RENORMALIZED NON–MODAL THEORY OF TURBULENCE OF PLASMA SHEAR FLOWS

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In our report, we present the results of the non-linear investigations of the temporal evolution and saturation of drift turbulence in shear flows, which have the non-modal approach as their foundation.

PACS: 94.05.Lk, 94.20.wf

1. INTRODUCTION

The discovery[1] of the L–H transition phenomena is one of the greatest successes in the investigation of the magnetized fusion related plasmas. That transition characterized by a sudden suppression of edge density- and magnetic turbulence, which follows by rapid drop of turbulent transport at the plasma edge that resulted in the development of steep edge gradients indicating the set up of a transport barrier in the outermost few cm of the confinement region. The experiments revealed that suppression of the drift turbulence conditioned by shear flows, which developed prior to the transition in the boundary layers of plasma. The discovery of the connection of the observed turbulence suppression with shear flows determined the development of the turbulence theory of shear flows as the one of the most important task in the theory of the controlled fusion and of the plasma theory in whole.

The contemporary theory of plasma shear flows turbulence meets with great obstacles in its development. That theory grounds on two approaches. The first is called as the normal mode or modal approach, in which perturbations of the fields and density, temperature, ets. are considered as spatially inhomogeneous in the direction of the flow shear and the application of the spectral transform in time is assumed. The solution obtained on this way in linear approximation has as a rule the singularities at the critical level, where phase velocity of the perturbations is equal to the local magnitude of the flow velocity. Because of that singularity plasma turbulence grounded on the modal approach is still absent. Even the simplest turbulence theory grounded on the weak interaction approximation is not developed yet because of the divergence of the power series expansions used in this approach. The phenomenological shear flow turbulence theory (in which the problem of the solutions secularity even not notice) was presented in Refs.[2, 3]. That theory bases on the suggestion, that observed suppression of the drift turbulence is the result of the enhanced decorrelation of the plasma displacements, which follows from the coupled action of the turbulent scattering and convection by shear flow. The experiments, however display the results, which are opposite to the prediction of that theory: the correlation times grow in plasma shear flow. In this report, we present the results of the development of the hydrodynamic and kinetic drift turbulence theory of the plasma shear flows. This theory is grounded on Kelvin's method of shearing modes or a so-called non-modal approach. The non-modal approach appears very effective in the development of the linear and weak nonlinear theories of plasma waves and instabilities in shear flow. This theory gives simple, exact, and uniformly bounded for all times, solutions, which are free from the problem of the singularities, which is inherent to modal approach. Particularly, the solution of the initial value problem, obtained in this approach, reveals that a drift wave in the shear flow gradually transformed into a convective cell and normal-mode solution is not the steady-state limit for the initial value problem considered.

2. RENORMALIZED HYDRODYNAMIC THEORY OF DRIFT TURBULENCE OF SHEAR FLOWS

We investigate the temporal evolution of drift modes in time-dependent shear flow using the Hasegawa–Wakatani equations for the dimensionless density $n=\tilde{n}/n_e$ and potential $\phi = e\varphi/T_e$ perturbations (n_e is the electron background density, T_e is the electron temperature),

$$\rho_s^2\left(\frac{\partial}{\partial t}+V_0\left(x,t\right)\frac{\partial}{\partial y}-\frac{c}{B}\left(\frac{\partial\phi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\phi}{\partial x}\frac{\partial}{\partial y}\right)\right)\nabla^2\phi = a\frac{\partial^2}{\partial z^2}\left(n-\phi\right),$$

$$\left(\frac{\partial}{\partial t}+V_0\left(x,t\right)\frac{\partial}{\partial y}-\frac{c}{B}\left(\frac{\partial\phi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\phi}{\partial x}\frac{\partial}{\partial y}\right)\right)n+v_{de}\frac{\partial\phi}{\partial y} = a\frac{\partial^2}{\partial z^2}\left(n-\phi\right),$$

where $V_0\left(x,t\right)$ is the velocity of the sheared flow. We transform these equations to new spatial variables ξ,η,

$$t=t,\qquad \xi = x,\qquad \eta = y-V_0{}'xt\quad,\quad z=z. \tag{1}$$

In these coordinates the linear convection terms are absent in above system:

$$\rho_s^2\left(\frac{\partial}{\partial t}-\frac{c}{B}\left(\frac{\partial\phi}{\partial\eta}\frac{\partial}{\partial\xi}-\frac{\partial\phi}{\partial\xi}\frac{\partial}{\partial\eta}\right)\right)\nabla^2\phi = a\frac{\partial^2}{\partial z^2}\left(n-\phi\right),$$

$$\left(\frac{\partial}{\partial t}-\frac{c}{B}\left(\frac{\partial\phi}{\partial\eta}\frac{\partial}{\partial\xi}-\frac{\partial\phi}{\partial\xi}\frac{\partial}{\partial\eta}\right)\right)n+v_{de}\frac{\partial\phi}{\partial\eta} = a\frac{\partial^2}{\partial z^2}\left(n-\phi\right). \tag{2}$$

It is interesting to note that transformation (1) conserves the $E \times B$ convective nonlinear derivative in in the form similar to one in a plasma without any flows. With new variables ξ_1, η_1 determined by the nonlinear relations

$$\xi_1 = \xi + \frac{c}{B}\int_{t_0}^{t} \frac{\partial \phi}{\partial \eta} dt_1, \qquad \eta_1 = \eta - \frac{c}{B}\int_{t_0}^{t} \frac{\partial \phi}{\partial \xi} dt_1, \qquad (3)$$

the convective nonlinearity in Eqs.(2) becomes of the higher order with respect to the potential ϕ. Omitting such nonlinearity, as well as small nonlinearity of the second order in the laplacian, resulted from the transformation to nonlinearly determined variables ξ_1, η_1, we come to linear equation with solution

$$\phi(\xi,\eta,t) = \int dk_\perp \int dl \phi(k_\perp,l,0) g(k_\perp,l,t) e^{ik_\perp \xi_1 + il\eta_1}, \qquad (4)$$

where wave numbers $k_\perp$, l are conjugate there to coordinates ξ_1, η_1 respectively. With variables ξ and η this solution has a form

$$\begin{aligned} &\phi(\xi,\eta,t) = \int dk_\perp \int dl \phi(k_\perp,l,0) \\ &\times g(k_\perp,l,t_1)\exp\left(ik_\perp\xi + il\eta - ik_\perp\tilde{\xi}(t_1) - il\tilde{\eta}(t_1)\right), \end{aligned} \qquad (5)$$

where

$$\tilde{\xi}(t) = -\frac{c}{B}\int_{t_0}^{t} \frac{\partial \phi(t_1)}{\partial \eta} dt_1, \quad \tilde{\eta}(t) = \frac{c}{B}\int_{t_0}^{t} \frac{\partial \phi(t_1)}{\partial \xi} dt_1. \quad (6)$$

Eq.(5) is in fact a nonlinear integral equation for potential ϕ, in which the effect of the total fourier spectrum on any separate fourier harmonic is accounted for. The functions $\tilde{\xi}(t)$ and $\tilde{\eta}(t)$ in the exponential of Eq.(5) involve through eq.(6) integrals of ϕ, which in turn, involve in their exponentials the integrals (5) and so on. This form of solution, however, appears very useful for the analysis of the correlation properties of the nonlinear solutions to Hasegawa-Wakatani system and for the development of the approximate renormalized solutions to Hasegawa-Wakatani system, which accounted for the effect of the turbulent motions of plasma on the saturation of the drift-resistive instability. We have obtained[4] the renormalized form of the potential (5), in which the average effect of the random convection is accounted for,

$$\begin{aligned} &\phi(\xi,\eta,t) = \int dk_\perp \int dl \phi(k_\perp,l,0) \\ &\times \exp\left(i\omega_d t + \gamma t - \int_0^t d\hat{t} C\left(k_\perp,l,\hat{t}\right) + ik_\perp\xi + il\eta\right). \end{aligned} \qquad (7)$$

The saturation of the instability occurs when $\partial\left(\phi(\xi,\eta,t)\right)^2 / \partial t = 0$, i.e. when

$$\begin{aligned} &\gamma(k_\perp,l) = C(k_\perp,l,t) = \frac{c^2}{B^2}\int dk_{1\perp}\int dl_1 \left|\phi(k_{1\perp},l_1,t)\right|^2 \\ &\times \left|[\mathbf{k}_\perp \times \mathbf{k}_{1\perp}]\right|^2 \frac{C(k_{1\perp},l_1,t)}{\omega_d^2(k_{1\perp},l_1)}. \end{aligned} \qquad (8)$$

From the double Eq.(8) we obtain the equation, which determines the level of the instability saturation

$$\gamma(k) = \frac{c^2}{B^2}\int dk_{1\perp}\int dl_1 \left|\phi(k_{1\perp},l_1,t)\right|^2 \left|[\mathbf{k}_\perp \times \mathbf{k}_{1\perp}]\right|^2 \frac{\gamma(k_1)}{\omega_d^2(k_1)}. \quad (9)$$

The sought-for value is a time t_{sat} at which the balance of the linear growth and nonlinear damping occurs for given initial disturbance $\phi(k_{1\perp},l_1,0)$ and dispersion. With obtained t_{sat} the saturation level will be equal to

$$\left|\phi(t_{sat})\right|^2 \sim \int dk_{1\perp}\int dl_1 \left|\phi(k_{1\perp},l_1,0)\right|^2 \times \exp\left(2\gamma(k_{1\perp},l_1)t_{sat}\right).$$

Also, the well known order of value estimate for the po-tential ϕ, in the saturation state is obtained easily from Eq.(9), $e\phi/T_e \sim (k_\perp L_n)^{-1}$. Obtained results show that the nonlinearity of the Hasegawa-Wakatani system of equations in variables ξ and η, with which frequency and growth rate are determined without spatially inhomogeneous Doppler shift and wave number is time independent, does not display any effects of the enhanced decorrelations provided by flow shear.

3. RENORMALIZED KINETIC THEORY OF DRIFT TURBULENCE OF SHEAR FLOWS

It was obtained in Ref.[5], that application the transformation (1) to Vlasov equation jointly with trans-formation of the velocity to convective set of reference resulted in Vlasov equation, in which inhomogeneities conditioned by shear flow are absent. With leading center coordinates, determined by relations

$$v_{\alpha x} = v_\perp \cos\phi, \qquad v_{\alpha y} = \sqrt{\eta} v_\perp \sin\phi,$$

$$X_\alpha = x_\alpha + \frac{v_\perp}{\sqrt{\eta}\omega_{c\alpha}}\sin\left(\phi_1 - \sqrt{\eta}\omega_{c\alpha}t\right)$$

$$Y_\alpha = y_\alpha - \frac{v_\perp}{\eta\omega_{c\alpha}}\cos\left(\phi_1 - \sqrt{\eta}\omega_{c\alpha}t\right) - V_0' t \frac{v_\perp}{\sqrt{\eta}\omega_{c\alpha}}\sin\left(\phi_1 - \sqrt{\eta}\omega_{c\alpha}t\right)$$

Vlasov equation has a form

$$\begin{aligned} &\frac{\partial F_\alpha}{\partial t_n} + \frac{e_\alpha}{m_\alpha}\frac{\sqrt{\eta}\omega_{c\alpha}}{v_{\perp n}}\left(\frac{\partial \varphi}{\partial \phi_n}\frac{\partial F_\alpha}{\partial v_{\alpha n}} - \frac{\partial \phi}{\partial v_{\perp n}}\frac{\partial F_\alpha}{\partial \phi}\right) \\ &+ \frac{e_\alpha}{m_\alpha}\frac{1}{\eta\omega_{c\alpha}}\left(\frac{\partial \varphi}{\partial X_\alpha}\frac{\partial F_\alpha}{\partial Y_\alpha} - \frac{\partial \phi}{\partial Y_\alpha}\frac{\partial F_\alpha}{\partial X_\alpha}\right) - \frac{e_\alpha}{m_\alpha}\frac{\partial \varphi}{\partial z_\alpha}\frac{\partial F_\alpha}{\partial v_{\alpha z}} = 0, \end{aligned} \qquad (10)$$

where perturbed electrostatic potential is determined as

$$\begin{aligned} &\varphi(\mathbf{r}_\alpha,t) = \int dk_x dk_y dk_z, \varphi(t,k_x,k_y,k_z) \\ &\times \exp\left(ik_x X + ik_y Y + ik_z z + i\mathbf{k}\delta\mathbf{r}(t)\right) \\ &\times \exp\left[-\frac{ik_\perp(t)v_\perp}{\sqrt{\eta}\omega_c}\sin\left(\phi_1 - \sqrt{\eta}\omega_c t - \theta(t)\right)\right]. \end{aligned} \qquad (11)$$

In (11) $i\mathbf{k}\delta\mathbf{r}(t)$ term,

$$\begin{aligned} &\mathbf{k}\delta\mathbf{r}(t) = k_x\delta X(t) + k_y\delta Y(t) - k_\perp(t)\omega_c^{-1} \\ &\times\left(\delta v_\perp(t)\sin(\phi - \theta(t)) + v_\perp\cos(\phi - \theta(t))\delta\phi(t)\right), \end{aligned} \qquad (12)$$

determines the nonlinear phase shift of the potential (11) due to turbulent scattering of ions in electrostatic turbulence. We find, that for the times $t < (V_0')^{-1}$ the main effect, which determines the nonlinear scattering of ions by long wavelength drift turbulence with $k_\perp\rho_i < 1$ is the scattering of the leading center coordinates, δX and δY. The non-modal effects are negligible at this time. At times $t > (V_0')^{-1}$ right the non-modal effects determine the nonlinear evolution of drift turbulence with dominant nonlinear phase shift due to scattering of the angle $\delta\phi$ in velocity space. For times $(V_0')^{-1} < t < t_s$ and for times $t > t_s$ we have, respectively

$$k_\perp \rho_i \delta\phi / k_x \delta X : \ k_y \rho_i (V_0' t)^3 \ ? \ 1,$$
$$k_\perp \rho_i \delta\phi / k_x \delta X : \ (V_0' t)^2 \ ? \ 1. \qquad (13)$$

We obtain for $(V_0')^{-1} < t < t_s$ the renormalized solution in the form

$$\varphi(\mathbf{k},t) = \varphi_0(\mathbf{k},t_0)\exp\left[-i\omega(\mathbf{k})\left(1-\frac{1+\tau}{a_i b_i}\frac{t^2}{3t_s^2}\right)t\right.$$
$$\left. +\left(\gamma(\mathbf{k})-\frac{t}{2a_i t_s^2}\right)t - \int_0^t C(\mathbf{k},t_1)\,dt_1\right], \qquad (14)$$

where $C(k,t)$ is determined by the equation

$$C(\mathbf{k},t) = \frac{c^2}{B^2}k_y^2\rho_i^2\frac{(V_0' t)^6}{8}$$
$$\times\int d\mathbf{k}_1 \left|\varphi(\mathbf{k}_1,t)\right|^2 C(\mathbf{k}_1,t)\frac{k_{1y}^4}{\omega^2(\mathbf{k}_1)}. \qquad (15)$$

If we omit linear non-modal terms in Eq.(14), the condition of the balance of the linear modal growth of the kinetic drift instability and non-linear non-modal dumping is determined by the equation $\gamma(k) = C(k,t)$. By using this equation in Eq.(15), we obtain the equation, which determines the time, at which that balance occurs,

$$\frac{\gamma(\mathbf{k})}{(V_0' t)^6} = \frac{c^2}{8B^2}k_y^2\rho_i^2\int d\mathbf{k}_1\left|\varphi(\mathbf{k}_1,t)\right|^2\gamma(\mathbf{k}_1)\frac{k_{1y}^4}{\omega^2(\mathbf{k}_1)}. \qquad (16)$$

The effect of the shear flow reveals in the reducing with time as $(V_0' t)^{-6}$ the magnitude of the growth rate in the left part of the balance equation (16). That causes rapid asselerated suppression of the drift turbulence. This balance does not correspond to the steady state for drift turbulence in shear flow. The evolution of drift turbulence continues on times $t \geq t_s$. It follows by strongly non-modal way, where Markovian approximation, which is appropriate for the solution Eq.(14), when the growth rate and non-modal terms are small with respect to the frequency $\omega(k)$, ceases be valid.

4. CONCLUSIONS

The results presented in this report prove that any "universal rules" or "paradigms", that thoroughly determines the turbulence suppression by shear flow, are absent. The suppression of turbulence by shear flows is a mode dependent process, which includes the sequence of different non-modal linear and non-linear processes with different time scales for different parts of the spectrum of the unstable waves. Presented nonlinear non-modal analysis of the resistive drift and kinetic (universal) drift instabilities reveals that non-modal effects lead to the decreasing the frequency and growth rate at time $t \leq t_2 = (V_0' k_y \rho_s)^{-1}$ and lead to rapid non-modal suppression of turbulence at time $t > t_2 = (V_0' k_y \rho_s)^{-1}$. The time dependence of the wavenumber $k_\perp(t)$ becomes the key element in the proper kinetic treatment of the long-time evolution of the perturbations in shear flow. In such kinetic analysis the nonlinear non-modal turbulent scattering of the phase angle of ion Larmor orbit is the dominant effect, which determines rapid suppression of the drift turbulence by shear flow.

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Article received 29.11.10

ПЕРЕНОРМИРОВАННАЯ НЕМОДАЛЬНАЯ ТЕОРИЯ ТУРБУЛЕНТНОСТИ СДВИГОВЫХ ТЕЧЕНИЙ ПЛАЗМЫ

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Представлены результаты нелинейных исследований временной эволюции и насыщения дрейфовой турбулентности в сдвиговых течениях, основанных на немодальном подходе.

ПЕРЕНОРМОВАНА НЕМОДАЛЬНА ТЕОРІЯ ТУРБУЛЕНТНОСТІ ЗСУВНИХ ТЕЧІЙ ПЛАЗМИ

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Подано результати нелінійних досліджень часової еволюції та насичення дрейфової турбулентності у зсувних течіях плазми, які базуються на немодальному підході.