CALCULATING REDUCED ELECTRIC FIELD IN DIFFUSION REGIME OF DC DISCHARGE POSITIVE COLUMN

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The present paper outlines an analytical model of the positive column of the direct current discharge in a diffusion mode. We consider the case when charged particles are produced through direct ionization of gas molecules via electron impact and the ambipolar escape of them to discharge tube walls is the sole mechanism of their loss. We solved the equation for the charged particles balance and obtained simple formulas for the reduced electric field *E/p* in the positive column in molecular gases. Results of our calculations for *E/p* values in nitrogen are in good agreement with experimental and theoretical data of other authors for low discharge current values.

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INTRODUCTION

Direct current discharge in long tubes is widely applied for pumping gas discharge lasers, separating isotopes etc. The positive column connecting the cathode portions and the anode and closing the discharge circuit is the most important part for these applications. The reduced electric field *E/p* (where *E* is the electric field strength and *p* is the gas pressure) is the important parameter describing the positive column. Transport of charged particles (drift, diffusion) and their collisions (elastic and inelastic) with gas molecules depend just on *E/p*. Therefore it is of interest to obtain simple analytical formulas for the reduced electric field *E/p* in the positive column. The available analytical and numerical models are usually complicated and they are based on treating the electron energy distribution function (EEDF). However it is possible to construct an analytical model employing such electron transport parameters as mobilities and diffusion coefficients of electrons and ions as well as the first Townsend coefficient for ionizing gas molecules via electron impact. These transport coefficients are integral functions of EEDF, and their application simplifies analytical treatment considerably.

This paper reports our analytical model of the diffusion mode of the positive column when charged particles are produced through direct ionization of gas molecules via electron impact, and the rate of their production has to compensate the rate of their escape to the discharge tube walls due to ambipolar diffusion. It is assumed that the concentration of charged particles is not large therefore one may neglect the volume recombination of electrons and positive ions. Balancing the ionization rate and diffusion loss we get the equation for the reduced electric field *E/p*. We found the approximation formulas for different cases that helped us to get the analytical formulas for the reduced electric field. *E/p* is a decreasing function of the product of the gas pressure and discharge tube radius *pR*. These formulas contain mobilities and free diffusion coefficients of electrons and positive ions as well as the first Townsend coefficient. We calculated the *E/p* values in the broad *pR* range for nitrogen which are in good agreement with the results of experiments [1-4] and calculations [1, 3].

1. DESCRIPTION OF THE ANALYTICAL MODEL

In this paper we outline the analytical model of the uniform positive column in the diffusion mode in a discharge tube of radius *R* in detail. We consider a case when direct ionization of gas molecules through electrons moving in the uniform constant electric field *E* is the sole process of charged particles production. Here we neglect the processes in which metastable molecules participate (stepwise and associative ionization) as well as those of gas heating. We also neglect volume losses of charged particles such as recombination and attachment. Let the escape of charge particles to the discharge tube walls due to ambipolar diffusion be the sole process of their loss.

According to papers by Schottky [5, 6], in the uniform positive column of the dc discharge the ionization rate ν_i and the ambipolar diffusion coefficient D_a are related as follows

$$\frac{\nu_i}{D_a} = \frac{1}{\Lambda^2} = \left(\frac{2.405}{R}\right)^2, \tag{1}$$

where Λ is the diffusion length [7], $\nu_i = \alpha \cdot V_{dr}$, α is the first Townsend coefficient (the number of ionizing collisions performed by an electron while moving along 1 cm path in the electric field), V_{dr} is the electron drift velocity. For the description of the ionization of molecular gases through electron impact one often employs the following empirical formula for the α coefficient obtained already by Townsend [8, 9]

$$\frac{\alpha}{p} = A \cdot \exp\left(-\frac{B}{E/p}\right), \tag{2}$$

where the *A* and *B* constants depend on the gas species. For the electron drift velocity we write the formula $V_{dr} = \mu_e \cdot E$, where μ_e is the electron mobility. As Schottky [5, 6] demonstrated, the coefficient of ambipolar diffusion depends on the coefficients of free diffusion of electrons D_e and ions D_i, as well as on the mobility of ions μ_i and electrons μ_e:

$$D_a = \frac{D_e \cdot \mu_i + D_i \cdot \mu_e}{\mu_i + \mu_e}. \quad (3)$$

Let us take into account that the electron mobility exceeds much that of ions, $\mu_e >> \mu_i$, then expression (3) can be cast in the form:

$$D_a \approx D_i + D_e \cdot \frac{\mu_i}{\mu_e}. \quad (4)$$

Let us introduce the mobility and diffusion coefficients for electrons and ions at the gas pressure of 1 Torr (μ_{e1}, μ_{i1}, D_{e1} и D_{i1}) to find out the dependence of ambipolar diffusion coefficient on gas pressure, and then we get the following expression

$$D_a = \frac{1}{p} \cdot \left(D_{i1} + D_{e1} \cdot \frac{\mu_{i1}}{\mu_{e1}} \right), \quad (5)$$

whereas the relation for the electron drift velocity assumes the form $V_{dr} = \mu_{e1} \cdot E/p$. Let us insert the expressions for the first Townsend coefficient (2), the ambipolar diffusion coefficient (5) and the electron drift velocity V_{dr} into the equation (1) and introduce an additional variable $z = B/(E/p)$, then we obtain the following equation:

$$z \cdot \exp(z) = \frac{A \cdot B \cdot \mu_{e1}}{D_{i1} + D_{e1} \cdot (\mu_{i1}/\mu_{e1})} \cdot \frac{(pR)^2}{(2.405)^2}. \quad (6)$$

Since the function $F(z) = z \cdot \exp(z)$ does not allow analytical solving the equation (6) with respect to z, we have to choose a suitable approximation formula for $F(z)$ to get such a solution. To this end one has to know the range of z variation beforehand. As it follows from the results of experiments and calculations [1-4], the reduced electric field values in the positive column in nitrogen lie in the range $E/p \approx 20…80$ V/(cm·Torr). The book by Raizer [10] gives the value $B = 342$ V/(cm·Torr) for nitrogen (we shall refine this value below). Therefore we obtain that the solutions are within the limits $z = B/(E/p) \approx 4…17$. The complicated behaviour of the $F(z) = z \cdot \exp(z)$ function makes it expedient to consider it in two different ranges separately. In the range of high values, $z = 10…100$, the $F(z)$ function can be described by the following approximation formula:

$$F(z) = z \cdot \exp(z) \approx Fa(z) = 8.84 \cdot \exp(z^{1.006}). \quad (7)$$

In the range of moderate values, $z = 2…10$, it is convenient to apply the formula

$$F(z) = z \cdot \exp(z) \approx Fa(z) = 3 \cdot \exp(z^{1.05}) - 9. \quad (8)$$

From Fig. 1 it is clear that formulas (7) and (8) furnish a good description of the function $F(z)$.

Substitution of expression (7) into relation (6) enables one after simple transformations to get the expression for the reduced electric field E/p in the range $z = 10...100$:

$$\frac{E}{p} = B \cdot \left\{ \ln\left[\frac{A \cdot B \cdot \mu_{e1}}{8.84 \cdot [D_{i1} + D_{e1} \cdot (\mu_{i1}/\mu_{e1})]} \cdot \frac{(pR)^2}{(2.405)^2} \right] \right\}^{-0.994}, \quad (9)$$

and in the range $z = 2…10$ one can find similarly from relations (6) and (8) that

$$\frac{E}{p} = B \cdot \left\{ \ln\left[3 + \frac{A \cdot B \cdot \mu_{e1}}{3 \cdot [D_{i1} + D_{e1} \cdot (\mu_{i1}/\mu_{e1})]} \cdot \frac{(pR)^2}{(2.405)^2} \right] \right\}^{-0.952}. \quad (10)$$

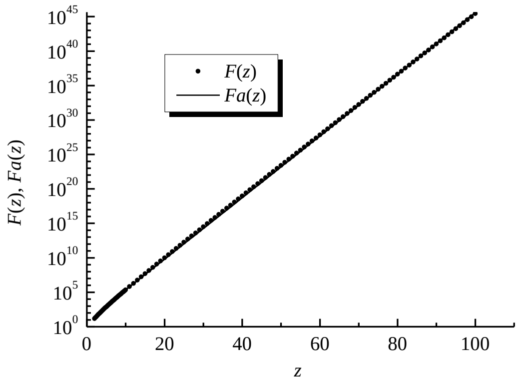

Fig. 1. F(z) function (points) and its approximation function Fa(z) (solid line) versus the ratio z = B/(E/p)

Relations (9) and (10) demonstrate, that in the positive column existing in the diffusion mode with direct ionization of gas molecules, the reduced electric field E/p is the sole function of the pR product depending on the gas species as well. Note also that to derive the relations (9) and (10) we applied minimum requirements to the gas species (direct ionization domination over step one, absence of strong attachment of free electrons to gas molecules), therefore these expressions can be used for E/p determination in the broad range of molecular gases.

2. CALCULATION DATA FOR NITROGEN

Let us now check the applicability of expressions (9) and (10) for describing the reduced electric field E/p in the positive column of the dc discharge in nitrogen. Nitrogen is an electropositive gas not producing negative ions. As it was shown in the paper [3], for low discharge current values, the direct ionization of molecules through electron impact dominates over step and associative ionization with the participation of metastable molecules, their concentration being small under these conditions. In this case the low concentration of charged particles enables one to neglect the dissociative recombination of electrons and positive molecular nitrogen ions as compared to their ambipolar escape to the tube walls.

Relations (9) and (10) contain the mobilities and diffusion coefficients of electrons and ions μ_{e1}, μ_{i1}, D_{e1} and D_{i1}. Electron mobility $\mu_{e1} = 4.2 \cdot 10^5$ cm^2 ·Torr/(V s) for nitrogen is determined from experimental values of the electron drift velocity V_{dr} presented in papers [11, 12]. From papers [13-16] we determine the coefficient of free electron diffusion $D_{e1} = 9 \cdot 10^5$ cm^2 ·Torr/s.

For positive nitrogen ions we employ in our calculations the following values of the mobility $\mu_{i1} = 1.54 \cdot 10^3$ cm^2·Torr/(V·s) and diffusion coefficient $D_{i1} = 39.7$ cm^2·Torr/s, which are taken from book [17]. To calculate the reduced electric field from expressions (9) and (10) we need A and B constants entering the formula for α/p (2). To find them we use the experimental data for

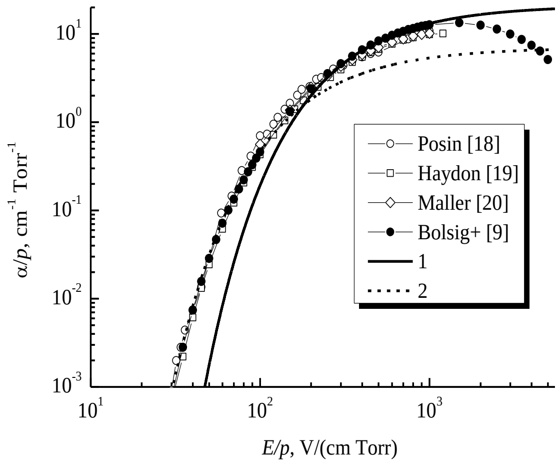

Fig. 2. Ratio α/p versus the reduced electric field E/p. Empty symbols correspond to experimental data from papers [18-20]. Filled symbols correspond to our calculation data using the Bolsig+ code [9]. Solid line is calculated according to formula (2) with the constants $A = 21\ cm^{-1}\cdot Torr^{-1}$ and $B = 469$ V/(cm·Torr) [9]. Broken line is calculated according to formula (2) with the constants $A = 7\ cm^{-1}\cdot Torr^{-1}$ and $B = 270$ V/(cm ·Torr)

the first Townsend coefficient α/*p* in nitrogen [18-20] in the broad range of the reduced field *E/p* = 30…1000 V/(cm·Torr). The book by Raizer [10] gives the values $A = 12\ cm^{-1}\cdot Torr^{-1}$ and *B* = 342 V/(cm ·Torr), but they provide a good description of the α/*p* pattern only in the range of large values *E/p* = 100 … 600 V/(cm·Torr). In paper [9], it was found that in the range of high values *E/p* = 200 … 1000 V/(cm·Torr) the α/*p* values demonstrate a good description by equation (2) with the constants $A = 21\ cm^{-1}\cdot Torr^{-1}$ and *B* = 469 V/(cm ·Torr) (see Fig. 2). However, as it was shown in papers [1-4], the reduced electric field values in the positive column in nitrogen are in the

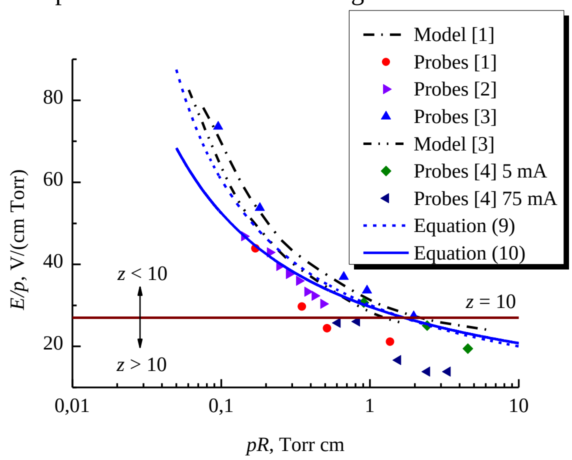

Fig. 3. Reduced electric field strength E/p versus pR product in nitrogen. Solid line depicts the calculation data according to formula (10). Broken line depicts the calculation data according to formula (9). Dots are the probe data from papers [1-4]. Dash-dot line is for data from the model [1], and dash-double dot line is for data from the model [3]

range *E/p* ≈ 20…80 V/(cm ·Torr). Therefore in this *E/p* range it is expedient to employ the constants $A = 7\ cm^{-1}\cdot Torr^{-1}$ and *B* = 270 V/(cm ·Torr).

In Fig. 3 we present our calculation data obtained from relations (9) and (10) as well as the measured data [1-4] and modelling ones [1, 3]. With the horizontal line we show the value *E/p* = 27 V/(cm ·Torr), for which we find *z* = *B*/(*E/p*) = 270/27 = 10. The values *E/p* for *z* < 10 have to be calculated with expression (10) whereas for *z* > 10 (lower reduced electric field values) one has to apply the expression (9). The dependence of the reduced electric field *E/p* versus the *pR* product we calculated is in good agreement with the registered data of papers [1-4] as well as with calculation data of papers [1, 3].

CONCLUSIONS

The present paper outlines the analytical model for the diffusion mode of the positive column of the dc discharge when the charged particles are produced under direct impact of electrons with gas molecules and lost due to ambipolar diffusion to discharge tube walls. The approximated expressions chosen enable us to solve the balance equation for charged particles and obtain simple expressions from which it follows that the reduced electric field depends on the product of the gas pressure and discharge tube radius ***pR*** as well as on the gas species. The results of our calculations for the positive column in nitrogen are in good agreement with the probe measurement data of papers [1-4] and calculated data of papers [1, 3].

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РАСЧЕТ ПРИВЕДЕННОГО ЭЛЕКТРИЧЕСКОГО ПОЛЯ В ДИФФУЗИОННОМ РЕЖИМЕ ПОЛОЖИТЕЛЬНОГО СТОЛБА РАЗРЯДА ПОСТОЯННОГО ТОКА

В.А. Лисовский, Е.П. Артюшенко, В.Д. Егоренков

Представлена аналитическая модель положительного столба разряда постоянного тока в диффузионном режиме. Рассмотрен случай, в котором заряженные частицы образуются вследствие прямой ионизации молекул газа электронным ударом, а единственным механизмом потерь является их амбиполярный уход на стенки разрядной трубки. Решено уравнение баланса заряженных частиц и получены простые формулы для приведенного электрического поля *E/p* в положительном столбе в молекулярных газах. Результаты наших расчетов для *E/p* в азоте хорошо согласуются с экспериментальными и теоретическими данными других авторов для случая низких разрядных токов.

РОЗРАХУНОК НАВЕДЕНОГО ЕЛЕКТРИЧНОГО ПОЛЯ В ДИФУЗІЙНОМУ РЕЖИМІ ПОЗИТИВНОГО СТОВПА РОЗРЯДУ ПОСТІЙНОГО СТРУМУ

В.О. Лісовський, К.П. Артюшенко, В.Д. Єгоренков

Представлено аналітичну модель позитивного стовпа розряду постійного струму в дифузійному режимі. Розглянуто випадок, коли заряджені частинки утворюються внаслідок прямої іонізації молекул газу електронним ударом, а єдиним механізмом втрат є їх амбіполярний вихід на стінки розрядної трубки. Розв'язано рівняння балансу заряджених частинок і отримані прості формули для зведеного електричного поля *E/p* у позитивному стовпі в молекулярних газах. Результати наших розрахунків для *E/p* в азоті добре узгоджуються з експериментальними і теоретичними даними інших авторів для випадку низьких розрядних струмів.