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CURRENT-CARRYING AND ELECTROSTATIC PLASMA-ELECTRON LENSES CONTROLLED BY THE EXTERNAL PROGRAMMED MAGNETIC FIELD

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1. Introduction

At present there is a significant requirement for the development of advanced devices for focusing of intense ion beams of middle and high energies for actual scientific and technological problems (inertial thermonuclear fusion on heavy and light ions, radiotherapy, high energy investigations, research of radiation resistance of materials, implantation metallurgy, etc.). Last time the attention was paid to elaboration of plasma lenses with large focusing force and charge compensation of the focused beams (e.g., see the review [1]). Under conditions of energy and current growth of accelerated beams, plasma lenses should replace the conventional ones. Several types of these lenses are known [1]: Gabor electron lens, Morozov electrostatic plasma lens, current-carrying (magnetic) plasma lens. Theoretical and experimental studies of the electrostatic plasma-electron lenses of Gabor-Morozov type were published in many works (e.g., see [2-6]. For high-energy ion beam focusing the magnetic plasma lenses were investigated (e.g., [7-11]). For focusing of ultra-high energy electron beams it was proposed and studied «passive» plasma lenses based on the magnetic self-focusing and wake-field focusing (e.g., [12-14]). Earlier, the mechanism of charge particles focusing by the fields that they radiate in a plasma was studied theoretically by V. B. Krasovitskii e.a. and experimentally by I.A. Soloshenko e.a.

In this work the studies of long uniform, and non-uniform modified adiabatic plasma lenses of different types are performed that intended for middle and high energy ion beam focusing. The advantages of these lenses are as follows. 1) One can minimize the spherical aberrations by the condition that the focusing fields forces are proportional to the focused particle deviation from the axis through the most part of the lens. 2) The force of the long plasma lens is greater on 1-2 orders then of the short one with the same parameters. 3) For the further enhancement of focusing efficiency of the non-uniform lenses, one can use the external magnetic field that changes along the device length so that the focusing channel radius was close to the focused beam radius, and it is decreased with beam radius decreasing. (At this consideration it is used the fact that focusing channel radius is inversely proportional to the square root of magnetic field strength. It should be noted that in the works [10, 13] the criterion of adiabatic property of plasma lenses was small changing of its parameters over focusing length (slight non-uniformity). In this work the another case is studied when that criterion is small changing of the adiabatic lens parameters on much smaller «cyclotron length» which is equal to the product of electron cyclotron frequency by longitudinal velocity of plasma electrons (the condition of the drift approximation). For this case plasma parameters change essentially on the focusing length (strong non-uniformity).

The common approach for the problem studying was developed, and it is described in the work for several kinds of such lenses. Under the term «lens», the long focusing channel will be understanded in this paper (i.e., «plasma focuser», as this term was used in [13]). In the Part 2 the long non-uniform magnetic (current-carrying) lenses are studied. The combined electrostatic-magnetic (charge-current) lens is described in the Part 3. The uniform and non-uniform long plasmaoptic focusing devices of Morozov type were investigated in the separate work [15].

2. Long non-uniform magnetic lens

Let us consider the problem of ion beam focusing by the azimuthal magnetic field of the longitudinal current in a plasma. We investigate the case that the current radius is determined by the external non-uniform longitudinal magnetic field. The problem is being solved at the paraxial approximation. In this case the equation of the magnetic surfaces is as follows:

$$a^2(z)=\frac{a^2(0)B_z(0)}{B_z(z)}, \qquad (1)$$

where $a(z)$ is the variable radius of the magnetic surface, $B_z(z)$ is the longitudinal magnetic field on the axis, $B_z(0)$ and $a(0)$ are determined by the boundary conditions at $z = 0$. We assume that in the case of the strong magnetic field the electrons which transport the current in plasma are moving along the cylindrical magnetic surfaces enclosed one into another. The boundary conditions are defined as it follows: at z=0, $a(0) = b$, where b is the radius of an electrode that supply the current in the plasma (e.g., it is the inner electrode of the plasma gun). From Eq.(1) it is follows: if the equidistantness of the magnetic surfaces is set in some cross-section, so it conserves in any other one. As a result, if the current density is homogeneous in the electron emitter region, so it will be homogeneous in any other current channel cross-section. It is necessary for focusing without spherical aberration, because Lorentz force F_m focusing an ion toward the axis is proportional to distance of the ion from the axis:

$$F_m=-\frac{e}{c}vB_\varphi=-\frac{ev}{c}\frac{2I}{cr}=-\frac{2evI}{c^2a^2(z)}r \qquad (2)$$

As a result, the equation for the focused ion trajectories will take the form:

$$r'' + k^2 \frac{B_z(z)}{B_z(0)} r = 0\,, \quad k^2 = \frac{2I\,e}{Mc^2 v b^2} \tag{3}$$

In Eqs.(2), (3), I is the current in plasma, e and M are the charge and mass of the ion (i.e., the proton), c is the light velocity, v is the ion velocity, $B_z(0)$ is the magnetic field intensity in the region of the plasma gun output, B_φ is the azimuthal magnetic field of the current.

Under condition $B_z(z)$ = const (or 0) from Eq.(3) we have: $r = r_0 \,\mathbf{cos}\, kz$, and the focusing distance in the plasma: $L_f = \pi / 2k$. For a lens of length $l < L_f$:

$$L_f = l + k^{-1} ctg(kl)\,,$$

whence at $kl << 1$ it is easy to receive the expression for a thin lens: $L_f = (k^2 l)^{-1}$.

In general case, the trajectories of focused particles are calculated with help of a computer . For some cases the Eq.(3) have analytic solution, e.g., for the «bell-liked» distribution of the magnetic field :

$$B_z(z) = B_z(0)\left[1 + (z/d)^2\right]^{-2} \tag{4}$$

In this case the Eq.(3) takes the form:

$$r'' + k^2 r\left[1 + (z/d)^2\right]^{-2} = 0 \tag{5}$$

The solution of the Eq.(5), that is known from the electron optics, can be written as it follows:

$$r = \frac{r_0}{\sqrt{1 + k^2 d^2}} \frac{\mathbf{sin}\left(\sqrt{1 + k^2 d^2}\ \mathbf{arcctg}\, z/d\right)}{\mathbf{sin}\left(\mathbf{arcctg}\, z/d\right)} \tag{6}$$

The coordinate of the ion beam focus corresponds to the condition r=0, and is defined by the expression:

$$z_f = d\,\mathbf{ctg}\frac{\pi}{\sqrt{1 + k^2 d^2}} \tag{7}$$

The calculations based on the above formulae show: due to compression of the current channel by the external magnetic field, the one order decrease of the focusing current can be reached. In this case, the focusing of intense proton beams (of MeV range energy) can be realized on the distance about 100 cm, in the steady state regime.

During the ion focusing and compression of the current channel by the magnetic field of a solenoid, some ions (with large injection radius) can move partly out of the current channel. They also deflected to the axis but not get to the common focus. The moving equation for them has the form:

$$r'' + \frac{\kappa}{r} = 0\,, \text{ where } \kappa = \frac{2eI}{c^2 Mv} \tag{8}$$

To put together all ions in the focus, it is needed an optimization of the external magnetic field distribution. For this aim we can determine the form of the magnetic surface that limit the current channel. Then we can calculate the parameters of the solenoid (for producing such magnetic surface) and determine the focusing ions trajectories. The calculation can be carried out for paraxial ion trajectories and paraxial magnetic surfaces where particles and magnetic force lines go through input and output butt-endes (faces) of a long cylindrical focusing device (“lens”).

The limiting magnetic surface is determined from the condition that its radius (R) coincides with the current channel radius (a) and the radius of the focused beam. The functions $R(z)$ and $B_z(z)$ are determined from the equation similar to (8):

$$R'' + \frac{\kappa}{R} = 0\,, \quad \kappa = \frac{2eI}{c^2 Mv} \tag{9}$$

The solution of the Eq.(9) (with initial conditions: $R = R_0\,, R' = R_0'$ at z=0) has the form:

$$z = \pm \int_{R_0}^{R} \frac{dr}{\sqrt{R_0'^2 - 2\kappa\,\mathbf{ln}\, R/R_0}} \tag{10}$$

Using the substitution:

$$t^2 = \frac{R_0'^2}{\kappa} - 2\,\mathbf{ln}\frac{R}{R_0} \tag{11}$$

and the definition of the tabulated function (the probability integral):

$$\Phi(R) = \sqrt{\frac{2}{\pi}} \int_0^R e^{-\frac{1}{2}t^2}\, dt\,, \tag{12}$$

we reduce the solution (10) to the form:

$$\begin{aligned} z = {} & \sqrt{\pi/2\kappa}\, R_0\, \mathbf{exp}\left(R_0'^2/2\kappa\right) \times \\ & \times \left[\Phi\left(\sqrt{R_0'^2/\kappa - 2\,\mathbf{ln}\left(R/R_0\right)}\right) - \Phi\left(R_0'/\sqrt{\kappa}\right)\right] \end{aligned} \tag{13}$$

In the case of the parallel ion beam injection, at z=0 we have $R_0' = 0$, besides, in the focusing region $z > 0$. As a result, the Eq.(13) takes the form:

$$z = \sqrt{\frac{\pi}{2\kappa}} R_0 \Phi_0\left(\sqrt{2\,\mathbf{ln}\frac{R_0}{R}}\right) \tag{14}$$

In the real experiment the current channel compression leads to the certain value R_g (not equal to zero) that corresponds to the coordinate z_g. At this place the current channel is finished (by a wire mesh or metallic foil). Later on the inertial focusing of ions (which are charge compensated) in the focal spot takes the place. This point’s coordinate is defined as follows:

$$z_f = \sqrt{\frac{\pi}{2\kappa}} R_0 \Phi_0\left(\sqrt{2\,\mathbf{ln}\frac{R_0}{R_g}}\right) + \frac{R_g}{\sqrt{2\kappa\,\mathbf{ln}\left(R_0/R_g\right)}} \tag{15}$$

The numerical calculations of the Eq.(8) with help of the PC give the result for z_f coinciding with the (15).

The results of calculations are being compared with the experimental ones [16] in the case of the 5 MeV proton beam focusing by the current-carrying plasma lens placed in the external non-uniform magnetic field. In the Fig. 1 the calculated proton trajectories are presented for the typical experimental conditions. In the Fig. 2 the calculated distribution of the proton current density is presented for the z-coordinate where the fluorescent screen had been placed. The calculated and measured sizes of the proton beam cross-section show the satisfactory coincidence.

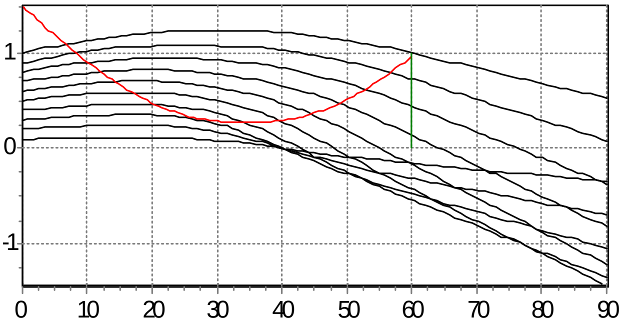

Fig. 1 The calculated proton trajectories for the typical experimental conditions: the proton beam energy is 5 MeV, the proton beam current is 10 mA, the initial proton beam radius r_b=1 cm, the beam divergence is 0.015 rad· r/r_b , the focusing current is 1 kA, the initial current channel radius is 1.5 cm, the final current channel radius is 1.0 cm, the wire mesh cathode coordinate is 60 cm, the solenoid face coordinate is 25 cm, the solenoid length is 19 cm, the solenoid inner radius is 7.5 cm, the solenoid outer radius is 12.5 cm. The parabola type curve presents the current channel boundary. The horizontal and vertical scales are given in *cm.*

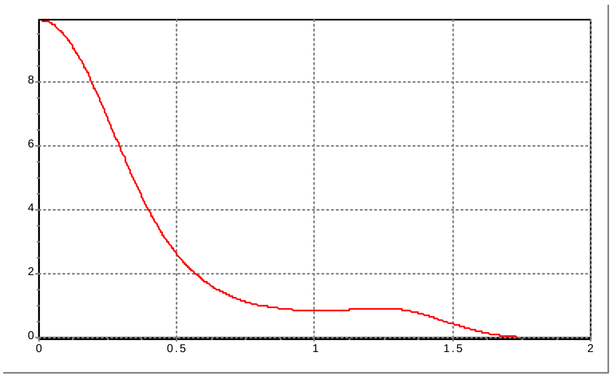

Fig. 2. The calculated proton current density (mA/cm^2) versus the radius (cm) for the fluorescent screen coordinate.

In conclusion of this part, we add the following remark. For vacuum magnetic lenses the focusing length $L_f \propto \kappa^{-2}$, but for short magnetic plasma lenses $L_f \propto \kappa^{-1}$, i.e., it is much less. In this work it is shown that for long plasma magnetic lenses $L_f \propto (\kappa^{-1})^{1/2}$, i.e., it is some more less. The suitable compression of the focusing channel gives additional gaining of several times over.

3. Combined electrostatic-magnetic lens

In this Part we are studying briefly the case of ion beam focusing by the counter-stream intense electron beam. As it is known, the electron concentration in such beams can reach to $10^{12} – 10^{13}$ cm $^{-3}$, therefore its using for ion beam focusing can have good prospects.

The expression for the focusing force have the form:

$$F_r = F_e + F_m = -2\pi n e^2 r - 2\pi e j v c^{-2} r\,, \qquad (16)$$

where n is the electron concentration, j is the current density. Let us to express n and j by the current I, radius a and velocity v_e of the electron beam:

$$n(z) = \frac{I}{e v_e \pi a^2(z)}\,,\ \ j(z) = \frac{I}{\pi a^2(z)} \qquad (17)$$

The expression for the focusing force takes the form:

$$F_r = -\frac{2eI}{a^2(z) v_e}\left(1 + \frac{v v_e}{c^2}\right) \qquad (18)$$

The equation for focused ion trajectories (with account of Eq.(1)):

$$r'' + \frac{B_z(z)}{B_z(0)} k_e^2 r = 0\,,\ k_e^2 = \frac{2eI\left(1 + v v_e c^{-2}\right)}{M v^2 v_e a_0^2} \qquad (19)$$

In the uniform or non-uniform cases it can be used similar methods and formulae (with the own k_e) as it have considered above in the case of the magnetic lens.

The uniform and non-uniform long plasmaoptic focusing devices of Morozov type were investigated in the separate work [15].

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