ON POSSIBILITY OF PRESSURE PERTURBATION RESONANT EXCITATION BY AN EXTERNAL LOW FREQUENCY HELICAL FIELD NEAR EDGE PLASMA

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In the frame of one-fluid MHD a possibility of the pressure perturbation resonant excitation by external low frequency helical magnetic perturbations near the plasma edge is shown. The plasma rotation plays the key role in this phenomenon. The plasma response has being taken into account. These pressure perturbations may affect on the ballooning and peeling modes stability.

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1. INTRODUCTION

Control of Edge Localized Modes (ELMs) is a critical issue of the present day large tokamaks and future ITER operation. ELMs are short bursts of particles and energy at tokamak edge plasma observed in H-mode operation [1, 2]. Melting, erosion and evaporation of divertor target plates may occur as results of these bursts.

Many experiments in DIII-D have shown that ELMs can be suppressed by small external low frequency helical magnetic perturbations [3, 4]. Until now, understanding of the underlying physics of ELMs and their suppressions has been far from complete.

In Ref. [5] the influence of an external helical field on the equilibrium of ideal plasma was investigated in the frame of MHD theory. A perfect shielding of the external resonant field was assumed.

In the present paper, a possibility of the pressure perturbation resonant excitation by external low frequency helical magnetic perturbations near the plasma edge is shown. A perfect shielding is not assumed. This pressure perturbation resonant excitation strongly depends on a plasma rotation. The plasma response takes into account.

Note, that ELM frequency dependence on the toroidal rotation in JT-60U was shown [6].

2. BASIC EQUATIONS

We start from the one-fluid MHD equations

$$\rho\frac{d\mathbf{V}}{dt} = -\nabla p - \nabla\cdot\boldsymbol{\pi}_i + \frac{1}{c}[\mathbf{J}\times\mathbf{B}], \qquad (1)$$

$$\frac{dp}{dt} + \gamma_0 p\, div\mathbf{V} = 0, \qquad (2)$$

the Maxwell's equations

$$rot\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},\quad rot\mathbf{B} = \frac{4\pi}{c}\mathbf{J},\quad div\mathbf{B} = 0, \qquad (3)$$

$$div\mathbf{J} = 0, \qquad (4)$$

and Ohm's Law (σ - conductivity)

$$\mathbf{J} = \sigma\left(\mathbf{E} + \frac{1}{c}[\mathbf{V}\times\mathbf{B}]\right), \qquad (5)$$

where ρ is the plasma mass densities, p is the plasma pressure, $\mathbf{J}$ is the current density, $\boldsymbol{\pi}_i$ is the ion gyroviscosity tensor, respectively.

We consider a current carrying toroidal plasma with nested equilibrium circular magnetic surfaces (ρ_0 is the radius of the magnetic surfaces, ω_0 is the poloidal angle in the cross-section $\varsigma = const$, ς is the toroidal angle). Each magnetic surface is shifted with respect to the magnetic axis (ξ is the shift, R is the radius of the magnetic axis). The equilibrium toroidal contravariant component of the magnetic field, $B_0^{\varsigma} = \Phi'/\left(2\pi\sqrt{g}\right)$, is large with respect to the poloidal one, $B_0^{\theta} = \chi'/\left(2\pi\sqrt{g}\right)$, Φ' and χ' are the radial derivatives of toroidal and poloidal fluxes, respectively; $q(a) = \Phi'/\chi'$ is the safety factor, $\mu = 1/q$. The known expressions for metric tensor are used [7].

On each magnetic equilibrium surface (see, e.g. [7]) we introduce a straight magnetic field line coordinate system (a, θ, ς) $\rho_0 = a$, $\omega_0 = \theta + \lambda(a)\sin\theta$

$$\lambda(a) = -\xi'(a) - a/R, \qquad (6)$$

$$\xi'(a) = \frac{1}{aR}\left(\frac{\chi'(a)}{2\pi R}\right)^{-2}\int_0^a\left[16p_0(b) + \left(\frac{\chi'(b)}{2\pi R}\right)^2\right]bdb. \qquad (7)$$

Assuming that perturbation $B^{\varsigma} \approx 0$ we get equations for perturbations (*m >>1, nq >>1*). From Eq.(4) by usual way [7] in a linear approximation in *1/R* one finds

$$\frac{1}{\sqrt{g}}L_{\|}\left[\left(-\frac{\partial}{\partial\theta}g_{11} + \frac{\partial}{\partial a}g_{12}\right)B^a + \left(-\frac{\partial}{\partial\theta}g_{12} + \frac{\partial}{\partial a}g_{22}\right)B^{\theta}\right] + \frac{8\pi^2}{\Phi'\sqrt{g}}\frac{\partial\sqrt{g}}{\partial\theta}\cdot\frac{\partial p}{\partial a}$$
$$-\frac{2\Phi'}{p_0'\sqrt{g}}\frac{\partial p}{\partial\theta}\left[W(a,\theta) - \frac{p_0'}{c\Phi'}L_{\|}\left(\frac{\nu}{p_0'}\right)'\right] + \frac{4\pi}{c}\left(B^a\frac{\partial}{\partial a} + B^{\theta}\frac{\partial}{\partial\theta}\right)\frac{J' + \partial\nu/\partial\theta}{\Phi'} = 0, \qquad (8)$$

where

$$L_{\|} = \mu(\partial/\partial\theta) + (\partial/\partial\varsigma),$$

$$\chi'(\partial\nu/\partial\theta) = 4\pi^2 c p_0' \left((\sqrt{g})_{(0)} - \sqrt{g}\right),$$

$$J_0^{\varsigma} = \left(J' + \partial\nu/\partial\theta\right)/2\pi\sqrt{g},$$

$$W(a,\theta) = W_0(a) + W_1(a,\theta),$$

$$W_0(a) = \frac{2p_0'}{RB_{0\varsigma}^2(a)}\left(\mu^2 - 1 + \frac{ap_0'}{B_{0\omega_0}^2} - \frac{SR}{a}\xi'\right),$$

$$W_1(a,\theta) = \frac{2p_0'S}{aB_{0\varsigma}^2(a)}\cos\theta, \quad S = a\frac{q'}{q}.$$

Equilibrium parameters denotes by the subscript 0.

Assuming periodicity in both θ and ζ, we take the perturbations in the form

$$X(a,\theta,\zeta,t) = \sum_{m,n} X_{mn}(a)\exp[i(m\theta - n\zeta - \omega t)], \quad (9)$$

where ω is the frequency of the external perturbation.

In our consideration all poloidal harmonic amplitudes of perturbations have finite values. The amount of poloidal harmonics with finite values of amplitudes depends on the antenna spectrum (external perturbation).

Early the case was studied for one dominant poloidal external mode and neighboring poloidal modes were considered as small [8].

Using Eq. (9), Eq. (8) (derivatives with respect to radius a denote by the prime) becomes

$$F_m(a)\left[i(a^2B_m^{\theta})' + mB_m^a\right] + \frac{4\pi SqR}{B_{0\varsigma}^2(a)}p_0'(B_{m-1}^a + B_{m+1}^a) + \frac{4\pi iqR}{B_{0\varsigma}^2(a)}p_0'a(B_{m-1}^{\theta} - B_{m+1}^{\theta}) + \frac{2aR}{c}\frac{B_m^a}{B_{0\varsigma}(a)}(J'/a)' -$$
$$-\frac{8\pi im}{B_{0\varsigma}(a)}\frac{a}{R}\left(\mu^2 - 1 + \frac{ap_0'}{B_{0\omega_0}^2(a)} - \frac{R}{a}S\xi'\right)p_m - \frac{4\pi i}{B_{0\varsigma}(a)}(ap_{m-1}' - ap_{m+1}') + \frac{4\pi i}{B_{0\varsigma}(a)}\left[(m-1)p_{m-1} + (m+1)p_{m+1}\right] = 0. \quad (10)$$

In Eq. (10) $F_m(a) = m\mu - n$, $B_{0\varsigma}(a) = \Phi'/2\pi a$, $B_{0\omega_0}(a) = \chi'/2\pi R$, $(aB_m^a)' + imaB_m^{\theta} = 0.$

In the similar way from Eq. (2), using the parallel (with respect to equilibrium magnetic field) momentum from Eq. (1), we get for pressure perturbation ($div\,\mathbf{V}_{\perp} \neq 0$) the next equation:

$$p_m = -\frac{i}{\Omega_m^2}\left\{\frac{c_s^2}{R}\frac{B_{\varsigma 0}}{B_0}F_m(a)\left(\rho_0 V_{0\|}' V_{Em}^a + p_0'\frac{B_m^a}{B_0}\right) + \omega_m p_0' V_m^a + \frac{\omega_m \rho_0 c_s^2}{R}\left[\frac{(aV_{m-1}^a)'}{m-1} - \frac{(aV_{m+1}^a)'}{m+1} - (V_{m-1}^a + V_{m+1}^a)\right]\right\}. \quad (11)$$

Here

$$\Omega_m^2(a) = \omega_{im}\omega_m - \frac{c_s^2}{R^2}F_m^2(a), \quad c_s^2 = \gamma_0\frac{p_0}{\rho_0}, \quad (12)$$

$$\omega_m = \omega - \frac{B_{0\varsigma}}{B_0}\frac{F_m(a)}{R}V_{0\|} + \frac{B_{0\varsigma}}{B_0}\frac{m}{a}c\frac{E_{0a}}{B_0}, \quad (13)$$

$$\omega_{im} = \omega - \frac{B_{0\varsigma}}{B_0}\left[\frac{F_m(a)}{R}V_{0\|} + \frac{m}{a}\left(c\frac{p_{0i}'}{en_0B_0} - c\frac{E_{0a}}{B_0}\right)\right]. \quad (14)$$

We took into account the equilibrium poloidal plasma rotation due to the existence of an equilibrium radial electric field E_{0a} and the ion diamagnetic drift; and the parallel with respect to equilibrium magnetic field plasma rotation with a velocity $V_{0\|}$. Recall that $V_m^a = V_{Em}^a + V_{pm}^a$ (see, e.g. [7, 9]).

From the radial component of Faraday's Law and Ohm's Law Eq. (5) we find

$$\omega_{im}B_m^a = -F_m(a)\frac{B_{0\varsigma}}{R}V_m^a - \frac{ic^2m}{4\pi\sigma a^2}\left[i(a^2B_m^{\theta})' + mB_m^a\right]. \quad (15)$$

Equations (10), (11) and (15) describe the affect of an external helical field on the ballooning and peeling modes due to the direct change of the edge plasma pressure. This affect has a resonant character.

3. DISCUSSIONS

When

$$\Omega_m^2(a) = \omega_{im}\omega_m - \frac{c_s^2}{R^2}F_m^2(a) \approx 0, \quad (16)$$

the resonant excitation of pressure perturbation takes place.

Note, that during the plasma eigenmodes stability analysis for a pressure perturbation (Eq. (1.98) of [10]) the nonresonant term $\gamma^2 + (c_s^2/R^2)F_m^2(a)$ occurred.

For more typical situation, when

$$\frac{a}{mR}F_m(a)c_s << c\frac{E_{0a}}{B_0},$$

the resonant excitation takes place, if

$$\omega \approx \frac{B_{0\varsigma}}{B_0}\frac{F_m(a)}{R}V_{0\|} - \frac{B_{0\varsigma}}{B_0}\frac{m}{a}c\frac{E_{0a}}{B_0} \quad (17)$$

or

$$\omega \approx \frac{B_{0\varsigma}}{B_0}\left[\frac{F_m(a)}{R}V_{0\|} + \frac{m}{a}\left(c\frac{p_{0i}'}{en_0B_0} - c\frac{E_{0a}}{B_0}\right)\right]. \quad (18)$$

Resonant conditions (17) and (18) useful to present in next form, respectively:

$$\omega \approx k_{\|}V_{0\|} - k_{\perp}c\frac{E_{0a}}{B_0}, \quad (19)$$

$$\omega \approx k_{\|}V_{0\|} + k_{\perp}(c\frac{p'_{0i}}{en_0 B_0} - c\frac{E_{0a}}{B_0}). \quad (20)$$

In the case $\omega = 0$ (e.g. for DIII-D) resonant conditions (17) and (18) convenient to present the next way:

$$\frac{a}{R}(\mu(a) - n/m)V_{0\|} = c\frac{E_{0a}(a)}{B_0}, \quad (21)$$

$$\frac{a}{R}(\mu(a) - n/m)V_{0\|} = (c\frac{E_{0a}}{B_0} - c\frac{p'_{0i}}{en_0 B_0}). \quad (22)$$

Note, that for DIII-D edge plasma parameters [3, 4] the resonant conditions (21) and (22) may be realized.

4. CONCLUSIONS

A possibility of the pressure perturbation resonant excitation by external low frequency helical magnetic perturbations near the plasma edge is shown. This phenomenon occurs during the plasma rotation. It may affect on the ballooning and peeling modes excitation because of a plasma pressure change.

The equations that describe this influence on the ballooning and peeling modes excitation are derived on the basis of MHD equations when all poloidal harmonic amplitudes of external perturbations have finite values. Plasma rotation and plasma response are taken into account.

Expected result may be used for interpretation of the plasma stability experiments in tokamaks JET, DIII-D, TEXTOR and future ITER operation.

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О ВОЗМОЖНОСТИ РЕЗОНАНСНОГО ВОЗБУЖДЕНИЯ ВОЗМУЩЕНИЯ ДАВЛЕНИЯ ВНЕШНИМ НИЗКОЧАСТОТНЫМ ВИНТОВЫМ ПОЛЕМ ВБЛИЗИ КРАЯ ПЛАЗМЫ

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В рамках одножидкостной МГД показана возможность резонансного возбуждения возмущений давления у края плазмы внешними низкочастотными винтовыми возмущениями магнитного поля. Вращение плазмы играет ключевую роль в этом явлении. Учтен отклик плазмы. Эти возмущения давления могут влиять на устойчивость баллонных и пилинг-мод.

ПРО МОЖЛИВІСТЬ РЕЗОНАНСНОГО ЗБУДЖЕННЯ ЗБУРЕННЯ ТИСКУ ЗОВНІШНІМ НИЗЬКОЧАСТОТНИМ ГВИНТОВИМ ПОЛЕМ БІЛЯ КРАЮ ПЛАЗМИ

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У рамках однорідинної МГД показана можливість резонансного збудження збурень тиску біля краю плазми зовнішніми низькочастотними гвинтовими збуреннями магнітного поля. Обертання плазми відіграє ключову роль у цьому явищі. Враховано відгук плазми. Ці збурення тиску можуть впливати на стійкість балонних та пілінг-мод.