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ION CYCLOTRON RESONANCE FOR FAST MAGNETOSONIC WAVES IN ADIABATIC TRAPS

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The absorption of the electromagnetic waves in low-pressure plasma has been investigated in the axial-symmetric adiabatic trap under the cyclotron-resonance condition. The plasma radius is assumed to be greater than or equal to, to the order of magnitude, the transverse wavelength. In this case the solution to the linearized kinetic equation for resonance ions has been obtained as a power series in ion Larmor radius of the order of ρ_i^{2n-2} as well as the expressions for the contribution from resonance ions into the wave electric current density and RF power absorbed by the plasma (n is the resonance multiplicity).

1. INTRODUCTION

Cyclotron resonance $\omega = n\omega_{ci}$ (at the fundamental (n=1), as well as the second (n=2) harmonic of the ion cyclotron frequency) is applied successfully for heating plasma in tokamaks (see. e.g., [1]). It was also suggested to apply the resonance at the second harmonic for heating plasma in a gas dynamic trap [2]. This resonance can also be applied for separating isotopes [3]. With the growth of the resonance multiplicity n the absorption of the RF power decreases. But it might be important for the traps with the sufficiently large pressure $\beta = 8\pi n_{0i}T_i/B_0^2 < 1$. Straight systems [2,3], as compared with tokamaks, usually possess comparatively small transverse dimensions of plasma a_p, it being less than or order of the transverse wavelength of fast magnetosonic waves $\lambda\!\!\!^{-}_{\perp} \sim \omega/V_A \sim c/\omega_{pi}$ or the characteristic size over which the electric field of forced oscillations generated by the antenna located outside the plasma varies. The cyclotron resonance $\omega = n\omega_{ci}$ in such straight systems was considered in paper [4] for the uniform plasma cylinder (for n≥2), and for n=2 in paper [5] for the plasma cylinder with nonuniform density and temperature, and in paper [2] for the nonuniform cylinder in the nonuniform longitudinal magnetic field.

The present report considers the arbitrary multiple resonance in a straight plasma cylinder with the nonuniform density and temperature in the uniform axial magnetic field. Plasma pressure is assumed to be small compared with the magnetic one. In this case the ion Larmor radius is small compared with $\lambda\!\!\!^{-}_{\perp}$, and therefore for solving the kinetic equation for the perturbed distribution function the perturbation theory is used. The expressions for current density of resonant ions and the RF power absorbed by plasma are obtained.

2. EQUILIBRIUM STATE

We will assume that in the equilibrium state the plasma is immersed in the uniform axially symmetric magnetic field $\mathbf{B_0}$. In this case the ion distribution function will be uniform along the Z-axis (the axis OZ || $\mathbf{B_0}$). The radial electric field is assumed to be absent. Then, introducing the cylindrical coordinates (r,ϑ,z) and ($v_\perp$,φ,v_z), we find from the condition of equilibrium

$$v_x \frac{\partial F_{0i}}{\partial x} + v_y \frac{\partial F_{0i}}{\partial y} - \omega_{ci}\frac{\partial F_{0i}}{\partial \phi} = 0 \qquad (1)$$

where $v_x = v_\perp \cos\phi$, $v_y = v_\perp \sin\phi$, $x = r\cos\vartheta$, $y = r\sin\vartheta$, that the equilibrium distribution function F_{0i} depends on the particle energy $\varepsilon = \frac{m_i}{2}v^2$, $v^2 = v_\perp^2 + v_\parallel^2$, and the second integral of motion

$$X = r^2 + v_\perp^2/\omega_{ci}^2 + 2r(v_\perp/\omega_{ci})\sin(\phi - \vartheta) \qquad (2)$$

3. PERTURBED ION DISTRIBUTION FUNCTION

Let us develop the electric field of the electromagnetic perturbation generated by the antenna in which the electric current varies with the frequency ω, in the Fourier integral

$$\mathbf{E}(r,\vartheta, z, t) = \mathrm{Re}\iint dk_x dk_y\, \mathbf{E}(k_x,k_y)\, e^{i(k_x x + k_y y)},$$

where $\mathbf{E}(k_x,k_y) \sim \exp(ik_\parallel z - i\omega t)$. Then, integrating the Vlasov equation along unperturbed trajectories [6], we find that the perturbed ion distribution function is determined by the real part of the expression

$$\tilde{f}(r,\vartheta,z,v_\perp,\phi,v_z,t)=-\frac{i}{2}e_i\iint dk_x dk_y e^{i(k_x x+k_y y)}\times$$
$$\times\frac{\partial F_{0i}}{\partial\varepsilon}v_\perp\sum_{n,n'}J_n J_{n'}e^{i(n-n')(\phi-\psi)}\times \quad (3)$$
$$\times\left\{e^{-i\phi}R_{n+1}(E_x+iE_y)+e^{i\phi}R_{n-1}(E_x-iE_y)\right\}$$

where $R_n=(\omega-k_\parallel v_\parallel-n\omega_{ci})^{-1}$, $\psi=\mathrm{arctg}\frac{k_y}{k_x}$, $J_n=J_n\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right)$ is the Bessel function, $k_\perp=\sqrt{k_x^2+k_y^2}$. On obtaining (3), we have neglected the small component of the electric field E_z and kept in (3) the transverse components of the electric field $E_{x,y}(k_x,k_y)$.

4. DENSITY OF RESONANT ELECTRIC CURRENTS

As the contribution of the resonant part of the current density of resonant ions into the RF field absorption is determined by its left-hand polarized component

$$j_x+ij_y=e_i\int(v_x+iv_y)\tilde{f}\,d\mathbf{v}, \quad (4)$$

which, as is known, is proportional, with the account of the expression (3), to the small parameter $J_{n-1}^2\sim(k_\perp v_\perp/\omega_{ci})^{2n-2}$, the right-hand polarized component of the current is $(\omega_{ci}/k_\perp v_{Ti})^2$ times less than the left-hand polarized component, then in the expression for $F_{0i}(\varepsilon,X)$ one also has to retain in the series in powers of $(v_\perp/\omega_{ci})$ the necessary number of terms. Therefore we develop $F_{0i}(X)$ with the account of small terms $\sim v_\perp/\omega_{ci}$ in (3) in the series

$$F_{0i}(X)=\sum_{p=0}^{\infty}\sum_{j=0}^{p}e^{-i\frac{\pi}{2}p}(-1)^j\frac{1}{p!}\left(\frac{v_\perp}{\omega_{ci}}\right)^p\times\frac{\partial^p F_{0i}(X_0)}{\partial X_0{}^p}\times$$
$$\times C_p^j(x-iy)^{p-j}(x+iy)^j e^{i(p-2j)\phi} \quad (5)$$

where $C_p^j=\frac{p!}{j!(p-j)!}$ is the binomial coefficient, $X_0=r^2=x^2+y^2$.

Let us choose as F_{0i} the Maxwellian distribution with the density and temperature depending on the integration constant X, $F_{0i}\sim\frac{n_{0i}(X)}{(2\pi)^{3/2}m_i^{3/2}T_i^{3/2}(X)}\exp(-\varepsilon/T_i(X))$.

Then inserting the development (5) into (3) and performing in (4) integration in the velocity space over the azimuth angle ϕ, one can prove accounting for the small parameter $(k_\perp v_{Ti}/\omega_{ci})<<1$, that the terms with j=0 make the largest contribution to the sum (5). Keeping these terms and performing the integration over $v_\perp$ and $v_\parallel$, we obtain for (j_x+ij_y) the expression

$$j_x+ij_y=\sqrt{\frac{\pi}{2}}\frac{n}{2^n n!}\sum_{p=0}^{n-1}e^{-i\frac{\pi}{2}p}\binom{n}{p}(x-iy)^p\left(\frac{1}{r}\frac{\partial}{\partial r}\right)^p\frac{\omega_{pi}^2}{|k_\parallel|v_{Ti}}\times$$
$$\times\rho_i^{2n-2}W(z_n)\iint dk_x dk_y e^{ik_x x+ik_y y}k_\perp^{2n-2-p}(E_x+E_y)e^{ip\psi} \quad (6)$$

where $\binom{n}{p}=\frac{n(n-1)...(n-p)}{p!}$, $\omega_{pi}^2=\frac{4\pi e_i^2 n_{0i}(r)}{m_i}$, $W(z)=e^{-z^2}\left(1+\frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right)$, $z_n=\frac{\omega-n\omega_{ci}}{\sqrt{2}|k_\parallel|v_{Ti}}$, $v_{Ti}=\sqrt{\frac{T_i}{m_i}}$, $\rho_i=\frac{v_{Ti}}{\omega_{ci}}$.

The operator $\left(\frac{1}{r}\frac{\partial}{\partial r}\right)^p$ acts on the functions $n_{0i}(r)$ and $T_i(r)$.

Now we can transform from cartesian to cylindrical coordinates. Let us take into account that $x-iy=e^{-i\vartheta}r$ and

$$k_\perp^{2n-2-p}\to(-1)^{n-1-p}\Delta_\perp^{n-1-p}k_\perp^p$$
$$k_\perp^{2n-2-p}e^{ip\psi}\to(-1)^{n-1-p}\Delta_\perp^{n-1-p}(k_x+ik_y)^p\to \quad (7)$$
$$\to(-1)^{n-1-p}\Delta_\perp^{n-1-p}L^p$$

where there are introduced the operators acting on the electric field of the wave

$$\Delta_\perp=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\vartheta^2},$$
$$\hat{L}=-i\frac{\partial}{\partial x}+\frac{\partial}{\partial y}=e^{i\vartheta}\left(-i\frac{\partial}{\partial r}+\frac{1}{r}\frac{\partial}{\partial\vartheta}\right) \quad (8)$$

Then we ultimately get for $j^+=j_r(r,\vartheta)+ij_\vartheta(r,\vartheta)=e^{-i\vartheta}(j_x+ij_y)$, that

$$j^+=\sqrt{\frac{\pi}{2}}\frac{n}{2^n n!}\sum_{p=0}^{n-1}e^{-i\frac{\pi}{2}p}\binom{n}{p}r^p\left(\frac{1}{r}\frac{\partial}{\partial r}\right)^p\times$$
$$\times\frac{\omega_{pi}^2}{k_\parallel v_{Ti}(r)}\rho_i^{2n-2}W(z_n)e^{-i(p+1)\vartheta}\Delta_\perp^{n-1-p}\hat{L}^p e^{i\vartheta}E^+(r,\vartheta) \quad (9)$$

where $E^+(r,\vartheta)\equiv E_r(r,\vartheta)+iE_\vartheta(r,\vartheta)=e^{-i\vartheta}(E_x(r,\vartheta)+iE_y(r,\vartheta))$ is the left-hand polarized component of the electric field.

CONCLUSIONS

For the uniform plasma cylinder there remains in (9) only one term with p=0, in this case the expression (9) coincides with one given in paper [4] for axially symmetric waves. For the resonance at the second harmonic n=2 one should keep in (9) the terms with p=0 and p=1, in this case expression (9) coincides with the expressions given in [2,5].

To the order of magnitude, at $\lambda_\perp \sim \grave{a}_p$ the quantity j^+ and the RF power absorbed in plasma

$$P = \mathrm{Re}\int rdr \int d\vartheta j^+(r,\vartheta)\left(E^+(r,\vartheta)\right)^* \qquad (10)$$

are the same as in the uniform plasma. In this case the terms with $p \neq 0$, accounting for the nonuniformity of density and temperature, possess the same order of magnitude with the term $p = 0$.

The expression (9) may be used for numerical calculations of the excitation and cyclotron absorption of waves with antennas located outside the plasma.

For the resonant excitation when the frequency of currents in the antenna is close to the natural frequency of fast magnetosonic waves, $\omega \approx \omega^{(n)}(k_\parallel, m)$, where n is the radial number of the resonant mode, m is the azimuthal number, the field of the wave increases by $\omega^{(n)}/\gamma$ compared with the nonresonant case (γ is the damping rate), and the power absorbed in plasma increases by $\left(\omega^{(n)}/\gamma\right)^2$.

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