

# DIPOLAR ELECTROMAGNETIC WAVES IN COAXIAL STRUCTURE FILLED BY DISSIPATIVE PLASMA WITH AZIMUTH MAGNETIC FIELD

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This report is devoted to the investigation of dispersion properties and spatial attenuation coefficient of the dipolar high-frequency electromagnetic wave that propagates along the coaxial magnetized waveguide structure with non-uniform azimuthal magnetic field, partially filled by uniform collisional plasma. The influence of effective collision frequency and the value of the direct current on the phase characteristics and the spatial attenuation coefficient of the considered wave is studied. It was shown that it is possible to control effectively the dispersion properties and the spatial attenuation of the considered wave by varying the value of the direct current.

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## 1. INTRODUCTION

Electrodynamic properties of coaxial plasma-metal structures are the subject of intensive theoretical and experimental studies at present time. Such structures are widely used as the waveguide structures in the devices of plasma electronics [1] and also as the discharge chambers for gas discharge sustaining [2, 3]. The realized experimental study of the coaxial waveguide structures with a central metallic rod have shown that properties of electromagnetic waves and gas discharge plasma maintained by these waves, differ considerably from the corresponding properties of cylindrical plasma – metal waveguide structures without the central conductor [1, 4].

It is necessary to note that in spite of good plasma parameters obtained in experimental devices with coaxial structures, theoretical study of eigen waves properties of coaxial waveguide structures and efficiency of such structure usage in various applications is insufficient. This especially relates to the theoretical study of propagation and spatial attenuation of the electromagnetic eigen waves with complex azimuthal wave field structure that propagate along the coaxial structure with central metallic conductor partially filled by dissipative plasma. These circumstances greatly determine the urgency of presented work.

## 2. PROBLEM FORMULATION

Let's consider a high-frequency electromagnetic wave with azimuthal wavenumber  $m=1$  that propagates in cylindrical the coaxial magnetized waveguide structure, partially filled by dissipative plasma. The waveguide structure consists of the metallic rod of radius  $R_1$ , which is placed at the axis of plasma column. The direct current  $J_z$  flows along this rod, creating radially non-uniform azimuthal magnetic field  $H_0(r)$ . This rod is surrounded by the cylindrical plasma layer of radius  $R_2$ . The vacuum region ( $R_2 < r < R_3$ ) separates the cylindrical plasma layer from waveguide metallic wall with radius  $R_3$ . It was assumed, that plasma density is radially uniform (averaged over the plasma column cross-section) and varies slightly along the plasma column. Such approach is widely used for theoretical description of a gas discharge sustained by the travelling surface waves [5, 6]. Plasma

was considered in the hydrodynamic approach as a cold dissipative medium. The collisions were characterized by the effective electron collisional frequency  $\nu$  that is constant in the whole volume of the cylindrical plasma layer and is supposed to be small ( $\nu/\omega < 1$ , where  $\omega$  is wave frequency).

The permittivity tensor of magnetized plasma  $\epsilon$  can be written in the form [7]:

$$\epsilon = \begin{pmatrix} \epsilon_1 & 0 & -i\epsilon_2 \\ 0 & \epsilon_3 & 0 \\ i\epsilon_2 & 0 & \epsilon_1 \end{pmatrix}, \quad (1)$$

where  $\epsilon_1 = 1 - \frac{\omega_p^2 \omega'}{\omega[(\omega')^2 - \omega_c^2(r)]}$ ,  $\epsilon_2 = \frac{|\omega_c(r)|\omega_p^2}{\omega[(\omega')^2 - \omega_c^2(r)]}$ ,

$\epsilon_3 = 1 - \frac{\omega_p^2}{\omega\omega'}$ ,  $\omega' = \omega + i\nu$ ,  $\omega_p$  and  $\omega_c(r)$  are electron plasma and cyclotron frequencies, respectively.

Propagation of electromagnetic wave in the waveguide structure is governed by the system of Maxwell's equations. In the case considered the solution of the system of Maxwell's equation in cylindrical coordinate system ( $r, \phi, z$ ) (let assume that  $z$  – axis is directed along the axis of waveguide structure) can be found in the form:

$$E, H = E(r), H(r) \exp[i(k_3 z + m\phi - \omega t)], \quad (2)$$

where  $k_3$  is the complex axial wavevector, real part of it determines the wavenumber and imaginary part determines the wave spatial attenuation coefficient.

Taking into account (2) from the system of Maxwell's equations one can find, that in the region of cylindrical plasma layer the ordinary differential equations that govern the dipolar wave components can be written in the following form:

$$\begin{cases} \frac{dE_\theta}{dr} = -\frac{E_\theta}{r} + C_1 H_\phi - C_2 E_z + C_3 H_z \\ \frac{dH_\theta}{dr} = -\epsilon_1 C_1 E_\theta + C_4 H_\phi + C_5 E_z - C_2 H_z \\ \frac{dE_z}{dr} = -C_6 H_\theta - k_3 \frac{\epsilon_2}{\epsilon_1} E_z - C_7 H_z \\ \frac{dH_z}{dr} = -C_8 E_\theta + \epsilon_1 C_7 E_z \end{cases}, \quad (3)$$

where  $C_1 = i \frac{m k_3}{r k \varepsilon_1}$ ,  $C_2 = \frac{m \varepsilon_2}{r \varepsilon_1}$ ,  $C_3 = i \left( k - \frac{m^2}{r^2} \frac{1}{k \varepsilon_1} \right)$ ,

$C_4 = k_3 \frac{\varepsilon_2}{\varepsilon_1} - \frac{1}{r}$ ,  $C_5 = \frac{i}{k} \left( \frac{m^2}{r^2} + k^2 \frac{\varepsilon_2^2 - \varepsilon_1^2}{\varepsilon_1} \right)$ ,  $C_7 = i \frac{m k_3}{r k \varepsilon_1}$ ,

$C_6 = \frac{i}{k \varepsilon_1} (k_3^2 - k^2 \varepsilon_1)$ ,  $C_8 = \frac{i}{k} (k_3^2 - k^2 \varepsilon_3)$  and  $k = \omega / c$  is

the vacuum wavenumber. In the case considered this system for arbitrary external parameters can be solved only with the help of numerical methods.

In the cylindrical vacuum region the system of Maxwell equations possesses the analytical solutions expressed in the terms of modified Bessel functions. These solutions contains the wave field constants, that can be obtained with the help of boundary conditions consisting in continuity of the tangential wave field components at the plasma – vacuum interface. From such conditions these constants can be written in the form:

$$\begin{cases} A_1 = -A^1 E_z^p(R_2) - A^2 H_z^p(R_2) + A^3 H_\phi^p(R_2) \\ A_2 = A^4 E_z^p(R_2) + A^5 H_z^p(R_2) - A^6 H_\phi^p(R_2) \\ A_3 = iA^2 E_z^p(R_2) - A^3 E_\phi^p(R_2) - A^1 H_z^p(R_2) \\ A_4 = -A^5 E_z^p(R_2) + A^6 E_\phi^p(R_2) + A^4 H_z^p(R_2) \end{cases}, \quad (4)$$

where  $A^1 = \kappa_\nu R_2 K_m'(\kappa_\nu R_2)$ ,  $A^2 = i \frac{m k_3 K_m(\kappa_\nu R_2)}{k}$ ,

$$A^3 = i \frac{\kappa_\nu^2 R_2 K_m(\kappa_\nu R_2)}{k}, \quad A^5 = i \frac{m k_3 I_m(\kappa_\nu R_2)}{k},$$

$A^4 = \kappa_\nu R_2 I_m'(\kappa_\nu R_2)$ ,  $A^6 = i \frac{\kappa_\nu^2 R_2 I_m(\kappa_\nu R_2)}{k}$ ,  $\kappa_\nu^2 = k_3^2 - k^2$ ,

prime denotes the derivative with respect to argument and  $E_z^p(R_2)$ ,  $H_z^p(R_2)$ ,  $E_\phi^p(R_2)$ ,  $H_\phi^p(R_2)$  are the wave fields at the plasma – vacuum interface, obtained by the numerical integration of the system of ordinary differential equations (3).

The boundary conditions for  $E_z(r)$  and  $E_\phi(r)$  wave field components at the waveguide metallic wall  $r = R_3$  gives the dispersion equation that can be written in the following form:

$$\begin{cases} A_1 I_m(\kappa_\nu R_3) + A_2 K_m(\kappa_\nu R_3) = 0 \\ A_3 I_m(\kappa_\nu R_3) + A_4 K_m(\kappa_\nu R_3) = 0 \end{cases}, \quad (5)$$

### 3. RESULTS AND DISCUSSION

It is necessary to mention that, in the case when the external current flows along the propagation direction of the wave considered the dispersion equation (5) possesses two solutions with different frequency values for the fixed value of the dimensionless wavenumber  $\text{Re}(k_3)R_1$ . One of them with comparatively higher frequency will be called further high frequency (HF) wave, and other – low frequency (LF) wave.

Properties of these waves are substantially determined by the direct current value. The influence of the direct current value on the LF and HF dispersion properties is shown on the Fig. 1. The dimensionless parameters

$\mu = \omega / \omega_p$  and  $x = \text{Re}(k_3)R_1$  were used to represent the obtained results. The increase of the direct current value leads to the decrease of the HF wave frequency and to the increase of the LF wave frequency. So, for rather high dimensionless direct current value ( $j = eJ_z / (2mc^3) > 1.5$ ) the frequencies of HF and LF waves for rather high  $\text{Re}(k_3)R_1$  values are close. In the limiting case, when the azimuthal magnetic field  $H_0(r)$  trends to zero the LF wave vanishes.

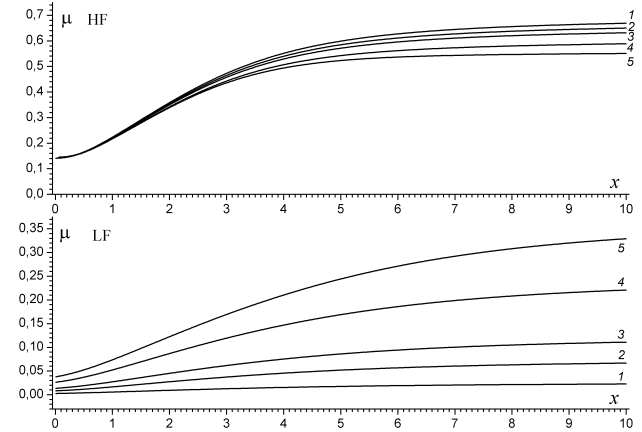


Fig. 1. The dependence of the wave frequency  $\mu$  on the wavenumber  $x$  for the LF and HF waves. Numbers on the curve corresponds to different  $j$  values: 1 –  $j = 0.1$ , 2 –  $j = 0.3$ , 3 –  $j = 0.5$ , 4 –  $j = 1.0$ , 5 –  $j = 1.5$ . Other parameters are equal:  $R_1 \omega_p / c = 4.0$ ,  $R_2 \omega_p / c = 5.0$ ,  $R_3 \omega_p / c = 6.0$ ,  $\nu / \omega = 0.001$

The influence of the effective electron collisional frequency  $\nu$  on the LF and HF spatial attenuation is shown on the Fig. 2. The dimensionless parameter  $\alpha = \text{Im}(k_3)R_1$  was used to represent the obtained results. It was obtained that the increase of  $j$  value leads to the increase of attenuation coefficient  $\alpha$  of HF wave and to the decrease of attenuation coefficient of LF wave. For rather low direct current values ( $j < 0.1$ ) the LF wave strongly damps and therefore cannot propagate. It is necessary to mention that the attenuation coefficient  $\alpha$  for the HF and LW waves has different behaviour in the areas of rather high and rather low wavenumbers  $\text{Re}(k_3)R_1$ . When  $\text{Re}(k_3)R_1$  is rather low ( $\text{Re}(k_3)R_1 < 0.3$  for HF wave and  $\text{Re}(k_3)R_1 < 0.5$  for LF wave) the attenuation coefficient decrease with the increase of  $\text{Re}(k_3)R_1$ . In the next region the attenuation coefficient increases with the increase of  $\text{Re}(k_3)R_1$  value. Such complicated behaviour may be very important for the determination of the frequency range, where the considered wave can maintain the stable discharge [6].

The influence of the effective collisional frequency value on the dispersion and attenuation properties of the HF and LW waves was also studied. Dispersion of the HF and LF waves depends on  $\nu / \omega$  parameter rather weakly. The increase of  $\nu / \omega$  value leads to the increase of the

attenuation coefficient  $\alpha$  for the HF and LF waves. It is necessary to note that the value of the attenuation coefficient of the LF wave for the small  $\nu/\omega$  values ( $\nu/\omega < 0.001$ ) is approximately of one order greater than the value of attenuation coefficient of the HF wave. For rather high  $\nu/\omega$  values ( $\nu/\omega \approx 0.01$ ) the attenuation coefficients for the LF and HF waves are of one order. It was obtained that the HF and LF waves in the region of moderate  $\text{Re}(k_3)R_1$  values ( $0.5 < \text{Re}(k_3)R_1 < 4$ ) have rather small attenuation coefficient value. In other region the value of the attenuation coefficient for both waves are rather large.

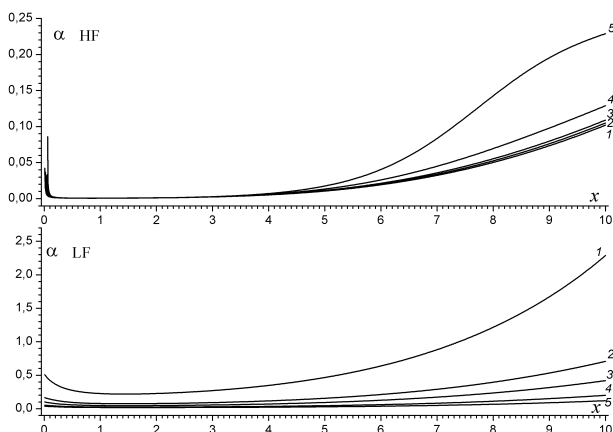


Fig. 2. The dependence of the attenuation coefficient  $\alpha$  on the wavenumber  $x$  for the LF and HF waves. Problem parameters are the same as for the Fig. 1

#### 4. CONCLUSIONS

It was studied the influence of the effective collision frequency and the value of the direct current on the phase

characteristics and the attenuation coefficient of the electromagnetic dipolar wave that propagates along the coaxial waveguide structure. It was shown that it is possible to control effectively the dispersion properties and the spatial attenuation of the dipolar wave by varying the value of the direct current. The influence of the dimensionless collision frequency on the dispersion and attenuation properties of the HF and LF waves was study as well. It was shown that the LF dipolar wave attenuates more effectively than the HF wave.

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### ДИПОЛЬНЫЕ ЭЛЕКТРОМАГНИТНЫЕ ВОЛНЫ В КОАКСИАЛЬНОЙ СТРУКТУРЕ, ЗАПОЛНЕННОЙ ДИССИПАТИВНОЙ ПЛАЗМОЙ С АЗИМУТАЛЬНЫМ МАГНИТНЫМ ПОЛЕМ

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Исследуются дисперсионные свойства и коэффициент пространственного затухания дипольной высокочастотной электромагнитной волны, распространяющейся в коаксиальной магнитоактивной волноводной структуре с радиально неоднородным азимутальным магнитным полем, частично заполненной радиально однородной столкновительной плазмой. Изучено влияние эффективной частоты столкновений электронов и величины внешнего аксиального постоянного тока на фазовые характеристики и коэффициент пространственного затухания дипольной волны. Показана возможность эффективного управления фазовыми характеристиками и коэффициентом пространственного затухания дипольной волны с помощью внешнего постоянного тока.

### ДИПОЛЬНІ ЕЛЕКТРОМАГНІТНІ ХВИЛІ В КОАКСІАЛЬНІЙ СТРУКТУРІ, ЩО ЗАПОВНЕНА ДИСИПАТИВНОЮ ПЛАЗМОЮ З АЗИМУТАЛЬНИМ МАГНІТНИМ ПОЛЕМ

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Розглянуто дисперсійні властивості та коефіцієнт просторового загасання дипольної високочастотної хвилі, що поширюється в коаксіальній магнитоактивній хвилеводній структурі з радіально неоднорідним азимутальним магнітним полем, яка частково заповнена радіально однорідною плазмою з зіткненнями. Досліджено вплив ефективної частоти зіткнень електронів та величини зовнішнього аксіального постійного струму на фазові характеристики та коефіцієнт просторового загасання дипольної хвилі. Показана можливість ефективного керування фазовими характеристиками та просторовим загасанням дипольної хвилі за допомогою зовнішнього постійного струму.