THE NONLINEAR THEORY OF THE ELECTROMAGNETIC FIELD EXCITATION IN ORBITRON

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(Received July 6, 2009)

The nonlinear theory of electromagnetic waves generation in orbitron is developed. The set of the equations including the equations of field excitation and the equations of 2-dimential motion is constructed and numerically solved. It is shown, that mechanism of electron bunching and energy exchange of electrons with the wave in orbitron and in magnetron has much in common. For the fixed parameters of orbitron from the point of view of generated energy and electronic efficiency there is some optimal value of electron density in the interaction region.

PACS: 84.30.Jc, 85.10.Jz

1. INTRODUCTION

In [1] the original generator of millimeter waves - so called "orbitron" has been proposed, that represents a coaxial structure, which inner cylinder is the thin metal string. Advantage of such generator consists in simplicity of a design, absence of slowing down system and an external magnetic field. At applying to the string (as an anode) of positive potential of several kV electrons are pulled out from an inner surface of the cylindrical resonator, being as a cathode. Multiply spreading on molecules of residual gas, they get an azimuthal component of the velocity V_φ that allows electrons to exchange energy with waves traveling in an azimuthal direction. It has been shown [2], that frequencies of rotating eigenwaves are determined by the formula

$$\omega_{mn} \approx \frac{\pi c}{b}\left(n + \frac{|m|}{2} + \frac{1}{4}\right), \qquad (1)$$

where c is speed of light, b is inner radius of an external casing of coaxial structure, $m = 0, \pm1, \pm2, ...$ and $n = 1, 2, ...$ - are azimuthal and radial numbers of harmonics of eigenwave accordingly. As frequencies of eigenwaves are discrete, we shall conventionally name the considered coaxial system as a resonator. From the formula (1) follows, that phase velocity of eigenwave $v_{ph} = \omega_{mn} r/m$ is less than speed of light under condition of $r \ll b$, where r is a distance from the resonator axis. Thus, the wave appears slowed down near the string. Just in this area electromagnetic waves generation takes place that corresponds to experimental data [1]. We shall emphasize, that slowing down of a wave in orbitron occurs in absence of the special slowing down system. The linear stage of generation in orbitron has been investigated in [2-4] where the conditions of instabilities originating have been found and formulas for their increments have been obtained. In [5] in of the given field approximation the nonlinear dynamics of nonrelativistic electrons in orbitron has been considered at small amplitudes of the wave. It is of interest to carry out more general nonlinear consideration with refuse from the assumptions made in [5], simplifying the picture of wave generation in orbitron. In the present work the nonlinear theory of electromagnetic waves excitation in orbitron is developed, allowing to study the generation process starting from the field fluctuation amplification.

2. DERIVATION OF THE EQUATIONS OF THE NONLINEAR THEORY

Let's consider the high Q coaxial cylindrical resonator, unbounded along an axis z (the cylindrical system of coordinates r, φ, z is used). Radius of the charged string, which creates an electrostatic field $\vec{E} = 2\vec{e}Q/r$ where Q is linear charge density of the string, is equal $a << b$. The following two-dimentional non-stationary problem is being solved that simulates the generation process in orbitron. At absence of electrons in orbitron there is some fluctuation of an electromagnetic field having components H_z, E_φ, E_r (H -wave). At the initial moment of time nonrelativistic electrons are uniformly distributed along the circle of radius r_0 . By virtue of azimuthal symmetry they have equal initial speeds $V_{r0}, V_{\varphi o}$. At the following moments of time electrons start to move in plane r, φ in the electrostatic field of the string and in the fluctuation field, giving up its energy to the fluctuation. As a result of fluctuation amplification the electromagnetic field is generated in orbitron. We find the time-dependent field of the wave in the form of expansion on eigen waves of the

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resonator that forms a full set of functions

$$\vec{E}(r,\varphi,t) = Re\sum_{m=-\infty}^{\infty}\sum f_{mn}\vec{e}_{mn}(r,\varphi),$$
$$\vec{H}(r,\varphi,t) = Re\sum_{m=-\infty}^{\infty}\sum g_{mn}\vec{h}_{mn}(r,\varphi)\,, \tag{2}$$

where $\vec{e}_{mn}(r,\varphi)$,$\vec{h}_{mn}(r,\varphi)$ is intensity electric and a magnetic field of eigen harmonics of the wave

$$e_{mn,r} = -\frac{mR_{mn}(k_{mn}r)}{k_{mn}r\sqrt{N_{mn}}}exp(im\varphi),$$
$$e_{mn,\varphi} = -i\frac{mR'_{mn}(k_{mn}r)}{\sqrt{N_{mn}}}exp(im\varphi), \tag{3}$$
$$h_{mn,r} = \frac{R_{mn}(k_{mn}r)}{\sqrt{N_{mn}}}exp(im\varphi),$$

where
$R_{mn} = J_m(k_{mn}r)N'_m(k_{mn}a) - J'_m(k_{mn}a)N_m(k_{mn}r)$,
$R'_{mn} = J'_m(k_{mn}r)N'_m(k_{mn}a) - J'_m(k_{mn}a)N'_m(k_{mn}r)$,
k_{mn}/c, $J_m(x)$ and $N_m(x)$ are Bessel and Neumann functions (the stroke means differentiation by argument x),

$$N_{mn} = \{[(k_{mn}b)^2 - m^2]R^2_{mn}(k_{mn}b) - [(k_{mn}a)^2 - m^2]R^2_{mn}(k_{mn}a)\}/(4k^2_{mn})$$

is a normalizing multiplier. In (2) the field of a spatial charge is not taken into account. In the further the harmonic with $m = 0$ will not be considered, as it does not lead to bunching of electrons. Eigen functions (3) are normalized by the following way:

$$\int_a^b drr\int_o^{2\pi}d\varphi\vec{h}_{mn}\vec{h}^*_{m'n'} = \int_a^b drr\int_o^{2\pi}d\varphi\vec{e}_{mn}\vec{e^*}_{m'n'} = 4\pi\delta_{mm'}\delta_{nn'}\,. \tag{4}$$

Substituting relation (2) in Maxwell equations

$$rot\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t},$$
$$rot\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}, \tag{5}$$

and using conditions of normalization (4), we shall obtain the following equations of excitation for time-dependent amplitudes of the expansion (2) :

$$\frac{df_{mn}(t)}{dt} + i\omega_{mn}g_{mn}(t) = -2K_{mn}(t),$$
$$\frac{dg_{mn}(t)}{dt} + i\omega_{mn}f_{mn}(t) = 0, \tag{6}$$

$$f_{mn}|_{t=0} = f_{0mn} = |f_{0mn}|exp(i\Phi_{0mn}),$$
$$g_{mn}|_{t=0} = g_{0mn} = |g_{0mn}|exp(i\Phi^M_{0mn}), \tag{7}$$

where function of time

$$K_{mn}(t) = \int_a^b drr\int_0^{2\pi}d\varphi\vec{j}(r,\varphi,t)\vec{e^*}_{mn}(r,\varphi)\,, \tag{8}$$

makes sense a coefficient of coupling of eigen (cold) wave with a flow of electrons. The current $\vec{j}$ in formulas (5,8) is formed by electrons emitted from the cathode. By means of (3) formula (8) can be represented in the form

$$K_{mn}(t) = \frac{1}{\sqrt{N_{mn}}}\{K_{1mn}(t) - K_{4mn}(t) + i(K_{2mn}(t) + K_{3mn}(t))\}, \tag{9}$$

where

$$K_{1mn} = \int_a^b drr\int_0^{2\pi}d\varphi j_\varphi R'_{mn}(k_{mn}r)sin(m\varphi),$$
$$K_{2mn} = \int_a^b drr\int_0^{2\pi}d\varphi j_\varphi R'_{mn}(k_{mn}r)cos(m\varphi), \tag{10}$$

$$K_{3mn} = \int_a^b drr\int_0^{2\pi}d\varphi j_r\frac{mR_{mn}(k_{mn}r)}{k_{mn}r}sin(m\varphi),$$
$$K_{4mn} = \int_a^b drr\int_0^{2\pi}d\varphi j_r\frac{mR_{mn}(k_{mn}r)}{k_{mn}r}cos(m\varphi). \tag{11}$$

As in absence of electrons there is no chosen direction in azimuth, it is natural to assume, that initial fluctuations of the field are standing waves in an azimuthal direction. Therefore following relations between amplitudes and phases of the fluctuations should be fulfilled

$$|g_{0mn}| = |g_{0,-mn}| = |f_{0mn}| = |f_{0,-mn}|,$$
$$\Phi_{0mn} = \Phi^g_{0mn} = \Phi_{0,-mn} = \Phi^g_{0,-mn}\,. \tag{12}$$

The solutions of the equations of excitation (6) with initial conditions (7) are convenient to present in an integrated view

$$f_{mn} = f_{0mn}cos(\omega_{mn}t) - ig_{0mn}sin(\omega_{mn}t) - exp(-i\omega_{mn}t)\int_0^t dt'exp(i\omega_{mn}t')K_{mn}(t') - exp(i\omega_{mn}t)\int_0^t dt'exp(-i\omega_{mn}t')K_{mn}(t'), \tag{13}$$

$$g_{mn} = -if_{0mn}sin(\omega_{mn}t) + g_{0mn}cos(\omega_{mn}t) - exp(-i\omega_{mn}t)\int_0^t dt' exp(i\omega_{mn}t')K_{mn}(t') + exp(i\omega_{mn}t)\int_0^t dt' exp(i\omega_{mn}t')K_{mn}(t')\,. \tag{14}$$

Relations(10-14) allow to present any harmonic of the electric field of the wave (2) in the form

$$E_{mn,r}(r,\varphi,t) = Re(f_{mn}(t)e_{mn,r}(r,\varphi) + f_{-mn}(t)e_{-mn,r}(r,\varphi)) = \frac{2mR_{mn}(k_{mn}r)}{N_{mn}k_{mn}r}\{L_{1mn}(t)cos\Phi^+_{mn} + L_{2mn}(t)sin\Phi^+_{mn} + L_{3mn}(t)cos\Phi^-_{mn} + L_{4mn}(t)sin\Phi^-_{mn} - \frac{\sqrt{N_{mn}}}{2}|f_{0mn}|(cos(\Phi^+_{mn} + \Phi_{0mn}) + cos(\Phi^-_{mn} - \Phi_{0mn})\}\,, \tag{15}$$

$$E_{mn,\varphi}(r,\varphi,t) = Re(f_{mn}(t)e_{mn,\varphi}(r,\varphi) + f_{-mn}(t)e_{-mn,\varphi}(r,\varphi)) = -\frac{2R'_{mn}(k_{mn}r)}{N_{mn}}\{L_{5mn}(t)cos\Phi^+_{mn} + L_{6mn}(t)sin\Phi^+_{mn} + L_{7mn}(t)cos\Phi^-_{mn} + L_{8mn}(t)sin\Phi^-_{mn} - \frac{\sqrt{N_{mn}}}{2}|f_{0mn}|(sin(\Phi^+_{mn} + \Phi_{0mn}) - sin(\Phi^-_{mn} - \Phi_{0mn})\}\,, \tag{16}$$

where

$$L_{1mn} = -\int_0^t dt'\{\cos\theta' K_{4mn}(t') + sin\theta' K_{3mn}(t')\}, L_{2mn} = \int_0^t dt'\{\sin\theta' K_{4mn}(t') - cos\theta' K_{3mn}(t')\}, \tag{17}$$

$$L_{3mn} = \int_0^t dt'\{\sin\theta' K_{3mn}(t') - cos\theta' K_{4mn}(t')\}, L_{4mn} = -\int_0^t dt'\{\sin\theta' K_{4mn}(t') + cos\theta' K_{3mn}(t')\}, \tag{18}$$

$$L_{5mn} = \int_0^t dt'\{\cos\theta' K_{2mn}(t') + sin\theta' K_{1mn}(t')\}, L_{6mn} = \int_0^t dt'\{\cos\theta' K_{1mn}(t') - sin\theta' K_{2mn}(t')\}, \tag{19}$$

$$L_{7mn} = \int_0^t dt'\{\cos\theta' K_{2mn}(t') - sin\theta' K_{1mn}(t')\}, L_{8mn} = \int_0^t dt'\{\cos\theta' K_{1mn}(t') + sin\theta' K_{2mn}(t')\}, \tag{20}$$

$$\theta' = \omega_{mn}t', \Phi^+_{mn} = m\varphi - \omega_{mn}t, \Phi^-_{mn} = m\varphi + \omega_{mn}t. \tag{21}$$

From formulas (15,16,21) it is visible, that partial waves of the field in orbitron represent waves traveling in an azimuthal direction. To find electron current $\vec{j}(r,\varphi,t)$, which determines functions (8-11), it is necessary to solve nonrelativistic equations of motion in cylindrical coordinates with corresponding initial conditions

$$\frac{dV_r}{dt} = \frac{V_\varphi^2}{r} - \frac{V_Q^2}{r} - \frac{e}{m_e}E_r(r,\varphi,t), \frac{dV_\varphi}{dt} = -\frac{V_rV_\varphi}{r} - \frac{e}{m_e}E_\varphi(r,\varphi,t), \frac{dr}{dt} = V_r, \frac{d\varphi}{dt} = \frac{V_\varphi}{r}, \tag{22}$$

$$V_r|_{t=0} = V_{r0}, V_\varphi|_{t=0} = V_{\varphi 0}, r|_{t=o} = r_0, \varphi|_{t=o} = \varphi_0. \tag{23}$$

In the equations (22) $-e < 0$ and m_e are charge and mass of electron, $V_Q^2 = 2eQ/m_e$ is square of a certain scale velocity of electrons, and components of the electric field of the wave are determined by formulas (2). The set of equations (6,7,22) describes self-consistently the process of electromagnetic waves excitation in orbitron. For their solution we shall present electron current in the form

$$\vec{j}(r,\varphi,t) = -e\sum_i^{\tilde{N}} \frac{1}{r}\delta(r - r_i(t))\delta(\varphi - \varphi_i(t)), \quad (24)$$

where $r_i(t), \varphi_i(t), \vec{V}_i(t)$ are the solutions of the equations (22,23) for i electron, $\widetilde{N}$ is full amount of electrons in the interaction region. For application of the method of macroparticles it is necessary to write down the current (24) in the following view ,

$$\vec{j}(r,\varphi,t) = -e\mu_e\sum_j^{N} \frac{1}{r}\delta(r - r_j(t))\delta(\varphi - \varphi_j(t)), \quad (25)$$

where j is number of macroparticle, N is full amount of macroparticles, μ_e is mass of macroparticle, determined by the amount of electrons in macroparticle. We note, that representation of the current in the form (24,25) allows to pass simply from Euler coordinates in (10,11) to Lagrange coordinates of macroparticle, which are the solutions of the set of equations (22,23). At that there are absent, from the computing point of view, laborious process of distribution of a charge in cross-points of Euler grids of coordinates and interpolation of force in points of particles locations. Substituting (25) into formulas (10,11), we shall obtain

$$K_{1mn} = -e\mu_e\sum_j^N V_{j\varphi}(t)R'_{mn}(k_{mn}r_j(t))sin(m\varphi_j(t)), K_{2mn} = -e\mu_e\sum_j^N V_{j\varphi}(t)R'_{mn}(k_{mn}r_j(t))cos(m\varphi_j(t)), \quad (26)$$

$$K_{3mn} = -e\mu_e\sum_j^N V_{jr}(t)\frac{mR_{mn}(k_{mn}r_j(t))}{k_{mn}r_j(t)}sin(m\varphi_j(t)), K_{4mn} = -e\mu_e\sum_j^N V_{jr}(t)\frac{mR_{mn}(k_{mn}r_j(t))}{k_{mn}r_j(t)}cos(m\varphi_j(t)). \quad (27)$$

For convenience of calculations the equation deduced above have been led to a dimensionless form. For conciseness they are not presented. The formula for energy of an excited field of the wave, corresponding to unit of orbitron length, can be obtained from expansion (2). It has a view ,

$$\varepsilon(t) = \sum_{m=1}^{\infty} \varepsilon_{mn}(t), \varepsilon_{mn}(t) = \frac{1}{4}\{|f_{mn}(t)|^2 + |f_{-mn}(t)|^2 + |g_{mn}(t)|^2 + |g_{-mn}(t)|^2\}. \quad (28)$$

The solution of the problem is determined by the following main parameters: the ratio of radii b/a ; amount of electrons, corresponding to unit of coaxial length l , i.e. $\tilde{N}/l$; voltage U in kV applied to the string; coefficient of decreasing of particle energy $\alpha = m_eV_0/(4eQln(b/a)) < 1$, which characterizes of its energy losses due to collisions with molecules of residual gas and is equal to the ratio of its kinetic energy in the point r_0 to the maximal possible kinetic energy which the particle would gain, having passed a way from the cathode up to the anode without scattering; parameter of synchronism of particles with a harmonic (m,n) of initial fluctuation of the wave $a_{smn} = (V_{\varphi 0} - v_{phmn}/v_{phmn})$; angle $_{sc}$ which is formed by initial velocity of the particle $\vec{V}_0$ with radial direction, at that $tg\varphi_{xc} = -V_{\varphi 0}/V_{r0}$. The values $\beta_{\varphi 0} = V_{\varphi 0}/c$ and $\rho_0 = r_0/a$ are determined by parameters α , U , a_{smn} , at that at their fixed values $\beta_{\varphi 0}$ grows together with ρ_0 . Parameters of initial fluctuation $|f_{0mn}|, \Phi_{0mn}$ are chosen small enough that final results did not depend on them. Accuracy of calculations is determined by amount of particles N and by step of integration in time Δt . Electron in orbitron possesses not only kinetic W_k , but also potential W_p energy which are given by the relations

$$W_k = W_{k\varphi} + W_{kr}, \quad W_{k\varphi} = \frac{m_eV_\varphi^2}{2}, \quad W_{kr} = \frac{m_eV_r^2}{2}, \quad W_p = 2eQln\frac{r}{r'}\,, \quad (29)$$

where r' is reference point of potential. The electron efficiency for orbitron is determined as follows. Let's consider firstly the case when electrons fall down only on the string. Let in the initial moment of time their kinetic and potential energies are equal W_{k0} and W_{p0} , and energy of initial fluctuation of an electromagnetic field is $\varepsilon(0)$. After the termination of generation process when all particles will fall down on the string, corresponding values are equal $W_{kf}, W_{pf}, \varepsilon_f$. From the law of energy conservation follows, that

$$W_{k0} + W_{p0} - W_{pf} = \varepsilon_f - \varepsilon(o) + W_k \,. \qquad (30)$$

Work of the external source creating a voltage between the anode and the cathode is spent on the initial kinetic energy of electrons and their potential energy relating to the string and consequently is equal $W_{k0} + W_{p0} - W_{pf}$. From (30) it is visible, that at the end of generation the work of the external source transforms to the energy of the field and the energy of the anode heating-up, which is equal W_{kf} . The electron efficiency is equal

$$\eta = \frac{\varepsilon_f - \varepsilon(0)}{W_{k0} + W_{p0} - W_{pf}} = 1 - \frac{W_{kf})}{W_{k0} + W_{p0} - W_{pf}}. \qquad (31)$$

For magnetrons the relation $W_{k0} \approx 0$ is fulfilled. In magnetron case formulas (31) transorm into efficiency for magnetron [6-8]. Under certain conditions in orbitron subsidence of some part of particles on the cathode is possible. The similar phenomenon takes place in magnetrons too [6-7]. In this case the part of work of an external source is spent for increase in potential energy of this part of particles which should be added in the right part of equality (30) and in numerator of the second formula (31). Subsidence of electrons on the cathode leads to reduction of the value η .

3. RESULTS OF CALCULATIONS

The numerical solution the equations obtained above yielded following results. Values ε, η as functions of the angle φ_{sc} have a maximum at $\phi_{sc} = \pi/2$. Deviations of the angle φ_{sc} from this value leads to sharp reduction of energy of the generated field and efficiency of its excitation. It means, that only those electrons effectively interact with the wave, which trajectories are close to circular and which components of velocity are subjected to the condition

$$V_\varphi >> V_r. \qquad (32)$$

It means, that electron exchanges with the wave only by an azimuthal part $W_{k\varphi}$ of its kinetic energy. Power, transferred to the wave by an electron, by virtue of (32) and relation $E_\varphi \approx E_r$ [5] is equal

$$P = -e(V_\varphi E_\varphi + V_r E_r) \approx -eV_\varphi E_\varphi. \qquad (33)$$

Besides for interaction of electrons with a harmonic (m, n) , similarly to magnetron [6-8], should be satisfied the condition of synchronism of angular velocity of electrons ω_0 and angular phase velocity ω_{phmn} of this harmonic

$$\omega_0 \approx \omega_{phmn}, \quad \omega_0 = \frac{V_\varphi}{r}, \quad \omega_{phmn} = \frac{\omega_{mn}}{m}. \qquad (34)$$

It is known [8], that in the field of the charged cylinder the frequency of radial fluctuations of electron more than $\sqrt{2}$ times exceeds its angular velocity. The incommensurability of frequencies of radial and azimuthal motion of electron leads to that under condition (34) there is no synchronism of components of the field E_r with radial motion of an electron. Therefore, and also by virtue of (33) it is possible to consider, that the electron bunching and waves generation in orbitron slightly depend on E_r and mainly are determined by component E_φ During some initial interval of time electrons, being uniformly distributed on phase, do not exchange energy with initial fluctuation of the field (2,7). In the further under action of azimuthal nonuniformity of the wave one part electrons gets in decelerating phases, where $E_\varphi > 0$, and another part gets in accelerating phases, where $E_\varphi < 0$. Electrons of the first part, being slowed down, approach to the string as the balance of centrifugal force and force of an attraction to the string is broken. These electrons give to the wave a part of their energy consisting from $W_{k\varphi}$ and W_p

$$W' = W_{k\varphi} + W_p. \qquad (35)$$

Electrons the second part, being accelerated, gain a part of energy from the wave. At that the part W'' of their energy is increased and they approach to the cathode. Thus, electrons in orbitron exchange with the wave not only by azimuthal part $W_{k\varphi}$ of their kinetic energy, but also by potential energy. In this respect orbitron reminds magnetron, in which however only potential energy of electrons is transferred to the wave[6-8]. The bunching of electrons in orbitron is determined by dependence of their angular speed on time. Using the formula for ω_0 (34) and second equation of (22), we obtain the following relation

$$\frac{d\omega_0}{dt} = -\frac{1}{r}(\frac{2V_r V_\varphi}{r} + \frac{e}{m_e}E_\varphi) \approx -\frac{2V_r V_\varphi}{r^2}. \qquad (36)$$

In (36) it is used the fact, that in RF-devices of small and moderate power the amplitude of an excited field E_φ is less than electrostatic fields. From (36) follows, that electrons, being in decelerating phase where $E_\varphi > 0, V_r < 0$ have positive angular velocity. Meanwhile electrons, being in accelerating phase where $E_\varphi < 0, V_r > 0$ have negative angular velocity. Therefore in an azimuthal direction electrons move contrary to the force acting on them, and being displaced on radius. Sometimes this phenomenon is named the effect of "negative mass" [9]. As a result electrons are bunching on an azimuth at transition from the phase of deceleration to the phase of acceleration. Electrons trapped by the wave move together with the wave. For them the condition of synchronism (34) is satisfied. If parameters of orbitron are those, that electrons, being in decelerating phase more than in accelerating phase the electromagnetic waves will be generated in orbitron. Let's consider an electron of the bunch, being in decelerating phase. Having given to the wave the part of its kinetic $W_{k\varphi}$ and potential W_p energy, electron leaves synchronism (34). At that it is decelerated in an azimuthal direction, but owing to the effect of "negative mass" its angular velocity increases. As a result electron again gets in synchronism with the wave and gives to it the next portion of the energy W' , gradually approaches to the string. Electron, being in accelerating phase gains a part of its energy from the wave and also leaves synchronism (34). At that its $W_{k\varphi}$ and W_p are increased. Acceleration in an azimuthal direction results owing to the effect of "negative mass" to that angular velocity of electron decreases. Again it gets in synchronism with the wave (34) and gains from the wave the next portion of energy, gradually approaches to the cathode. Thus, during the process of energy exchange with the wave in orbitron electron restores the angular velocity ω_0 and does not leave synchronism (34). In magnetron electrons also continuously restore a condition of synchronism (34) during interaction with the wave [6]. A condition for increments dV_φ and dr , at which angular velocity is restored after interaction of electron with the wave it is possible to obtain from the formula (34) for ω_0 . It has the view

$$\frac{dV_\varphi}{V_\varphi} = \frac{dr}{r}. \tag{37}$$

If by means of (29) to pass in the ratio (37) to variables W_p and $W_{k\varphi}$, then the expression (37) takes the following view:

$$\frac{dW_p}{dW_{k\varphi}} = \frac{V_Q^2}{V_\varphi^2}. \tag{38}$$

The formula (38) gives the ratio of a part of potential energy to a part of kinetic energy of electron lost or gained by electron during its interaction with the wave. If there are too much electrons in the interaction region, the intensive energy exchange of the wave with electrons it can be occurred electron bunching. At that the amount of electrons, being in decelerating phase and in accelerating phase are approximately equal. The decelerated electrons quickly excite the wave of very big amplitude and at once settle on the string. Accelerated electrons, being in the field of high amplitude, gain energy from the wave and quickly settle on the cathode. Finally energy of the field in orbitron appears close to zero. The similar phenomenon takes place also in magnetron [7]. According to experimental data [1] the density of electrons in orbitron is rather low, as their plasma frequency approximately is much less than the frequency of generated waves. In [2] it is shown, that increments of cold waves growth in orbitron quickly decrease with growth of azimuthal number of a harmonic $|m|$. It allows to consider, that the main contribution to expansion (2) will be given by harmonics with $|m| = 1$ and by several first numbers n of radial harmonics. Firstly the calculations for a single wave have been carried out with $|m| = 1$ and $n = 1$. Energy of a wave ε everywhere is presented in terms of erg /cm. Calculations which results are presented below, are executed at the following parameters:

$$\frac{b}{a} = 100, \quad a_{s11} = -0.1, \quad \varphi_{sc} = \frac{\pi}{2}. \tag{39}$$

In Fig. 1 it is shown how energy ε (erg /cm) of the excited wave in orbitron changes in time at the following values parameters:

$$\frac{\tilde{N}}{l} = 10^{11}, U = 1.5, \alpha = 0.2, \beta_{\varphi 0} = 0.0343, \rho_0 = 2.07. \tag{40}$$

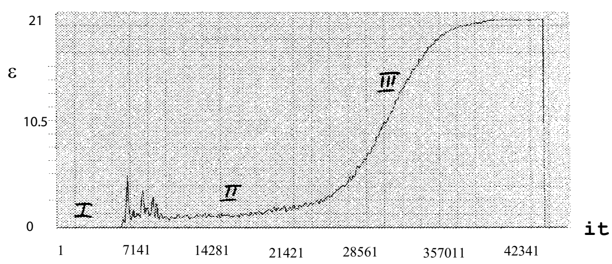

Fig.1. *Dependence energy ε from temporal step it*

Along the axis of abscissa the amount of steps in time is marked. The value of the step in time Δt is equal

$$\Delta t = 0.05 \frac{a}{V_{\varphi 0}} \tag{41}$$

and amount of particles $N = 800$. On time interval I initial fluctuation (7,12) amplifies. On time interval II bunching of electrons takes place. In the beginning of this interval there is a small splash in energy of the wave, which is explained by that approximately the half of non bunched electrons, being in decelerated phase, gives energy to the wave, and other half of electrons, which is being in accelerating phase, gains energy from the wave. At a stage III delays electrons bunched in decelerating phase give energy to the wave and settle on the string. Dependence of the amount of particles which settle on the string on time is similar to the dependence ε and for brevity is not presented. In Fig. 2-5 it is shown how values η in % (continuous lines) and ε in erg/cm (dashed lines) depend on ρ_0 at various values of parameters $\tilde{N}/l, U$. First of all we note, that the wave is effectively excited by electrons, which initial radial coordinates ρ_0 lay in a cylindrical layer in immediate proximity from the string (anode).

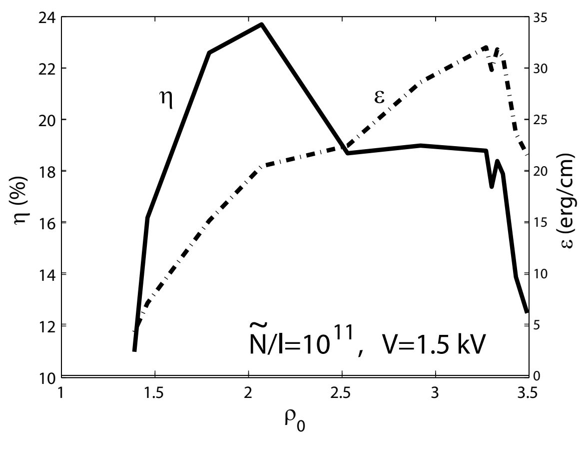

Fig.2. *Dependence ϵ, η from ρ_0*

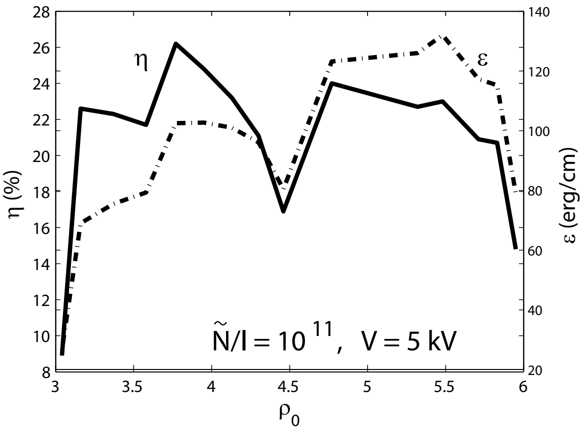

Fig.3.*Dependence ϵ, η from ρ_0*

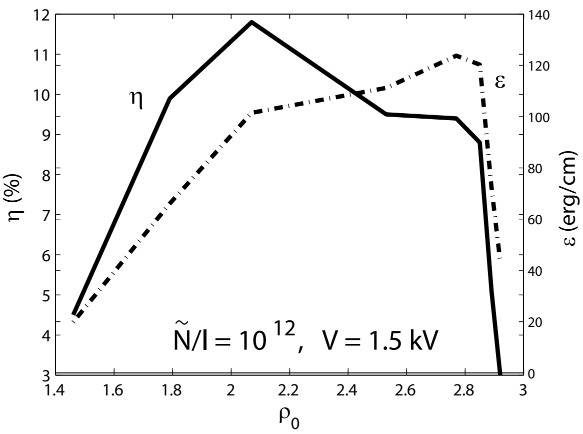

Fig.4. *Dependence ϵ, η from ρ_0*

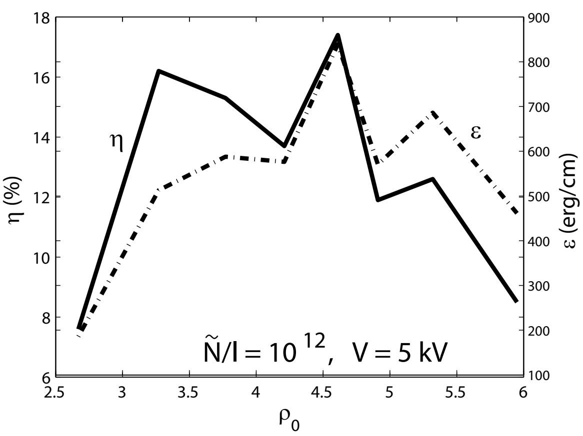

Fig.5. *Dependence ϵ, η from ρ_0*

Thickness of this layer is small in comparison with radius b of the external cylinder of the coaxial. It is in the consent with experimental data [1], and assumptions of work [2] according to which near to the string the field generation in orbitron takes place. Outside of this layer the values ε, η decrease. The bottom border of the layer is determined by that electrons, located near to it, have the small initial energy W' and cannot excite significant energy of the field. Besides because of deviations of orbits from circular one these electrons can quickly settle on the string, without exciting the wave. Electrons, located near to the right border of the layer ρ_0, have greater azimuthal velocity. Therefore significant part of them gets on the cathode that reduces ε, η . Comparing Fig. 2, 4 on the one hand and Fig. 3, 5 on the other hand, it can see that, having increased a voltage U at constant electron density, it is possible to increase considerably the energy of the excited field without reduction of efficiency of its excitation.

It is explained by that for existence of trajectories of electrons, close to circular ones, with U growth $V_{\varphi 0}$ should be increased that leads to increase in energy which electron can give to the wave. From comparison Fg. 2, 3 with Fig. 4, 5 it is visible that the increase in density of electrons at a constant voltage leads to reduction of efficiency of the field generation in orbitron. It is connected with that the increase of amount of electrons in the interaction region leads to increase in the wave amplitude. At that electrons, being in accelerating phase, settle on the cathode that leads to reduction of η value. If one takes electron density even greater then not only wave amplitude increases, but increment of its growth increases too. At that nonbunched electrons interact with the wave. They do not excite the field, about what it was spoken above. This case is illustrated by Fig.6 where calculations with a set of parameters (39-41), in which the density of electrons is increased up to the value $\tilde{N}/l = 10^{13}$, are presented.

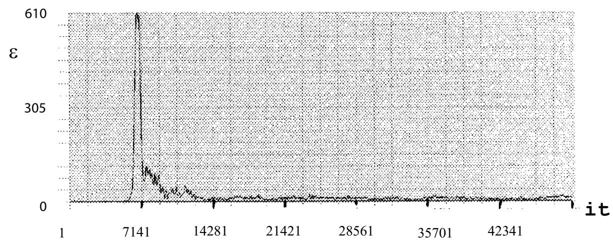

Fig.6. *Dependence energy ε from temporal step it*

Thus, from the point of view of values η, ε, there is some optimal value of electron density in the interaction region. Calculations under the formula (38) showed, that in the initial moment of time for various parameters $dW_p/dW_{k\varphi} = V_Q^2/V_{\varphi 0}^2 \approx 0.5...1.0$. In process of transforming energy to the wave V_φ^2 decreases, and it leads to increase in a share of potential energy in the energy V_φ^2 , given to the wave. The part W_p' of potential energy of electrons transforms to kinetic energy W_{kr}' of their radial motion which goes on a warming up of the anode. We note that in magnetron the magnetic field turns trajectories of electrons in such a way, that their radial motion transforms into azimuthal one. As a result W_{kr} transforms into energy $W_{k\varphi}$ which is given to the wave. Therefore in magnetron the efficiency is higher, than in orbitron. The analysis lead above has shown, that mechanisms of electromagnetic waves generation in orbitron and in magnetron have much in common.The calculations with taking into account higher radial harmonics were also carried out. At that parameters (39-41) were used. Calculations with taking into account two and three radial harmonics gave the following values of field energy and efficiency: at $n = 1$ it is obtained ε =20.3, η =24%; at $n = 1, 2$ ε=17.5, η =20%; at $n = 1, 2, 3$ ε =18.2,η =21%. The results obtained with two and three harmonics, differ by several percents, therefore there is a saturation of results at increase of n .

4. THE CONCLUSION

The nonlinear theory of electromagnetic waves generation in orbitron is developed. The set of the equations including the equations of field excitation and the equations of 2-dimensional motion is constructed and numerically solved. It is shown, that mechanism of electron bunching and energy exchange of electrons with the wave in orbitron and in magnetron has much in common. For the fixed parameters of orbitron from the point of view of generated energy and electronic efficiency there is some optimal value of electron density in the interaction region.

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НЕЛИНЕЙНАЯ ТЕОРИЯ ВОЗБУЖДЕНИЯ ЭЛЕКТРОМАГНИТНОГО ПОЛЯ В ОРБИТРОНЕ

Ю.В. Кириченко, И.Н. Онищенко

В работе развита нелинейная теория генерации электромагнитных волн в орбитроне. Построена и численно решена система уравнений возбуждения и уравнений движения. Показано, что механизмы группировки и обмена энергией электрона с волной в орбитроне и магнетроне имеют много общего. Для фиксированных параметров орбитрона имеется некоторое оптимальное с точки зрения генерируемой энергии и электронного коэффициента полезного действия значение плотности электронов в пространстве взаимодействия. Достаточно точное описание процесса возбуждения волн в орбитроне можно получить, ограничиваясь основной собственной гармоникой.

НЕЛІНІЙНА ТЕОРІЯ ЗБУДЖЕННЯ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ В ОРБІТРОНІ

Ю.В. Кириченко, І.М. Оніщенко

В роботі розвинуто нелінійну теорію генерації електромагнітних хвиль в орбітроні. Побудовано і чисельно розв'язано систему рівнянь , що включає рівняння збудження та руху. Показано, що механізми групування та обміну енергією електрона з хвилею в орбітроні та магнетроні мають багато спільного. Для фіксованих параметрів орбітрона щільність електронів у просторі взаємодії має оптимальне значення з точки зору енергії, що генерується, та коефіцієнта корисної дії. Достатньо точний опис процесу збудження хвиль в орбітроні можна отримати за допомогою основної власної гармоніки.