SHEAR-FLOW-DRIVEN ION CYCLOTRON INSTABILITY OF MULTICOMPONENT MAGNETIC FIELD-ALIGNED PLASMA FLOW

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The ion cyclotron instability of magnetic field aligned sheared plasma flow with two $H+$ and $O+$ ion species is investigated. The oxygen ions are assumed to be the active species while hydrogen ions are a background one, so that the frequency of oscillation approximately equals $O+$ cyclotron frequency. The threshold and growth rate of instability versus the flow velocity shear and relative concentration of oxygen ions are analyzed.

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1. INTRODUCTION

The investigations of the auroral region of the Earth's ionosphere have discovered the inhomogeneous structures of electrostatic potentials which are correlated with regions of the formation and acceleration of the magnetic field-aligned upward ion beams [1]. One of the main signatures of these beams is the gradient of the flow velocity across the magnetic field (flow velocity shear) V'_{0i} which can reach specifically for $O+$ ions the values of $6\omega_{ci}$ [2]. The upflowing ion beams are mainly composed of $H+$ and $O+$ ions the composition of which varies significantly from beam to beam [3]. These auroral ion beams are often correlated with electrostatic ion cyclotron (EIC) oscillations having the cyclotron frequencies of hydrogen and oxygen ions [4,5]. It was shown that the flow velocity shear along with the other mechanisms may be responsible for the excitation of EIC waves in the auroral ionosphere due to development of the shear-flow-driven EIC instability [6,7].

The shear-flow-driven EIC instability was studied in plasma with single ion species. However, the application of these results in ionosphere investigations requires taking into account the presence of several ion components, the relative concentrations of which are changed significantly with the altitude in ionospheric plasma. We have carried out the study of the shear-flow-driven EIC instability in the sheared magnetic field-aligned plasma flow with two, $H+$ and $O+$, ion species. The oxygen ions are assumed to be the main species, while hydrogen ions are a background one, so that the frequency of oscillation approximately equals the $O+$ cyclotron frequency. We have analyzed the dispersion equation for ion-hydrodynamic mode of the shear-flow-driven EIC instability assuming that the waves propagate nearly perpendicularly to the magnetic field but under the assumption that electrons are adiabatic.

2. THE INSTABILITY OF THE FIRST CYCLOTRON HARMONIC

The kinetic dispersion relation for homogeneous multi-ion component plasma with a flow velocity shear is given by [8]

$$\varepsilon(K,\omega)=1+\frac{1}{k^2\lambda_{De}^2}+\sum_{\alpha}\frac{1}{k^2\lambda_{D\alpha}^2}\left(1-\frac{k_y}{k_z}S_\alpha+i\sqrt{\pi}\times\right. \quad (1)$$

$$\left.\times\sum_{n=-\infty}^{\infty}W(z_{\alpha n})\Gamma_n(b_\alpha)\left(\frac{\omega-k_zV_{0\alpha}}{\sqrt{2}k_zV_{T\alpha}}-\frac{k_y}{k_z}S_\alpha z_{\alpha n}\right)\right)=0,$$

where $\lambda_{D\alpha}$ is the Debye length, $A_n(b)=e^{-b}I_n(b)$, $I_n(b)$ is the modified Bessel function, $b_\alpha=(k_\perp\rho_{T\alpha})^2$, $\rho_{T\alpha}=V_{T\alpha}/\omega_{c\alpha}$ is the thermal Larmor radius, $S_\alpha=V'_{0\alpha}(X)/\omega_{c\alpha}$ is the normalized flow velocity shear, $z_{\alpha n}=(\omega-n\omega_{c\alpha}-k_zV_{0\alpha})/\sqrt{2}|k_z|V_{T\alpha}$, $W(z)=e^{-z^2}\times\left(1+\left(2i/\sqrt{\pi}\right)\int\limits_0^z e^{\xi^2}d\xi\right)$. We study the heavy-ion cyclotron mode having the frequency $\omega(k)=n\omega_{ch}+k_zV_{0h}+\delta\omega(k)$ with $|\delta\omega(K)|\ll\omega_{ch}$. Assume, that both ion species have the equal flow velocities $V_{0h}=V_{0l}$ and equal magnitudes of velocity shear $V'_{0h}=V'_{0l}$, where indexes h and l mean the heavy $O+$ and light $H+$ ions.

We first analyze the instability of the main $n=1$ cyclotron harmonic. For the oscillations propagating almost across the magnetic field so that inequality $|z_{i1}|>1$ holds the asymptotic form of W - function for large argument $W(z_i)\approx\left(i/\sqrt{\pi}z_i\right)\left(1+1/2z_i^2\right)$ can be used. In this case the ion cyclotron damping can be neglected for both light and heavy ions. The dielectric permittivity of heavy ions can be written as

$$\delta\varepsilon_h\approx\frac{1}{k^2\lambda_{Dh}^2}\left(1-G_{h1}-\frac{\omega_{ch}}{\delta\omega}A_1(b_h)\right.$$
$$\left.+\frac{k_y}{k_z}\frac{k_z^2V_{Th}^2}{\delta\omega^2}S_hA_1(b_h)\right), \quad (2)$$

where $G_{h1}=A_1(b_h)+\left(1-A_0(b_h)\right)/b_h$. In the sum over cyclotron harmonics of light ions we retain only null summand because of significant difference in the masses of heavy and light ions. Then the dielectric permittivity of light ions becomes

$$\delta\varepsilon_l\approx\frac{1}{k^2\lambda_{Dl}^2}\left(1-A_0(b_l)+\frac{k_y}{k_z}\frac{S_lA_0(b_l)}{2z_{l0}^2}\right), \quad (3)$$

where $z_{l0} = \omega_{ch} / \sqrt{2} k_z V_{Tl}$, $S_l = S_h / \sqrt{\mu}$ and $\mu = m_h / m_l$. The dispersion equation (1) ultimately takes the form

$$\delta\omega^2 (K) - p\delta\omega(K) + q = 0 , \qquad (4)$$

where

$$p = \delta\omega_{1h} \left[1 + \tau/\alpha_h - G_{h1} + k^2 \lambda_{Dh}^2 \delta\varepsilon_l \left(k, \omega(k)\right) \right]^{-1} ,$$

$$q = \sigma_h^2 \left[1 + \tau/\alpha_h - G_{h1} + k^2 \lambda_{Dh}^2 \delta\varepsilon_l \left(k, \omega(k)\right) \right]^{-1} ,$$

$\delta\omega_{1h} = \omega_{ch} A_1 (b_h)$, $\sigma_h^2 = k_y k_z V_{Th}^2 S_h A_1 (b_h)$, $\alpha_h = n_h / n_e$ is the relative concentration of heavy ions, $\tau = T_i / T_e$ with $T_h = T_l = T_i$. The solution of Eq. (4) has the form

$$\delta\omega = \left(\delta\omega_{1h} \pm \Omega_{1h} \right) / 2\beta_{1h} , \qquad (5)$$

where $\Omega_{1h} = \left(\delta\omega_{1h}^2 - 4\sigma_h^2 \beta_{1h} \right)^{1/2}$, $\beta_{1h} = 1 - G_{1h} + \tau/\alpha_h +$ $+ (\alpha_l / \alpha_h) \left(1 - A_0 (b_l) + k_y k_z \rho_{Th}^2 S_h A_0 (b_l) \right)$. The solution (5) gives the shear-flow-driven EIC instability if inequality $4\sigma_h^2 \beta_{1h} > \delta\omega_{1h}^2$ is met. For the wave numbers such as $k_y \rho_{Th} \square 1$ and respectively $k_z \rho_{Tl} \square 1$ this condition can be written as $\lambda < \lambda_1$, where $\lambda = 1/k_z \rho_{Th}$ is the normalized wavelength parallel to the magnetic field, $\lambda_1 \square k_y \rho_{Th} S_h \beta_{1h} / A_1 (b_h)$ is the threshold wavelength of instability for $n = 1$ harmonic and $\beta_{1h} \square 1 - G_{1h} + \tau/\alpha_h$.

Then let us estimate the effect of the relative concentration of heavy ions on the condition $|z_{h1}| > 1$. From the definition of z_{h1} we have

$$|z_{h1}| = |\delta\omega| / \sqrt{2} k_z V_{Th} = \sqrt{k_y S_h A_1 (b_h) / 2 k_z \beta_{1h}} . \qquad (6)$$

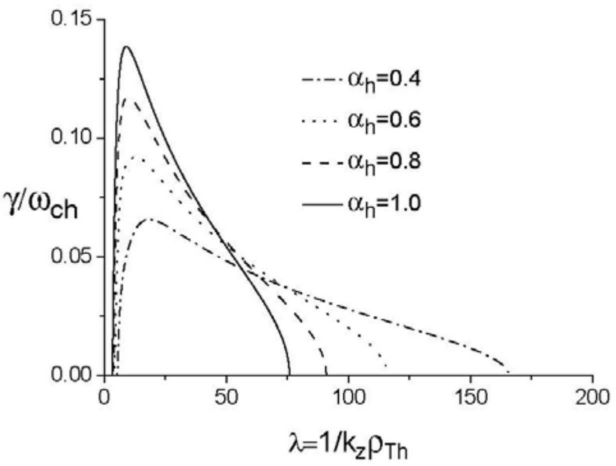
Taking into account that $\beta_{1h} \propto 1/\alpha_h$ and $A_1 (b_h) \approx 0.2$ we obtain from Eq. (6) that the inequality $|z_{h1}| > 1$ holds when $\alpha_h \square 10 \tau k_z / S_h k_y$.

Now we investigate the effect of α_h and S_h on the growth rate of the shear-flow-driven EIC instability. The growth rate of instability obtained from Eq. (5) is approximately

$$\gamma \square \left[(\lambda_1 / \lambda) - 1 \right]^{1/2} / 2\beta_{1h} . \qquad (7)$$

With a decrease of α_h the growth rate away from threshold decreases approximately as $\sqrt{\alpha_h}$, however, the magnitude of threshold wavelength increases as α_h, so that the longer waves become unstable. The dependence of the growth rate on the normalized shear S_h is expressed by the similar relation, i.e. $\gamma \propto \sqrt{S_h}$ and $\lambda_1 \propto S_h$. Thus the effects of relative concentration of oxygen ions on the growth rate and long-wavelength threshold is identical with the flow velocity shear.

We also numerically solved the dispersion equation (1) for the different values of relative concentration of oxygen ions and obtained the dependence of the growth rate versus the normalized wavelength along the magnetic field. The results of calculations for $S_h = 3$, $k_y \rho_{Th} = 1$ and $\tau = 1$ are shown in Figure. The maximum of the growth rate occurs at $|z_{1h}| \square 1$ what is a boundary of ion-hydrodynamic mode which is located to the right of the point of maximum. The Figure shows a decrease of the growth rate as well as an increase of the long-wavelength threshold with the decrease of α_h that is in a good agreement with analytical results.



The growth rate of instability vs the normalized wavelength along the magnetic field for different magnitudes of relative concentration of $O+$ *ions*

2. THE INSTABILITY OF THE HIGH CYCLOTRON HARMONICS

Now we investigate the instability of the high, $n \square 1$, cyclotron harmonics. Using the same assumptions as for the first harmonic we sum over cyclotron harmonics at $k_y \rho_{Th} \square n \square 1$ and obtain approximately the dielectric permittivity of heavy ions as

$$\delta\varepsilon_h \approx \frac{1}{k^2 \lambda_{Dh}^2} \left(1 - G_{hn} - \frac{n\omega_{ch}}{\delta\omega} A_n (b_h) + \frac{k_y}{k_z} \frac{k_z^2 V_{Th}^2}{\delta\omega^2} S_h A_n (b_h) \right), \qquad (8)$$

where

$$G_{hn} = 2 z_{\perp h} e^{-z_{\perp h}^2} \int_0^{z_{\perp h}} e^{t^2} dt + A_n (b_h)$$

and $z_{\perp h} = \omega(k) / \sqrt{2} k_\perp V_{Th} \approx n / \sqrt{2} k_\perp \rho_{Th}$. In the dielectric permittivity of light ions (3) we take into account that inequality $k_y \rho_{Th} \square 1$ holds, so that $k_y \rho_{Tl} > 1$ and then $\delta\varepsilon_l \approx 1 / k^2 \lambda_{Dl}^2$. In this case the dispersion equation (1) takes the similar form as for the first harmonic (4). Its solution is

$$\delta\omega = \left(\omega_{nh} \pm \Omega_{nh} \right) / 2\beta_{nh} , \qquad (9)$$

where $\delta\omega_{nh} = n\omega_{ch} A_n (b_h)$, $\Omega_{nh} = \left(\delta\omega_{nh}^2 - 4\sigma_h^2 \beta_{nh} \right)^{1/2}$, $\beta_{nh} \approx 1 - G_{nh} + \tau/\alpha_h + \alpha_l / \alpha_h$. The solution (9) gives the shear-flow-driven EIC instability if inequality $4\sigma_h^2 \beta_{nh} > \delta\omega_{nh}^2$ is met. This condition can be also written as $\lambda < \lambda_n$, where $\lambda_n \square k_y \rho_{Th} S_h \beta_{nh} / n^2 A_n (b_h)$ is the threshold wavelength of instability for $n \square 1$ harmonics.

Note that the function $A_n(b_h)$ at $k_y\rho_{Th} \square n \square 1$ has the asymptotic form $A_n(b_h) \square \left(1/\sqrt{2\pi}k_\perp\rho_{Th}\right)\exp\left(-n^2/2k_\perp^2\rho_{Ti}^2\right)$ and for $k_y\rho_{Th} = n$ we have $A_n(b_h) \approx 0.2/n$ that gives $\lambda_n \approx \lambda_1$. Thus the long-wavelength threshold is the same as for the first and high cyclotron harmonics.

Evaluating the effect of the relative concentration on the condition $|z_{hn}| > 1$, we conclude that for $k_y\rho_{Th} = n$ the condition on the α_h coincides with that of the main harmonic. Now we estimate the effect of α_h and S_h on the growth rate of high cyclotron harmonic of the shear-flow-driven EIC instability. The growth rate of instability obtained from Eq. (9) approximately equals

$$\gamma \square \left[(\lambda_n/\lambda)-1\right]^{1/2}/2\beta_{nh}. \qquad (10)$$

Since the thresholds λ_1 and λ_n are equal we obtain that the dependence of growth rate on the concentration and the shear is the same as for the main cyclotron harmonic. The numerical calculations confirm these results.

3. CONCLUSIONS

The presence of the light $H+$ ion species in the sheared plasma flow with $O+$ ions leads to a decrease of the growth rate of the shear-flow-driven EIC instability with $O+$ ion cyclotron frequency, whereas the long-wavelength threshold of instability is shifted toward longer wavelengths both for the main $n=1$ and high cyclotron harmonics. In so doing the effects of relative concentration of oxygen ions on the growth rate and long-wavelength threshold is identical to the flow velocity shear.

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ВОЗБУЖДАЕМАЯ ШИРОМ ПОТОКОВОЙ СКОРОСТИ ИОННАЯ ЦИКЛОТРОННАЯ НЕУСТОЙЧИВОСТЬ ПОТОКА МНОГОКОМПОНЕНТНОЙ ПЛАЗМЫ ВДОЛЬ МАГНИТНОГО ПОЛЯ

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Исследована ионная циклотронная неустойчивость сдвигового течения плазмы вдоль магнитного поля с двумя сортами, $H+$ и $O+$, ионов. Ионы кислорода считаются основным видом, тогда как ионы водорода являются фоном, так что частота колебаний приблизительно равна циклотронной частоте ионов $O+$. Анализируется зависимость порога и инкремента неустойчивости от градиента скорости течения и относительной концентрации ионов кислорода.

ЗБУДЖУВАНА ШИРОМ ПОТОКОВОЇ ШВИДКОСТІ ІОННА ЦИКЛОТРОННА НЕСТІЙКІСТЬ ПОТОКУ БАГАТОКОМПОНЕНТНОЇ ПЛАЗМИ ВЗДОВЖ МАГНІТНОГО ПОЛЯ

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Досліджено іонна циклотронна нестійкість зсуненого потоку плазми вздовж магнітного поля з двома видами, $H+$ і $O+$, іонів. Іони кисню вважаються основним видом, тоді як іони водню є фоном, так що частота коливань приблизно дорівнює циклотронній частоті іонів $O+$. Аналізується залежність порога та інкремента нестійкості від градієнта швидкості потоку і відносної концентрації іонів кисню.