# EFFECT OF MAGNETIC FIELD DISTRIBUTION ON ISOTOPES SEPARATION IN SYSTEM WITH ACUTE-ANGLE GEOMETRY OF MAGNETIC FIELD 

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By numerical solving of the particle motion equations the separation of carbon isotopes ${ }^{12} C$ and ${ }^{13} C$ in varies magnetic field configuration is considered. It is shown, that isotope separation can be improved considerably sampling of distribution of a magnetic field

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Theoretical analyses of isotopes separation in cusp and some experiment confirmation of the predictions of the theory were conducted in [1]. Typical configuration magnetic lines of force in system is shown in Fig.1. Because of nonadiabatic motion of a charged particle passing near the zero field region (a magnetic field component along $z$ axis is varying from its maximum magnitude at the ends of the system to zero at its center) the trajectories of particles experience appreciable changes.


Fig.1. Cusp magnetic field configuration
The particles, located at radius $r_{0}$ relative to symmetry axis in the left cusp boundary, start in axial direction with velocity $v_{0}$. After crossing the plane of zero magnetic field the particles having the same initial conditions but small mass difference begin to move on noticeably different trajectories. On the left part of the system these particles are moving performing small radial oscillations and they encircle
the axis in the right part of the system. In this region the most of their directed velocity converted to transverse velocity. After passing of the system the particles continue to move in homogeneous or near homogeneous magnetic field. The subsequent calculation allowed to evaluate the permissible values for the initial transverse velocities, exceeding of which leads to deterioration of isotopes separation, and dimension of the region of injection. It was shown that such system allows the separation both heavy and light isotopes. More complete description of these results is given in [2].

Although the trajectories of particles have been calculated in different configurations of magnetic field, as a rule, the isotopes of different elements were considered. And this imposes some difficulties for estimation the advantage of one magnetic field distribution in comparison to another one.

The trajectory of particle motion is characterized by two nondimensional parameters: $k r_{0}$ is a position of a particle initial location, where $k=\pi / 2 L$, $2 L$ is a distance between the planes where the maximum magnitude of magnetic field is reached; and $\eta=r_{0} / r_{l}$, where $r_{l}=M c v_{0} / e H_{0}$ is Larmor radius, which involves the initial particle velocity $v_{0}$ along $z$ axis, and a magnetic field strength $H_{0}$ at the position of particle location. If the magnetic field distribution was given a condition for optimal isotope separation can be found by changing of these parameters.

The purpose of this work is to compare the separation of isotopes of one and the same element in different magnetic field configurations. The carbon isotopes ${ }^{12} C$ and ${ }^{13} C$ were selected for that. In natural conditions carbon represents a mixture of two isotopes. And in this mixture it is the order of $1.1 \%$ isotope ${ }^{13} \mathrm{C}$ contained. But it has many applications in science and technologies.

[^0]As it is known [3] the solution for the magnetic field component $H_{z}$ and $H_{r}$ in such system can be expressed in cylindrical coordinates by the sum of harmonics. Each of these three harmonics is a product of trigonometric and Bessel's functions of imaginary argument and zero or first order. The coefficients at each harmonic can be found from boundary conditions at the ends of system. In early works, when the trajectories of particle motion were calculated, usually, the magnetic field distribution as a first term of a series had been taken, and it can be written in the following form:

$$
H_{z}=-H_{0} \sin (k z), \quad H_{r}=\frac{H_{0} r}{2} \cos (k z)
$$

It is assumed that condition $k r \ll 1$ was satisfied. As a result of calculation it was found that there is some critical meaning of parameter $\eta \quad\left(\eta_{c r}=0.72\right)$. When $\eta$ exceeds $\eta_{c r}$ the particle does not pass the system because of reflection. In subsequent calculations it was found that quantity of $\eta_{c r}$ can be much closer to 1 when the magnetic field distribution as a sum of several harmonics is taken. If we assume that at the ends of system radial uniformity of the magnetic field at a distance at least bounded by trajectory of a particle is fulfilled. Then for the magnetic field of three harmonics the following expression can be written

$$
\begin{aligned}
l H_{z}= & -1,17 I_{0}(k r) \sin (k z)-0,13 I_{0}(3 k r) \sin (3 k z)- \\
& -0,01 I_{0}(5 k r) \sin (5 k z) \\
H_{r}= & 1,17 I_{1}(k r) \cos (k z)+0,13 I_{1}(3 k r) \cos (3 k z)+ \\
& +0,01 I_{1}(5 k r) \cos (5 k z) .
\end{aligned}
$$



Fig.2. Distribution of carbon isotopes ${ }^{12} \mathrm{C}$ and ${ }^{13} \mathrm{C}$ over the cross section at $k z=1.54$ after passing the magnetic field of the first harmonic, general solution

The calculation of the trajectories of particles moving in a magnetic field distribution of the three or four harmonics shows that the ranges of variation of the dimensionless parameters for the particles
passing through system can be notably expended. It should be noted, that there are different ways for selection of separated isotopes after they pass the cusp. In some fixed plane $k z=$ const near the exit the particles of different masses will occupy different regions of azimuthal angles over the cross section in this plane. In axial direction the intervals where these isotopes fall are shifted along $z$-direction.

In Fig. 2 the distribution of carbon isotopes over the cross section at $k z=1.54$, after passing the magnetic field of the first harmonic, was shown. The dimensions of the region of azimuthal angles occupied by different isotopes are determined by initial radial component of velocity and radial dimensions of the region of injection. Parameters $k r_{0}$ and $\eta$ in this calculation were of such that for particles with initial radial velocity scattered in the range $0.02 v_{0}>\dot{r_{0}}>-0.02 v_{0}$ and radial dimension of injection region $\triangle r=0.002$ there were full isotope separation.

In Fig. 3 the regions of azimuthal angles for the same isotopes, which they occupy after passing the system with magnetic field distribution of three harmonics, were shown. One can see that regions occupied by each isotope shifted on more than $\pi / 2$. The heavier isotope ${ }^{13} C$ is finishing its first turn while a lighter one is traveling on the second turn. In this case the interval of initial radial velocity is two times greater than that in the case of the magnetic field of the first harmonic and the radial dimension of injection region is four times greater amounting to $\Delta r=0.008$. Radial intervals where isotopes fall after they pass the system also are different. For example in the plane $k z=2$ the heavier isotope ${ }^{13} C$ is within the radii $\Delta r=0.81 \ldots 0.94$, and lighter one is within the radii of $\Delta r=0.5 \ldots 0.85$.


Fig.3. Distribution of carbon isotopes ${ }^{12} \mathrm{C}$ and ${ }^{13} \mathrm{C}$ at $k z=2$ after passing the magnetic field of three harmonics

As it is possible to see from Fig. 4 after some shifting along $k z \quad(k z=2.2)$ the intervals of ra-
dial positions, where separated isotopes locate, are not overlapped. For ${ }^{12} C$ this interval is $\Delta r=$ $0.5 \ldots 0.7$, and for ${ }^{13} C$ it is $\Delta r=0.9 \ldots 1.0$.


Fig.4. Distribution ${ }^{12} \mathrm{C}$ and ${ }^{13} \mathrm{C}$ isotopes at $k z=2.2$ after passing the magnetic field of three harmonics

The displacement of the starting position of a particle for some angle in azimuthal direction does
not change the regions of isotopes disposition over cross section relative the one to another because of the symmetry of the system. The whole picture in the cross section of this plane is turning at the proper angle. The appreciable improvement of the separation of isotopes with heavy atomic weights in the magnetic field distribution of three harmonics is observed too.

## References

1. B.S. Akshanov, N.A. Khizhnyak. A new efficient method isotope separation // Letters in ZhTF. 1991, v.17(6), p.13-16 (in Russian).
2. A.G. Belikov, V.G. Papkovich. On possibilities of isotope separation in cusp magnetic field // Problems of Atomic Science and Technology. Ser. "Plasma electronics and new methods of acceleration". 2004, N4, p.58-63 (in Russian).
3. K.D. Sinelnikov, N.A. Khizhnyak, N.S. Repalov, et al. The investigation of charge particle motion in a magnetic trap of acute-angle geometry // Fizika plasmu i problemu upravlayemogo termojadernogo synteza. 1965, N4, Kyiv: "Naukova Dymka", p.383-402 (in Russian).

# ВЛИЯНИЕ РАСПРЕДЕЛЕНИЯ МАГНИТНОГО ПОЛЯ НА РАЗДЕЛЕНИЕ ИЗОТОПОВ В СИСТЕМЕ С ОСТРОУГОЛЬНОЙ ГЕОМЕТРИЕЙ ПОЛЯ 

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Численным решением уравнений движения рассмотрено разделение изотопов углерода ${ }^{12} C$ и ${ }^{13} C$ в двух различных конфигурациях магнитного поля. Показано, что разделение изотопов можно значительно улучшить выбором распределения магнитного поля.

## ВПЛИВ РОЗПОДІЛУ МАГНІТНОГО ПОЛЯ НА ПОДІЛ ІЗОТОПІВ У СИСТЕМI З ГОСТРОКУТНОЮ ГЕОМЕТРІЄЮ ПОЛЯ

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Чисельним рішенням рівнянь руху розглянуто поділ ізотопів вуглецю ${ }^{12} C$ i ${ }^{13} C$ у двох різних конфігураціях магнітного поля. Показано, що поділ ізотопів можна значно поліпшити вибором розподілу магнітного поля.


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