain conditions this radial wakefield leads to more quick shift of the bunches, in comparison with shift by longitudinal wakefield, from decelerating phases into accelerating phases. This shift leads to change of a sign of an energy exchange of electron with a wave and to plasma wakefield amplitude saturation.

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1. INTRODUCTION

Recently impressing results on electron acceleration by strong fields (up to 100GeV/m), excited in plasma by the driver, have been achieved. These drivers are a powerful short laser pulse (10^{15} W of femtosecond duration) [1] or a short electron bunch with a large charge (more than 10^{11} electrons in a bunch of length 10...20 μm) [2]. The alternative approach to such method is studied theoretically and experimentally in NSC KIPT. This approach is called wakefield. It is actually modification of charged particle acceleration by fields of the space charge, excited in plasma by non compensated electron bunch, proposed by Ya.B.Fainberg [3]. In this approach the long sequence of electron bunches (up to $6 \cdot 10^3$ for a macro-pulse of duration 2 μs) is used instead of a single bunch with a large charge. The sequence is produced by the linear conventional electron resonance accelerator. The wakefield is excited in plasma. If frequency of bunch repetition coincides with frequency of wakefield the wakefields of the separate bunches are added coherently. The net wakefield should be equal to a field, excited by the equivalent bunch of a large charge, more than on 3 orders exceeding a charge of one bunch of the sequence. I.e. the equivalent intensity of the driver, comparable to a bunch on SLAC [2], can be achieved.

However, as follows from experimental results [4], not all bunches of sequence participate in coherent excitation. The mechanisms, restricting growth of wakefield amplitude, are following. The first mechanism is determined by an energy dissipation of an excited wave (a Q-factor in a resonator concept). This mechanism leads to saturation of wakefield amplitude when the dissipation is compared to a wave excitation by sequence of bunches. The second mechanism of saturation of wakefield amplitude is related to a wave nonlinearity - change of its phase velocity and detuning of frequency of bunch repetition and wave frequency, i.e. destroying of Cerenkov resonance. The third mechanism consists in bunch trapping by an excited wave and stopping on the average an energy exchange of bunches with a wave.

As it is difficult in experiment to specify mechanisms for determination of the main contribution of one of them, the numerical simulation of plasma wakefield excitation by long sequence of electron bunches has been performed [5]. In this numerical simulation the wakefield dissipation is excluded, and other mechanisms were subject for interpretation.

In this paper the new mechanism of saturation is considered. This mechanism is related to influence of radial electrical wakefield on phase dynamics of bunches. The radial wakefield leads to bunch shift from decelerating phases to accelerating phases, hence to change of a sign of an energy exchange of bunches with a wave.

2. PROBLEM FORMULATION

For electrons with the relativistic factor $\gamma \gg 1$ the phase shift relative to excited wakefield $d\phi = d(\omega t - k_z z) = (\omega - k_z v_z)dt$ (ω, k_z are the frequency and longitudinal wave number, v_z is the electron velocity) by longitudinal electrical field E_z is difficult (the longitudinal velocity v_z changes small) because the longitudinal electron mass $m_z = m_0\gamma^3$ is large (m_0 is rest mass of electron). This allows to form accelerating structure of the linear resonance accelerators homogeneous ($v_{ph} = \omega/k_z = v_z$). Since transversal electron mass $m_\perp = m_0\gamma$ (in this direction the electron is "easy" in comparison to the longitudinal direction, where it is "heavy") the electron can be accelerated by transversal electric field E_r to essential velocity v_r . As velocity of the relativistic electron $v = (v_z^2 + v_r^2)^{1/2} \simeq c$, the growth v_r means

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corresponding decrease of v_z and corresponding appreciable shift of the longitudinal phase ϕ, caused by transversal electrical field E_r. It can lead to the situation when after N^* bunches, passing through plasma, the further bunches of sequence occur in an accelerating phase of an excited wave. Consequently amplitude growth of wakefield saturates.

Similar feature of two-dimensional dynamics of a moving relativistic electron under the acting of constant transversal field E_x was investigated in [6]. It was shown that the relativistic electron unlike the nonrelativistic case moves instead of a parabola $x = z^2 eE_\perp / 2mv_0^2$ along more abrupt catenary line

$$x = \frac{W}{eE_\perp} \operatorname{ch}\left(\frac{zeE_\perp}{p_0 c}\right) . \qquad (1)$$

Here W is the kinetic energy, p_0 is the initial momentum, directed along z. In [6] however the reason of such change of the relativistic electron trajectory is not discussed. It is easy to be convinced that it is caused by that for lack of longitudinal force the longitudinal momentum is conserved

$$p_z = m_0 \gamma v_z = p_{z0} = const . \qquad (2)$$

The relativistic factor $\gamma = \left(1 + \frac{(p_\perp^2 + p_{z0}^2)}{m^2c^2}\right)^{1/2}$ increases due to increase of the transversal momentum accordingly to

$$\frac{dp_\perp}{dt} = eE_\perp . \qquad (3)$$

As a result the longitudinal velocity decreases

$$v_z = \frac{p_{z0}}{m_0 \gamma} . \qquad (4)$$

It leads to that unlike the nonrelativistic case for which $v_z = const$, the relativistic electron trajectories have more abrupt view.

In the presence of E_z the electron motion on the longitudinal phase under the acting of transversal electric field $E_\perp$ can be more essential for large γ, than under the effect of longitudinal electrical field E_z. For determination of threshold on γ of more effective action of the transversal electrical field in comparison with longitudinal one, the solution of an exact problem on an electron phase dynamics in a two-dimensional electrical field is necessary.

3. PHASE DYNAMICS OF THE ELECTRON BUNCH IN THE PLASMA WAKEFIELD

As it has been shown in [7] an electron bunch, moving in plasma, excites wake in the form of a plasma wave fields $E_z^{(1)}$ and $E_r^{(1)}$, which is potential one without magnetic component $H_\theta = 0$

$$\begin{aligned}
E_z^{(1)} &= -\left(\frac{4\pi^{1/2}\eta_z I_0}{\sigma_r^2 \omega_p}\right) exp\left(-\frac{\eta_z^2}{4}\right) P_z(\eta) cos\xi , \\
E_r^{(1)} &= \left(\frac{4\pi^{1/2}\eta_z I_0}{\sigma_r^2 \omega_p}\right) exp\left(-\frac{\eta_z^2}{4}\right) P_r(\eta) sin\xi , \\
P_z(\eta) &= K_0(\eta) \int_0^\eta I_0(x) exp\left(-\frac{x^2}{\eta_b^2}\right) x dx + \\
&\quad + I_0(\eta) \int_\eta^\infty K_0(x) exp\left(-\frac{x^2}{\eta_b^2}\right) x dx , \\
P_r(\eta) &= -\partial_\eta P_z , \qquad (5)
\end{aligned}$$

$$\eta = k_p r, \quad k_p = \frac{\omega_p}{c}, \quad \eta_b = k_p \sigma_r ,$$
$$\eta_z = k_p \sigma_z, \quad \xi = k_p (z - ct) .$$

Here ω_p is the plasma frequency, $\sigma_z = ct_b$ is the bunch length, σ_r is the bunch radius, j_b is the beam current density. Function $P_z(\eta)$ monotonously decreases, and function $P_r(\eta)$ has a maximum in the region of bunch boundary $\eta \simeq \eta_b$ [7]. In the case of a thin bunch $\eta_b \ll 1$ in the region $\eta \le 1$ the radial electrical field exceeds the longitudinal field. The bunch of the large transversal dimension $\eta_b \gg 1$ excites preferentially longitudinal electrical field $P_z(\eta) \ge P_r(\eta)$, $\eta \le 1$ [7].

The exciting bunch (driver) occurs in the plasma wakefield, excited by this bunch and by all previous bunches of sequence

$$E_z^N = \sum_1^N E_z^{(1)}, E_r^N = \sum_1^N E_r^{(1)} . \qquad (6)$$

These wakefields and bunch dynamics in them have been calculated with hybrid $2.5D$ code LCODE [5]. The longitudinal momenta of 500 bunches as they pass the middle of the plasma ($z = 50cm$) are shown in Fig. 1 [5]. The amplitude of the on-axis longitudinal electric field as a function of the coordinate along the plasma and the number of bunches [5] is shown in Fig. 2. It is seen that after passing of about $100 \ldots 300$ bunches wakefield saturates.

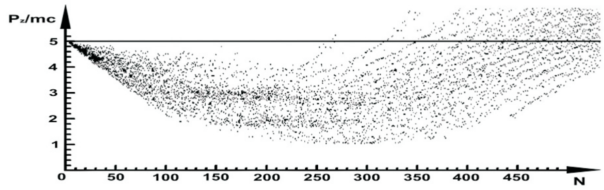

Fig.1. *Longitudinal momenta of 500 bunches as they pass the middle of the plasma (z=50 cm) [5]*

The longitudinal and radial electron dynamics of bunches in the plasma wakefield is described by the equation of motion [6]

$$\frac{d\vec{v}}{dt} = \frac{e}{m}\left(1 - \frac{v^2}{c^2}\right)^{1/2} \left[\vec{E} + \frac{[\vec{v}\vec{H}]}{c} - \frac{\vec{v}(\vec{v}\vec{E})}{c^2}\right] . \qquad (7)$$

As in plasma $H_\theta = 0$ (see (5)) we have

$$\frac{d\vec{v}}{dt} = \frac{e}{m}\left(1 - \frac{v^2}{c^2}\right)^{1/2} \left[\vec{E} - \frac{\vec{v}(\vec{v}\vec{E})}{c^2}\right] . \qquad (8)$$

The equations for longitudinal and radial velocities of electrons have the views:

$$\frac{dv_z}{dt} = [E_z - E_r\gamma^2\frac{v_r}{c}]\frac{e}{m\gamma^3}\,,$$
$$\frac{dv_r}{dt} = [E_r - E_z\frac{v_r}{c}]\frac{e}{m\gamma}\,. \quad (9)$$

At the condition

$$\frac{v_r}{c}\gamma^2 E_r > E_z \quad (10)$$

the phase shift of bunch electrons is determined mainly by the radial field E_r

$$\frac{dv_z}{dt} = \frac{e}{m\gamma}\frac{v_r}{c}E_r. \quad (11)$$

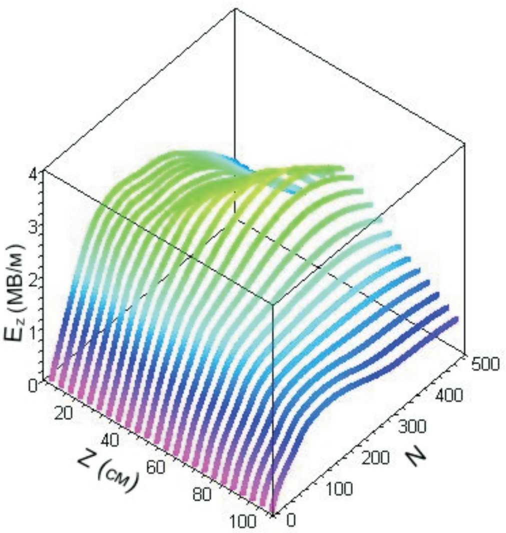

Fig.2. *The amplitude of the on-axis electric field as a function of the coordinate along the plasma and the number of bunches [5]*

This main influence of transversal field E_r is the result of large longitudinal mass $m_z = m_0\gamma^3$ of the relativistic electron and its small transversal mass $m_\perp = m_0\gamma$.

The ratio $\alpha = E_r/E_z$ depends on the bunch geometry [7] and for experimental parameters [4] $\alpha \sim 1$. Taking into account $\alpha \sim 1$, one can derive from (10) the condition of prevailing action of E_r on the electron shift along the longitudinal phase in the experiment [4]:

$$\frac{v_r}{c} > \frac{1}{\gamma^2} \quad (12)$$

or

$$eE_rL > \frac{m_0c^2}{\gamma} = \frac{W}{\gamma^2}\,. \quad (13)$$

Here L is the system length. This inequality is performed easily for the relativistic electrons.

It is necessary to note that for vacuum accelerators longitudinal and radial dynamics of bunch electrons is described by the equations

$$\frac{dv_z}{dt} = \frac{e}{m_0\gamma}[E_z\left(1-\frac{v_z^2}{c^2}\right) - \left(E_r\frac{v_z}{c} - H_\theta\right)\frac{v_r}{c}]\,,$$
$$\frac{dv_r}{dt} = \frac{e}{m_0\gamma}[E_r\left(1-\frac{v_r^2}{c^2}\right) - H_\theta\frac{v_z}{c} - E_z\frac{v_z v_r}{c^2}]\,. \quad (14)$$

From (14) two conclusions follow. First, because

$$H_\theta = E_r\frac{\omega}{ck_z}\,, \quad (15)$$

in (14) we have $(v_zE_r/c - H_\theta) = 0$,

$$\frac{dv_z}{dt} = E_z\frac{e}{m_0\gamma_0^3}\,. \quad (16)$$

Hence transversal fields do not act on longitudinal electron dynamics.

Second, from (14), (15) it follows in approximation $v_r \ll c$, $v_z \simeq c$

$$\frac{dv_r}{dt} = E_r\frac{e}{m_0\gamma_0^3}\,. \quad (17)$$

From (16), (17) it follows that the dependence of longitudinal and radial motions is of the same order on γ_0.

4 NUMBER OF BUNCHES OF COHERENTLY EXCITING WAKEFIELD

One can derive from (11) and second equation (9) in approximation $v_r << c$ that the longitudinal the longitudinal phase shift $\triangle\phi = \pi$, caused by wakefield E_r, is achieved at

$$\Delta\phi = k_z\int_0^{\frac{L}{v_z}} dt' \int_0^{t'} dt''\frac{ev_rE_r}{m\gamma c} = \pi\,, \quad (18)$$

where $v_r = \int_0^t dt'\frac{eE_r}{m\gamma}$. For coherent case (coincidence of plasma frequency and bunch repetition frequency) the total field according (6) is equal

$$E_r = NE_r^{(1)}\,. \quad (19)$$

Substituting (19) in (18) one can derive number of bunches, in which wakefield bunch shifts on longitudinal phase on π

$$N^* = \left(\frac{6\pi}{Lk_z}\right)^{1/2}\left(\frac{m\gamma c^2}{eLE_r^{(1)}}\right)\,. \quad (20)$$

The wakefield amplitude from single bunch in experiments [4] is equal

$$E_r^{(1)} = \frac{4I_0}{\sigma_r^2\omega_p} \simeq 0.3keV/cm\,. \quad (21)$$

Close value of $E_r^{(1)}$ has been observed in numerical simulation [5] and follows from analytical calculation. From (20), (21) the number of coherent bunches for $\gamma = 5$, $L = 50\,cm$,

$$N^* \simeq 130 \quad (22)$$

approximately coincides with numerical simulation [5].

5 CONCLUSION

Wakefield, excited in plasma by a long sequence of the relativistic electron bunches, has been considered. It has been shown that at plasma wakefield acceleration electron phase dynamics can be determined mainly by radial electrical wakefield. The radial wakefield can force shift of bunches from decelerating phases into accelerating phases. This shift changes a sign of an energy exchange of bunches with a wave and leads to saturation of its amplitude growth.

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О ФАЗОВОЙ ДИНАМИКЕ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОННЫХ СГУСТКОВ ПРИ ВОЗБУЖДЕНИИ КИЛЬВАТЕРНОЙ ВОЛНЫ В ПЛАЗМЕ

И.Н. Онищенко, В.И. Маслов

Проведено теоретическое исследование влияния радиальной компоненты поля кильватерной волны на фазовую динамику сгустков. Кильватерная волна возбуждается в плазме длинной последовательностью релятивистских электронных сгустков. Показано, что при определенных условиях эта радиальная компонента приводит к более быстрому смещению сгустков, чем под действием продольной компоненты, из тормозящих фаз в ускоряющие. Такое смещение приводит к смене знака энергообмена сгустков с волной и к насыщению ее амплитуды.

ПРО ФАЗОВУ ДИНАМІКУ РЕЛЯТІВИСЬКИХ ЕЛЕКТРОННИХ ЗГУСТКІВ ПРИ ЗБУДЖЕННІ КІЛЬВАТЕРНОЇ ХВИЛІ В ПЛАЗМІ

І.М. Онищенко, В.І. Маслов

Проведено теоретичне дослідження впливу радіальної компоненти поля кільватерної хвилі на фазову динаміку згустків. Кільватерна хвиля збуджується у плазмі довгою послідовністю релятивістських електронних згустків. Показано, що при визначених умовах ця радіальна компонента приводить до більш швидкого ссуву згустків, порівняно з ссувом під дією повздовжньої компоненти, із гальмуючих фаз в прискорюючі. Такий ссув призводить до зміни знаку енергообміну згустків з хвилею і до насичення її амплітуди.