Системы и интеллектуальное управление

WELL-POSED IDENTIFICATION OF NUCLEAR TYPE INFINITE AND MULTIDIMENSIONAL SYSTEMS

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Рассмотрены проблемы моделирования и идентификации сложных динамических систем. Для задач структурно-параметрической идентификации на основе описания в виде бесконечных разложений установлены и исследованы условия, при которых эти задачи являются корректно поставленными, а когда их решения становятся неустойчивыми и, следовательно, практически непригодными. Полученные результаты являются фундаментальными и дают более глубокое понимание процесса идентификации.

Ключевые слова: системная идентификация, некорректно-поставленная задача, конечно-частотная идентификация, идентификация на основе выделения ортогонального подпространства (4SID), регуляризация.

Розглянуто проблеми моделювання та ідентифікації складних динамічних систем. Для задач структурно-параметричної ідентифікації на основі опису у вигляді нескінченних розкладів встановлено та досліджено умови, за яких ці задачі є коректно поставленими, а коли їх розв'язки стають нестійкими і, отже, практично непридатними. Отримані результати є фундаментальними і дають більш глибоке розуміння процесу ідентифікації.

Ключові слова: системна ідентифікація, некоректно-визначена задача, кінцево-частотна ідентифікація, ідентифікація на основі виділення ортогонального підпростору (4SID), регуляризація.

INTRODUCTION

System identification problems belong to a class of inverse problems which have the feature that under certain conditions its solutions become unstable i.e. sensitive to the errors in input data. Especially it takes place in multidimensional cases when it is necessary to assume a model set which adequately represents the original system, then select a model order and find other parameters using only general information about unknown plant. Processes in such system may be so complex that on the base of fundamental laws and theoretical results it is often impossible to define even the model set which includes a model that completely describes the system that generates data. Also it is supposed that there is a causation between finite input and output of a plant. The only possible way in such cases is to use different infinite-dimensional expansion as model set [1, 2].

Computational mathematic methods serve as a foundation of such approach (e.g. Galerkin method and other iterational methods, methods based on Green function or impulse response for system with disturbed or lumped parameters [3–5]).

Example of such complicated system is a heating processes in power station. Fuel supply that affects vapor parameters is considered as an input. Processes of burning, heating and vaporizing are so complex that it is impossible to describe them in any other way besides approach proposed above, that is using infinite-dimensional model which links input (fuel supply) and output (vapor parameters). In this case input-output map is an impulse response that induces nuclear Hankel operator or equivalent non-rational transfer function.

The approximation of infinite-dimensional linear system by finite-dimensional ones is a subject of interest in a mathematical theory. In [2] using output normal realization the convergence of the finite-dimensional approximation to precise model was proven and error bounds on the truncated realization were given. These results are important for model reduction problem, i.e. conversion of the complex infinite-dimensional description into simple low-dimensional model [6].

In identification problem the reduced model order selection is defined primarily by the well-posedness, for which it is necessary to satisfy conditions providing the stability of the solution with respect to errors in the initial data. In this sense truncated model order will be the main regularization parameter which means that the dimension of rational approximation should be in agreement with the errors in available data. Hence regularization procedure should be incorporated in the existing methods of identification allowing to find the stable solution and to extend model order when errors tend to zero.

This paper shows how this problem may be solved correctly with selection of highest model order admissible by stability conditions using two popular identification methods. Besides numerous computational experiments that were conducted allowed us to clarify a cause of ill-posedness which is an essential to identification problem. Fundamental properties of the regularized solutions are established irrespectively to identification method used for plant that generated output data.

Systems with single input and single output (SISO) are considered because even in this case nontrivial results were revealed.

PROBLEM SETTING

For infinite-dimensional nuclear type linear SISO systems model set can be written as

$$\dot{x} = Ax + bu + \xi, \, y = cz + \eta, \tag{1}$$

where A is a linear operator mapping between infinite-dimensional linear vector spaces, b is a linear operator mapping one-dimensional space into infinite-dimensional one, c is a projecting operator. Under suitable assumptions they lead to existence of a non-rational transfer function $G(s) = c(sI - A)b$. Values ξ and η denote unknown perturbation in input and noise in output that may be present in real system.

Description (1) be also appropriate for multi-dimensional case when system

has finite but large and unknown dimension. In this case A is a matrix, b, c are vectors and transfer function is a rational. Such system is also a nuclear type if it eigenvalues belong to left half-plane.

Let description (1) be proper for unknown real system, i.e. is a nuclear type and (1) includes true model. It is required to find the finite-dimensional matrix A and vectors b, c using experimental data $(y(t), u(t))$ that would provide a rational approximation of the real system described by (1) so that difference between system output and model output would be either in accordance with errors ξ, η or defined by stability condition of the solution. In other words it means to solve identification problem correctly. For finite multidimensional case it means to construct the model with significantly less dimension than dimension of the real system. Here is a full analogy with so-called model reduction problem [6]. In this case rational approximation (1) can be represented as

$$\dot{x} = A_n x + b_n u, \; y = c_n^T x, \tag{2}$$

where A_n is a matrix $n \times n$, b_n is a column-vector, c^T is a row-vector (T is a transposition).

FINITE-FREQUENCY IDENTIFICATION WITH REGULARIZATION

Model reconstruction using Frequency method of identification is realized on frequency domain parameters, extracted from experimentally measured output when input is excited by the harmonic test signal. The classical frequency approach has a long history and here we don't give the review of these methods because readers can find it for example in the paper [7]. We note that in recent years the methods were significantly developed by Alexandrov and Orlov, see [7–11].

If (2) will be written in the form of Jordan realization it is easy to get the following input-output relation

$$
\begin{aligned}
y(t) = &\sum_{q=1}^{Q} g_q \int_0^t e^{-\lambda_q (t-\theta)} u(\theta) d\theta + \\
&+ \sum_{p=1}^{P} \int_0^t e^{-\alpha_p (t-\theta)} \left[f_p^c \cdot \cos \beta_p (t-\theta) + f_p^s \cdot \sin \beta_p (t-\theta) \right] u(\theta) d\theta.
\end{aligned}
\tag{3}
$$

For simplicity sake here it is accepted that initial state under t_0 is zero: $t_0 = 0$; eigenvalue multiplicity is missing; Q is a number of real and P is a number of complex-valued eigenvalues with $\lambda_q \geq 0, \alpha_p \geq 0$. Besides

$$
\begin{aligned}
g_q &= c_{nq} b_{nq}, & q = 1, Q \\
f_p^c &= c_{np}^c b_{np}^c + c_{np}^s b_{np}^s, & p = 1, P \\
f_p^s &= c_{np}^c b_{np}^s - c_{np}^s b_{np}^c,
\end{aligned}
\tag{4}
$$

c_{nq} and b_{nq} are components of the vectors b_n and c_n for real eigenvalues and

c_{np}^c, c_{np}^s, b_{np}^c, b_{np}^s are components of these vectors corresponding Jordan block
$$\begin{bmatrix} -\alpha_p & \beta_p \\ -\beta_p & -\alpha_p \end{bmatrix}.$$

From equations (4) it is possible to select different canonical form. For example, assuming $b_{nq}=1$, $b_{np}^c=1$, $b_{np}^s=0$ we receive the control canonical realization and for $c_{nq}=1$, $c_{np}^c=1$, $c_{np}^s=0$ we have observable canonical realization. By analogy balanced realization may also be obtained.

In frequency method the system and model inputs are usually exited by polyharmonic signal with K sinusoids of different frequencies ω_k and amplitudes u_k:

$$u(t) = \sum_{k=1}^{K} u_k \sin \omega_k t, \quad t \geq 0 . \tag{5}$$

For sample data the regression equation is derived if substitute (5) in (3) and integrate. As result we obtain

$$
\begin{aligned}
y(t) = \sum_{k=1}^{K} u_k \Bigg\{ &\sum_{q=1}^{Q} \frac{\omega_k g_q}{\lambda_q^2 + \omega_k^2} e^{-\lambda_q t} + \\
&+ \sum_{p=1}^{P} \Big[\big(\gamma_3 f_p^c + \gamma_2 f_p^s \big) \sin \beta_p t + \big(\gamma_2 f_p^c - \gamma_3 f_p^s \big) \cos \beta_p t \Big] e^{-\lambda_p t} + \\
&+ \Bigg[\sum_{q=1}^{Q} \frac{\lambda_q g_q}{\lambda_q^2 + \omega_k^2} + \sum_{p=1}^{P} \big(\gamma_1 f_p^c + \gamma_4 f_p^s \big) \Bigg] \sin \omega_k t + \\
&+ \Bigg[\sum_{q=1}^{Q} \frac{\omega_k g_q}{\lambda_q^2 + \omega_k^2} + \sum_{p=1}^{P} \big(\gamma_2 f_p^c - \gamma_3 f_p^s \big) \Bigg] \cos \omega_k t \Bigg\},
\end{aligned}
\tag{6}
$$

where $Q + 2P = n$,

$$\gamma_1 = \gamma_1(\alpha_p, \beta_p, \omega_k) = \frac{\alpha_p}{\alpha_p^2 + (\omega_k + \beta_p)^2} + \frac{\alpha_p}{\alpha_p^2 + (\omega_k - \beta_p)^2} ,$$

$$\gamma_2 = \gamma_2(\alpha_p, \beta_p, \omega_k) = \frac{\omega_k + \beta_p}{\alpha_p^2 + (\omega_k + \beta_p)^2} + \frac{\omega_k - \beta_p}{\alpha_p^2 + (\omega_k - \beta_p)^2} ,$$

$$\gamma_3 = \gamma_3(\alpha_p, \beta_p, \omega_k) = \frac{\alpha_p}{\alpha_p^2 + (\omega_k + \beta_p)^2} - \frac{\alpha_p}{\alpha_p^2 + (\omega_k - \beta_p)^2} ,$$

$$\gamma_4 = \gamma_4(\alpha_p, \beta_p, \omega_k) = \frac{\omega_k + \beta_p}{\alpha_p^2 + (\omega_k + \beta_p)^2} - \frac{\omega_k - \beta_p}{\alpha_p^2 + (\omega_k - \beta_p)^2} .$$

When $Q, P, \lambda_q, \alpha_p, \beta_p$ are known formula (6) becomes linear regression which can be used for g_q, f_p^c, f_p^s determination. In this regard three stages of identification

are proposed. At first maximal admissible by stability condition Q^* and P^* are defined. In other words a solution of the identification problem for Q^* and P^* should be weakly sensitive to errors in available data and further increase of order n will lead to scattering of the estimated parameters. Then Q and P may be considered as a regularization parameters. Here we have analogy with function recovery problem which is given in the form of approximately defined coefficients of Fourier expansion. At the second stage eigenvalues (i.e. $\lambda_q, \alpha_p, \beta_p$) are calculated for known model order. At the final stage parameters g_q, f_p^c, f_p^s are estimated from equations (4) using linear regression and then coefficients $b_{nq}, b_{np}^s, b_{np}^c, c_{nq}, c_{np}^s, c_{np}^c$ that correspond to Jordan realization in suitable canonical form are founded.

For finding solution on the first and second stages it is proposed to use the well known equations of the frequency identification method [7]

$$V(j\omega_k) - (\Phi_k + j\Psi_k)W(j\omega_k) = 0, \, k = \overline{1, K}, \tag{7}$$

where $\qquad j = \sqrt{-1}, \qquad G(j\omega_k) = \dfrac{V(j\omega_k)}{W(j\omega_k)} \qquad$ and

$V(j\omega_k) = (j\omega_k)^{n-1}v_{n-1} + (j\omega_k)^{n-2}v_{n-2} + \ldots + j\omega_k v_1 + v_0$,

$W(j\omega_k) = (j\omega_k)^{n}w_n + (j\omega_k)^{n-1}w_{n-1} + \ldots + j\omega_k w_1 + w_0$ are analytical expressions for numerator and denominator of transfer function represented as frequency response, $\Phi_k + j\Psi_k$ is a value of transfer function on imaginary axis which may be evaluated from measured in experiment output $y(t)$ according to [7, 9]

$$\Phi_k(\tau) = \frac{2}{u_k}\int_0^\tau y(t)\sin\omega_k t \, dt, \; \Psi_k(\tau) = \frac{2}{u_k}\int_0^\tau y(t)\cos\omega_k t \, dt, \; k = \overline{1, K}. \tag{8}$$

For large but finite τ Φ_k and Ψ_k are determined with errors. If $\tau \to \infty$ Φ_k and Ψ_k tend to precise values when certain requirements to the choice of ω_k are realized [7, 9]. Besides correct identification on the base (7) is achieved when $K \geq n$. Then using corresponding part equations of (7) parameters v_i $(i = \overline{0, n-1})$ may be expressed through w_i $(i = \overline{0, n})$ and after that we eliminate v_i from remaining equations. This procedure is correct because the relevant linear system equations has a Vandermonde determinant, composed from different ω_k only, i.e. it is a nonsingular and may be computed exactly. Details of this procedure one can find in paper [12]. Here we write the final result namely the linear equation of the system with respect to vector $w = (w_0, w_1, \ldots, w_n)$:

$$Hw = g, \tag{9}$$

where matrix H and g on the right side have a complicated dependence on Φ_k, Ψ_k, ω_k that is $H = H(\Phi_k, \Psi_k, \omega_k)$, $g = g(\Phi_k, \Psi_k, \omega_k)$. Their expressions are given in [12]. Large finite τ provides approximate Φ_k, Ψ_k. As a result system (9) has given approximate matrix H and right hand side of equation. Nuclear type systems generate the data which lead to ill-conditionality of the matrix H even for not large n, i.e. the problem (9) becomes ill-posed for multidimensional model. It was established in [12] that solution of (9) will be stable if

$$k(H)\left(\varepsilon_c + \frac{1+\varepsilon}{\tau} \right) < 1, \tag{10}$$

where $k(H)$ is a matrix H condition number; ε is a boundary (or variance) for noise and perturbation; ε_c defines level of computational error. Detailed researches of the solution stability problem were realized by means of numerical experiments. Wide class of systems with different structure parameters and noise have generated data for identification which thereupon were utilized in algorithms that estimates $\hat{\Phi}$, $\hat{\Psi}$ and matrix H. Model order n and therefore a dimension of a system equation (9) was significant parameter. So matrix H and vector g were computed for different n, i.e. we formed set of H_n and g_n for different n. Condition number of the matrices obtained in numerical experiments is evaluated by SVD decomposition

$$H_n = U_n \Sigma_n V_n^T, \tag{11}$$

where U_n, V_n are orthogonal matrices and Σ_n is a diagonal matrix with the singular values in non-increasing order on the diagonal. Then matrix H_n condition number in spectral norm $\|\cdot\|_2$ is defined by expression

$$\kappa_2(H_n) = \frac{\sigma_1(H^n)}{\sigma_n(H^n)}, \tag{12}$$

where σ_1 is the first and σ_n is the last singular values. Typical for nuclear type system dependence $\kappa_2(H_n)$ from approximate model order n illustrates Fig. 1. Increase of the model dimension on unit leads approximately to two order $\kappa_2(H_n)$ enlargement.

Now we address the stability problem that is how to determine the regularized solution of the identification problem. As it was said before model order n is regarded as regularization parameter. However it is more convenient to use condition number for numerical modeling of identification process instead n since condition number is roughly linked with n for each concrete system. In this case condition number may represent the class of system even with different n.

Estimations of $\hat{\Phi}$ and $\hat{\Psi}$ depend on observation interval τ. Obviously this dependence become tangible when τ is changed exponentially. So it is appropriate

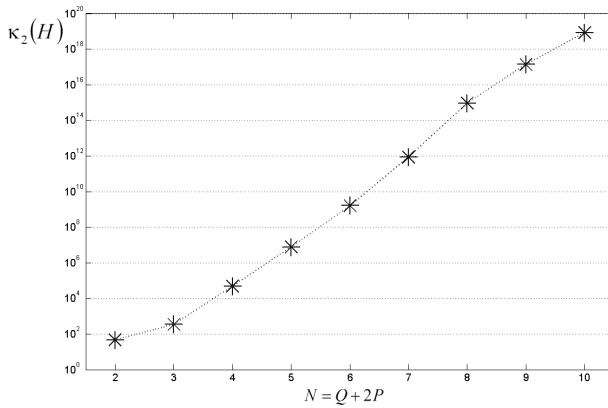to choose discrete time τ_k as $\tau_k = 10^k \tau_0$ where τ_0 is initial proper observation interval.

Fig. 1. Condition number values for approximate models of different order

We exploit small interval $[\tau_k, \tau_k + \Delta_k]$ near each τ_k for data collection and estimations of $\hat{\Phi}$ and $\hat{\Psi}$. We use set of matrices H obtained from these data for stability analysis. We can judge about appearing unstable solution by the scattering of eigenvalues obtained from equation $W(s) = 0$ with coefficients w_i ($i = \overline{1, n}$) calculated according to system (9). Fig. 2 summarizes the results of multiple numerical experiments for data generating systems with different structures and parameters. On plane of two parameters $\kappa_2(H)$ and τ are shown the stability domain A, range C where all systems were unstable and transition domain B where part of the systems provide the stable solution and another are unstable. Further τ increase leads to curves merging and after that joint curve tend to saturation that is explained by dominant influence of computational errors. As a result even for precise Φ, Ψ models with order $Q + 2P > 11$ lie in unstable domain. For models from transition domain the checking test on stability is required.
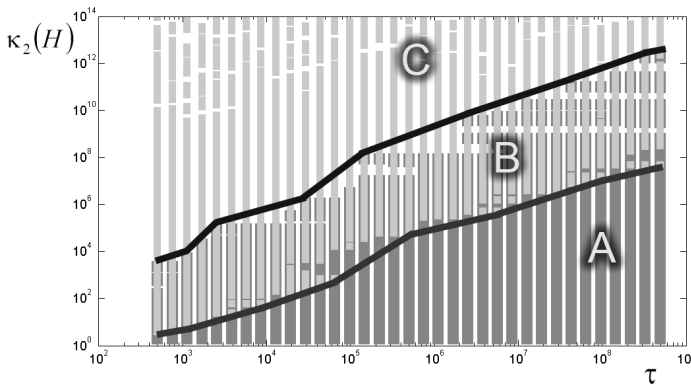

Fig. 2. Solution stability domains for frequency identification

REGULARIZED SUBSPACE (4SID) IDENTIFICATION METHOD

To build a finite-dimensional model of a plant with a modification of the 4SID method difference approximation of system (1) was selected as a model set. This model set may be written as

$$x_{t+1} = Ax_t + bu_t, \quad y_t = c^T x_t, \tag{13}$$

where A is a matrix $\alpha \times \alpha$, b is a vector $\alpha \times 1$, c^T is a vector $1 \times \alpha$, α is a model dimension, discretization Δt is fitted so that $\lim\limits_{\alpha \to \infty} \Delta t = 0$, i.e. description (13) should in limit lead to result coinciding with continuous approximation.

Subspace identification is a well developed method and description of it may be found for example in [13, 14]. Here we consider simplified version. Let input u_t be exited by square wave-form oscillator alternating with relaxation intervals

$$u_t = \begin{cases} u_i, t \in S_{2i} \\ 0, t \in S_{2i+1} \end{cases}, \quad i = 0, 1, 2, \ldots, N, \tag{14}$$

where $S_0, S_2, \ldots$ are exiting intervals, $S_1, S_3, \ldots$ are relaxation intervals. All S_{2i+1} ($i = 0, 1, \ldots, N$) have equal length M and duration of S_{2i} ($i = 0, 1, \ldots, N$) may be varied. Hankel matrix Y is formed from M measurements $y_{2i+1,1}, y_{2i+1,2}, \ldots, y_{2i+1,M}$ on each relaxation interval

$$Y = \begin{pmatrix} y_{11} & y_{31} & \cdots & y_{2N+1,1} \\ y_{12} & y_{31} & \cdots & y_{2N+1,2} \\ \vdots & \vdots & \vdots & \vdots \\ y_{1M} & y_{3M} & \cdots & y_{2N+1,M} \end{pmatrix}.$$

By analogy we form matrix X from vectors x_{2i+1} relevant to origin of each relaxation interval

$$X = \begin{pmatrix} x_{11} & x_{31} & \cdots & x_{2N+1,1} \\ x_{12} & x_{31} & \cdots & x_{2N+1,2} \\ \vdots & \vdots & \vdots & \vdots \\ x_{1M} & x_{3M} & \cdots & x_{2N+1,M} \end{pmatrix},$$

where $x_{2i+1,k}$, $k = \overline{1, \alpha}$ is a component of vector x_{2i+1}. Due to equations (13) and specifics of the matrix X, Y the following matrix equation may be written [13, 14]

$$Y = \Gamma_{M+1} \cdot X, \tag{15}$$

where $\Gamma_{M+1} = \begin{bmatrix} c^T \\ c^T A \\ \vdots \\ c^T A^M \end{bmatrix}$ is an observability matrix.

According to realization theory if $Y = Y_1 \cdot Y_2$ is any given full-rank factorization of the Hankel matrix, then Y_1 is the observability matrix and Y_2 is the matrix of initial state on the relaxation intervals for some state-space realization. Optimal way for realization of such factorization is to use SVD decomposition. Let the SVD of Y be given by

$$Y = Q\Sigma V^T, \tag{16}$$

where Q and V are orthogonal matrices and Σ is a diagonal matrix with singular values in non-increasing order on the diagonal. If the data generating system is n order, then in the ideal case (all y_m^{2i+1} are precisely given) Y is of rank n, so that only the n first singular values are non-zero provided that $\alpha \geq n, M > n, N > n$. But in reality instead of precise Y we would have a matrix $\widetilde{Y} = Y + Z$, where Y is a true matrix and Z is a matrix produced by perturbation of input and output noise. Then all singular values of matrix Σ will be positive. For multidimensional or infinite-dimensional systems condition number is evaluated as a ratio of singular values that grows exponentially when α incremented by one. So SVD decomposition (16) we write as follows

$$Y = Q_n\Sigma_n V_n^T + Q_r\Sigma_r V_r^T, \tag{17}$$

where $Q = (Q_n\ Q_r)$, $V = (V_n, V_r)$, $\Sigma = \begin{pmatrix} \Sigma_n & 0 \\ 0 & \Sigma_r \end{pmatrix}$.

In the framework of statistical approach for finite-dimensional system the first term corresponds to a "signal" and the second is a "noise". It is clear that $\Sigma_r = 0$ in the absence of errors. So the dimension of Σ_n defines first of all the system order n. In our case due to ill-conditioning of the matrix Y_n (first term) for large (or even for not large) n we consider partition (17) as regularization procedure. First term with maximal n defines the maximum order of approximate model when identification still gives the stable solution. Unlike to statistical approach the model order estimation depends on value $\left\| \widetilde{Y} - Y \right\|$. When $\left\| \widetilde{Y} - Y \right\| \to 0$, then following tendency holds $n \to n^*$, where n^* is an order of a model that corresponds to deterministic case. Value n^* is entirely defined by inevitable computational errors.

If partition in (17) is made correctly, which means the order of approximate model satisfy the stability condition, the next step is to find the model parameters. By setting $\Gamma_{n-1} = Q_n$ the matrix A for some realization can be obtained from the matrix equation

$$\Gamma_{1:M} \cdot A = \Gamma_{2:M+1}, \tag{18}$$

where $\Gamma_{2:M+1}$ is a submatrix derived from Γ_{M+1} crossing out the first row and $\Gamma_{1:M}$ is a submatrix derived from Γ_{M+1} crossing out the last row.

In non-singular case system in (18) is overdetermined and a least-squares or a total least-squares methods to solve the system with respect to A can be used. After obtaining matrix A from (18) we find its eigenvalues that allows to write matrix A in Jordan realization. For canonical controllable realization vector b is given and only vector c should be found. When canonical observable realization is preferable, then vector b should be found with given vector c. With given A and b from first equation in (13) and (14) it is easy to calculate x_t in each point t of the observation interval. Then the second equation in (13) allows to find vector c from overdetermined linear system of equations with y_t measured in experiment for the same input. Here we remark that problem estimation of c is more stable in comparison with determination of the matrix A eigenvalues. It means that well-posedness of the identification problem under consideration is completely defined by the splitting procedure (17) that is choise of term $Q_n \Sigma_n V_n^T$. Multiple numerical experiments show that condition number depends on n in just the same way as it reports in Fig. 1.

Stability property in dependence on n was studied by numerical modeling. Two main parameters define the solution stability, namely, $\varepsilon = \left\| \widetilde{Y} - Y \right\|$ and condition number κ_2 for spectral norm of the matrix $Q_n \Sigma_n V_n^T$, i.e. $\kappa_2(Q_n \Sigma_n V_n^T) = \dfrac{\sigma_1}{\sigma_n}$ where σ_1 is the first and σ_n is the last singular values of the matrix Σ_n. Instead of regularization parameter n we choose κ_2 on the same reason that was pointed out for frequency method. So the plane of these two parameters ε and κ_2 was used for demonstration of stability result obtained in multiple experiments with different system structures and parameters generating the data.
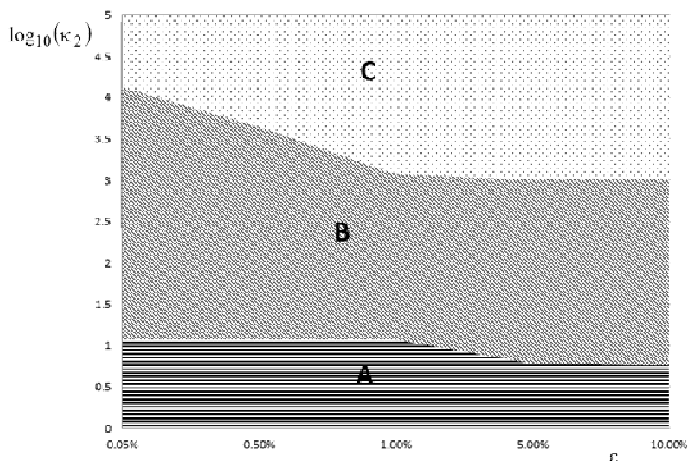

Fig. 3. Solution stability domains for subspace identification method

Domain A in Fig. 3 conforms to stability and C is unstable region. Between them transition zone B is located where part of the identified models are stable and the other are unstable. Here as in Fig. 2 the additional stability test is required

to use, for example set of data obtained by randomization. Fig. 2 and Fig. 3 can be used for model construction of reduced order or maximal order assumed by stability condition.

CONCLUSIONS

Identification problems of infinite or multidimensional systems may be fully treated as essentially ill-posed. Because of using only measured in experiment inaccurate data in many cases we not only look for a solution of the operator equation but also an operator itself. Hence the problem arises how to write the model set that includes true model of a plant. In such situation we propose in capacity of model set to represent it in the form of infinite expansion and to find finite approximation consistent with errors in available data using for this regularization. In other words the identification problem in such setting is always ill-posed. It is shown that this property is fundamental regardless what kind of method is applied.

It should be pointed out that approximate model obtained in identification is similar to real system with respect to output only. Moreover model parameters (eigenvalues and others) can essentially differ from the same parameters of the real system. For example, in most experiments each model eigenvalue represent whole cluster of system eigenvalues, i.e. is determined as averaged estimation.

It is also worse mentioning also about ill-posed identification of the finite-dimentional systems. All systems generating data with order corresponding to domain C in Fig, 2 and Fig. 3 lead to ill-posed identification. For such systems only approximate model of less order than at original system may be reconstructed.

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