STABILITY OF AXIALLY–SYMMETRIC LOWER HYBRID MODES OF SINGLE COMPONENT ELECTRON PLASMA WITH ADDITIVE OF BACKGROUND GAS IONS

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The low-frequency spectrum of axially-symmetric modes of oscillations of the nonneutral plasma completely filling a waveguide is evaluated. Plasma consists of cold electrons and a small additive of ions produced by ionization of neutrals of background gas by electron impact. Ions are described by the equilibrium distribution function adequately taking into account the peculiarity of their formation. The spectrum of oscillations consists of the family of lower hybrid modes and families of "modified" of ion cyclotron (MIC) modes. MIC modes intersecting lower hybrid modes are unstable within a wide range of changing of fields and plasma density due to anisotropy of the ion distribution function.

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1. The misconception has established after paper [1] in the theory of non-neutral plasma that a diocotron mode is the only low-frequency electron mode capable to interact with ions. The low-frequency volume (plasma) modes have escaped from the investigation and experimental results were interpreted exclusively in a spirit of paper [1]. However the direct graphings carried in [2, 3], have shown that the frequencies of volume electron modes of the waveguide completely filled with nonneutral plasma, equal to upper hybrid (**UH**) and lower hybrid (**LH**) frequencies with the Doppler shift [4], also get into the low–frequency range (Fig. 1). It occurs when the Doppler shift caused by rotation of electrons in crossed fields, is compensated by hybrid frequency. When $m=1$, only lower hybrid modes can be low-frequency. When $m \geq 2$ – both lower hybrid, and upper hybrid modes can be low-frequency. Presence even a small additive of ions of background gas, which always are present in plasma, leads to interaction of electron and ion modes and, probably, instabilities of plasma. Frequency spectra of such plasma for modes having azimuth numbers $m=1, 2$ have been evaluated in [2, 3] .

The axially-symmetric lower hybrid modes ($m=0$) do not contain the Doppler shift, but also pass through the low-frequency region at small values of the problem parameter $q=2\omega_{pe}^2/\omega_{ce}^2$ and at values of q of the order of Brillouin value $q \approx 1$ (regions I and II in Fig. 1). In the present paper the frequency spectrum of axially-symmetric modes of oscillations ($m=0$) of nonneutral plasma are evaluated using the dispersion equation derived in [2, 3]. Spectrum is found within the entire range of allowable values of strengths of electric and magnetic fields, for magnetized and unmagnetized ions.

2. We consider plasma completely filling a metal waveguide of radius a and consisting of "cold" magnetized electrons, homogeneously distributed on radius, and a small additive of ions produced by ionization of atoms, molecules of background gas by electron impact. Equilibrium distribution function of such ions is equal [2, 3]:

$$F(\varepsilon_\perp, M, v_z) =$$
$$= \frac{N}{T_i}\frac{m_i}{\omega_{ci}} Y\ e\Psi\ a\ -\varepsilon_\perp\ \ \delta\ \varepsilon_\perp - \omega_{rot} M\ \ \delta\ v_z\ , \quad (1)$$

where $N=const$ is a density of ion production, Y is the Heaviside function, δ is the Dirac delta function, m_i, $\varepsilon_\perp$, M, v_z are the mass, transverse energy, generalized momentum, and longitudinal velocity of ions, ω_{ci} is the ion cyclotron frequency, $T_i = 2\pi/\Omega_i = const$ is the period of radial ion oscillations in the crossed fields, $\Omega_i = \ \omega_{ci}^2 - 4eE_r/m_i r\ ^{1/2} = const$ is the MIC frequency, the coefficient $\omega_{rot} \equiv -c E_r/(Br) = const > 0$, $\Psi(r)$ is the electric potential, that is supposed to be a quadratic function of the radius $\Psi(r) = \Psi(a)(r^2/a^2)$, $\Psi(a) > 0$. The radial electric field is determined by the space charge of electrons and ions, $E_r = 2\ \Psi(a)/a^2\ r =$ $= -\ m_e/2e\ \omega_{pe}^2 (1-f) r < 0$.

The dispersion equation of oscillations of the considered plasma, that is contained in papers [2, 3], has been solved numerically for azimuth number $m=0$. The other parameters have the same values, as in [2, 3]: the ion mass equal to that of a nitrogen atom ($m_i = 14$ amu, $m_e/m_i \approx 4\cdot 10^{-5}$), the oscillations are strongly elongated along cylinder axis, $k_z a = 0.1$ (k_z is the longitudinal wave number), the ion density is small, $f = N/n_e = 0.01$ (n_e is a density of electrons). The results are presented in Figs. 2, 3 as dependencies of normalized frequencies of oscillations ω/Ω_i versus parameter q.

3. The spectrum of oscillations consists of the family of lower hybrid modes (Fig. 2) and families of MIC modes. Due to the low ion density MIC modes are located in close vicinities of harmonics of MIC frequencies and in a scale of Fig. 2 they are undistinguishable from these harmonics. Only the mode LH_1 passing near

to the second harmonic perturbs the mode MIC_1 considerably (see Fig. 2), and both modes remain stable. The behaviour of modes is traced in a more large-scale Fig. 3. Because of limited article volume we give patterns of behaviour of modes only near to the first harmonic of MIC frequency ($\omega/\Omega_i \approx 1$).

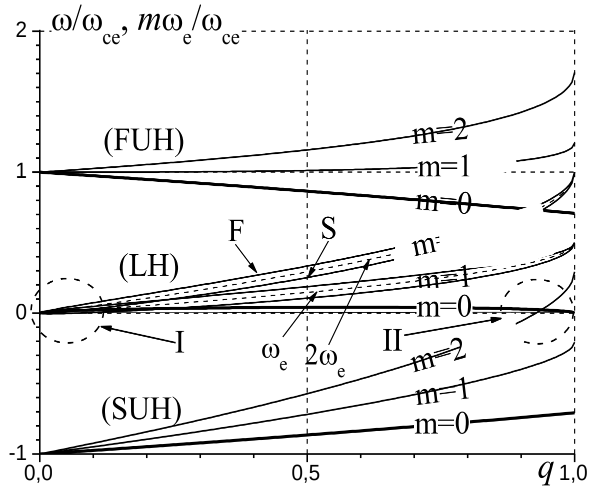

Fig. 1. Behaviour of fast (F) and slow (S) lower hybrid (LH) modes and upper hybrid (SUH, FUH) modes of single component electron plasma ($f = 0$) with azimuth numbers $m = 0, 1, 2$. In regions I and II electron modes are low-frequency

As it is seen from Figs. 2, 3,a; 3,b the first harmonic intersects with three radial lower hybrid modes – LH_0, LH_1, LH_2 – in regions I and II. In the region I the intersections with modes LH_0 and LH_1 occur in a weak radial electric fields ($q << m_e/m_i \approx 4\cdot 10^{-5}$) when ions of background gas are magnetized. The intersection with the mode LH_2 occurs in the field of the order of critical value ($q \sim m_e/m_i$) when ions are unmagnetized. In the region II intersections occur in extremely strong radial electric fields, of the order Brillouin value and even close to it.

Near to each harmonic of MIC frequency there are two MIC modes with the identical number of the radial mode. One mode is located above, another mode – below the harmonic (see Figs. 3,a; 3,b). MIC modes which are crossed with lower hybrid modes with the same radial number are unstable with slow growth rates within a wide range of changing of parameter q, located within a crossing interval.

This peculiarity of spectrum of MIC modes, their instability in a wide range of changing of parameter q are caused by anisotropy of the distribution function of ions of background gas (1). It shows itself only under the kinetic description of ions, both magnetized, and in a greater degree unmagnetized ions. For this cause the existence of instability of MIC modes in a wide range of fields changing could not be found out in [1] where the hydrodynamic description has been used for ions.

Radial mode MIC_2 has the fastest growth rate (both normalized, and absolute) in the region II near to the crossing with the radial electron mode (LH_2). It equals $\mathrm{Im}\,\omega \approx 0.004\Omega_i \approx 0.64\omega_{ci} \approx 0.9\cdot 10^{-2}\,\omega_{pi}$.

Near to the second harmonic of MIC frequency the mode MIC_1 in the region II has the fastest growth rate almost on two orders smaller, and in the region I – still on the order slower.

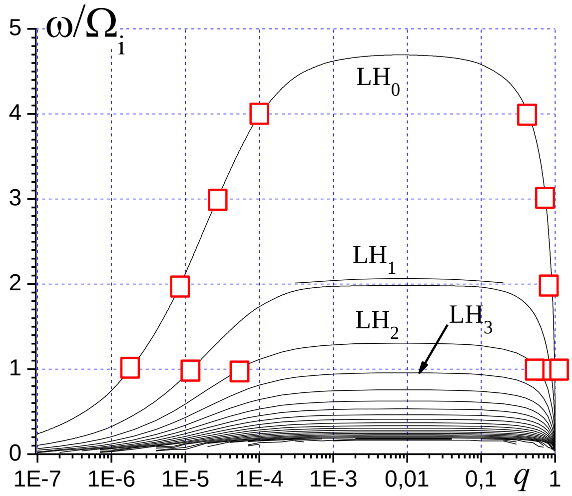

Fig. 2. Behaviour of the family of lower hybrid axially-symmetric modes. MIC modes on the scale of drawing coincide with harmonics MIC of frequency ($\omega/\Omega_i = n = 1, 2, 3, 4$). Square markers designate neighbourhoods of crossings of electron modes with MIC modes. Coefficients in labels of hybrid modes designate numbers of radial modes

Near to zero harmonic of MIC frequency ($\omega/\Omega_i \approx 0$) the ion modes are missing. Presence of ions leads to only small corrections to frequencies of electron modes.

4. Modes having $m = 0$ were observed in experiments with the electron beams propagating through residual gas (see, for example, [5]). Results of experiments were interpreted within the frame of the local theory. However ions of secondary plasma were unmagnetized in experiments and the local theory was inapplicable for their description. Bad accordance between a theory and experiment has been marked that has forced the author [5] to compare not spectra, but thresholds of excitation of unstable oscillations for which there has been better accordance.

During excitation of oscillations having asimuth numbers $m = 0, 1$ in the experiments described in [5], there was a multiple increase of transversal energy of secondary ions. It means that ions were in resonance ($\omega' \approx n\Omega_i$) with excited unstable MIC modes. When $m = 0$ - the frequency of mode ω is close to the first harmonic of MIC frequency ($n = 1,\ \omega \approx \Omega_i$), and when $m = 1$ the frequency ω' is close to a zero harmonic ($n = 0,\ \omega' = \omega - m\omega_+ \approx 0\cdot\Omega_i$), so the frequency ω is equal $\omega \approx m\omega_+ = -\omega_{ci} + \Omega_i\ /2$. Thus the frequency of mode with $m = 0$ appears greater than the frequency of mode with $m = 1$: $\omega(m = 0) > 2\omega(m = 1)$. The same correspondence takes place in an experiment (Fig. 3,b in [5]). Frequencies of modes are proportional to the MIC frequency Ω_i . The same dependencies of frequencies on the parameters of plasma and fields were observed in an experiment (Fig. 57 in [5]).

In general, experiments that were carried before 80th were interpreted as a rule on the basis of model of boundless plasma in cartesian geometry, i.e. plasma is not rotating, but is moving rectilineally. In such a model the relation takes place $\Omega_i = \omega_{ci}$, while the relation $\Omega_i >> \omega_{ci}$ is more often fulfilled in experiments. Such a model is unable to predict MIC modes excited in an experiment, and to interpret correctly the results of experiments. So, the interpretation of many such experiments must be revised in spirit of theory of nonneutral plasmas [4].

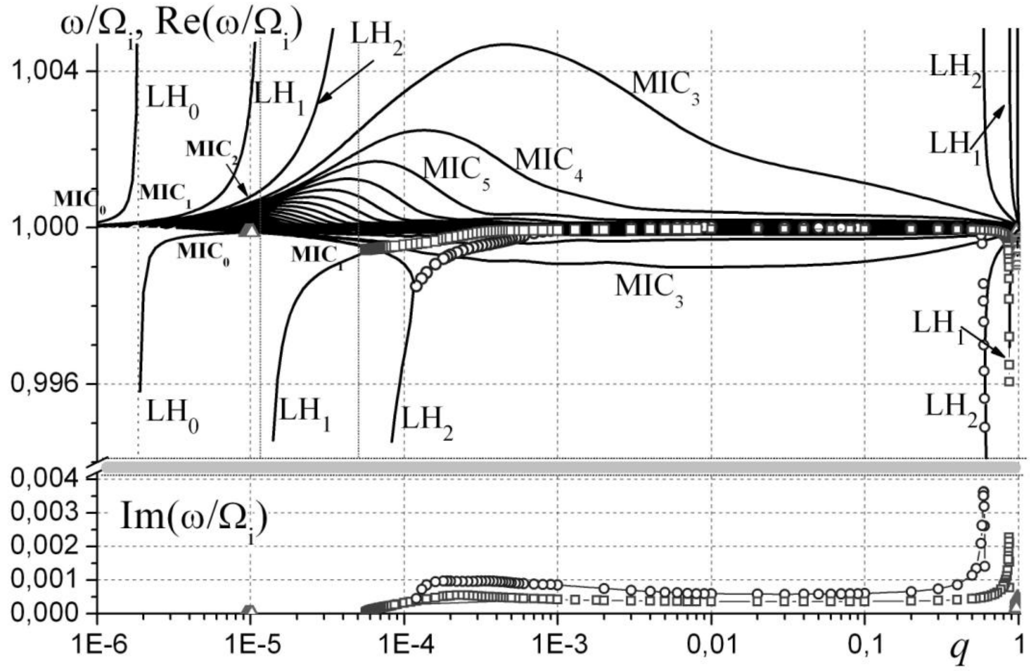

Fig. 3,a. Behaviour of frequencies and growth rates (in the bottom of Fig.) of modes near to the first harmonic of MIC frequency

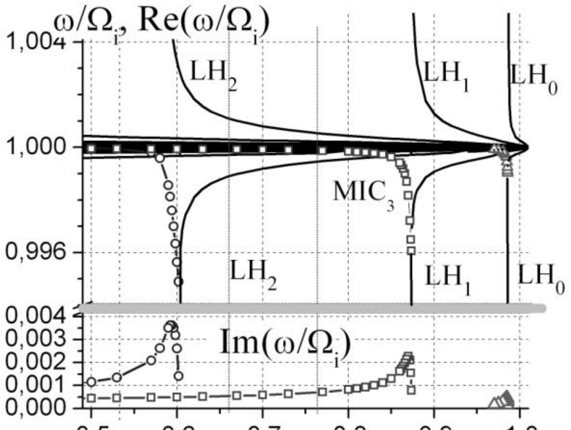

Fig. 3,b. The same, as in Fig. 3,a, but on the large scale in the region II ($q \sim 1$)

In [2, 3] and in the present paper stability of nonneutral plasma is considered within the framework of cylinder model, the adequate kinetic description of ions, produced by ionisation of neutrals of background gas by an electron impact, is given. It is valid for magnetized and unmagnetized ions, for long and short waves in longitudinal and transversal directions. It is applicable for interpretation of spectra of oscillations in these ([5]) and other similar experiments.

We will notice, secondary ions that appear as a result of charge exchange of ions of primary beam on neutrals of background gas [6], also form the distribution function (1), if the radial electric field is directed to the axis of ion beam (excess of electrons). Obtained in [2, 3] and in present paper results give description of such ions without any changes and determine their contribution to dispersion equation.

5. Not only ions can be unmagnetized, but also electrons. Such a situation takes place when a magnetic field is absent and the ion beam propagates through background gas, ionizes it, forming secondary plasma [7]. If the ion charge dominates in the beam channel, and mostly it so, secondary electrons move in a transversal plane on trajectories strongly elongated on radius, as well as considered ions of background gas. Therefore the used description of ions can be extended to these electrons.

Spectra of axially-symmetric oscillations of such plasma have been studied in [7] experimentally and theoretically. The kinetic equation for electrons has been solved by the same method, as in [1]. The solution has been obtained under strong restrictions on ω, k_z, electron temperature. When k_z is small, a reduction of an oscillation frequency has been found out in comparison with the electron Langmuir frequency (the formula (16) and Fig. 5 in [7]). According to authors opinion this phenomenon is caused by radial oscillations of electrons. Between theoretical and observational results [7] there is an appreciable discrepancy.

Obtained in [2, 3] expressions allow to determine the coefficient U of reduction of oscillations frequency due to radial oscillations, introduced in [7], for the model of homogeneous beam, completely filling a waveguide, without any restrictions used in [7]. It is determined by the summand in a diagonal matrix element A_{11} (see [2, 3]), in which it is necessary to put $m = p = 0$, that corresponds to the axially-symmetric mode and its low frequency ($\omega/\Omega_e << 1$). A coefficient of reduction of oscillation frequency is equal

$$U = \sqrt{I_z} k_z a \Big/ \left[(k_z a)^2 + \kappa_{0,1}^2 \right]^{1/2}, \qquad (2)$$

where $I_z \approx 1\big/ (N_0^1)^2 \int_0^1 x dx\, J_0^4 (\kappa_{0,1} x/2) \approx 1{,}9$, $\kappa_{0,1} \approx 2{,}4$, $(N_0^1)^2 = (J_1(\kappa_{0,1}))^2 / 2$.

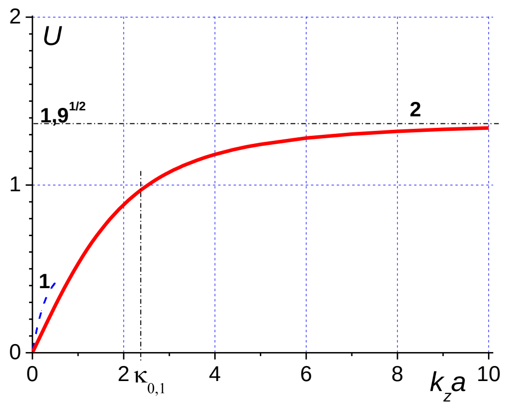

Fig. 4. The behaviour of the coefficients U of reduction of oscillation frequency due to radial oscillations: curve 1 is the theoretical result of paper [7], cirve 2 is based on formula (2) of present paper

Its behavior versus a parameter $k_z a$ is presented in Fig. 4 (curve 2). Here it is also presented the behavior of coefficient U, obtained theoretically in [7] (curve 1). Comparing to the experimentally measured curve dependence (see Fig. 4, 5 in [7]) testifies, that curve 2 correctly approximates behavior U both in the area of small $k_z a$ and in the area of $k_z a \gtrsim 1$. A conversion from one dependence to another takes place at $k_z a \sim 2 \sim \kappa_{0,1}$, as well as in an experiment [7]. However at $k_z a > \kappa_{0,1}$ coefficient $U > 1$, so that reduction of frequency due to radial oscillations of electrons does not take place in the considered model of homogeneous plasma completely filling a waveguide. During radial oscillations every particle submerges into the region, where the field of wave E_z is stronger. This increases the induced space charge of particles comparing to a case, when particles are magnetized and rotate, being on the same radius.

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УСТОЙЧИВОСТЬ АКСИАЛЬНО-СИММЕТРИЧНЫХ НИЖНЕГИБРИДНЫХ МОД ОДНОКОМПОНЕНТНОЙ ЭЛЕКТРОННОЙ ПЛАЗМЫ С ДОБАВКОЙ ИОНОВ ФОНОВОГО ГАЗА

Ю.Н. Елисеев

Численно определены низкочастотные спектры аксиально-симметричных мод колебаний заряженной плазмы, полностью заполняющей волновод. Плазма состоит из холодных электронов и малой добавки ионов, которые образовались ионизацией нейтралей фонового газа электронным ударом. Ионы описываются равновесной функцией распределения, адекватно учитывающей эту особенность их образования. Спектр состоит из семейства нижнегибридных мод и семейств «модифицированных» ионных циклотронных (МИЦ) мод. МИЦ-моды, которые пересекаются с нижнегибридными модами, неустойчивы в широком диапазоне изменения полей и плотности плазмы из-за анизотропии функции распределения ионов.

СТІЙКІСТЬ АКСІАЛЬНО-СИМЕТРИЧНИХ НИЖНЬОГІБРИДНЫХ МОД ОДНОКОМПОНЕНТНОЇ ЕЛЕКТРОННОЇ ПЛАЗМИ З ДОБАВКОЮ ІОНІВ ФОНОВОГО ГАЗУ

Ю.М. Єлісеєв

Чисельно визначені низькочастотні спектри аксіально-симетричних мод коливань зарядженої плазми, що повністю заповнює хвилевід. Плазма складається з холодних електронів і малої добавки іонів, які утворилися іонізацією нейтралів фонового газу електронним ударом. Іони описуються рівноважною функцією розподілу, що адекватно враховує цю особливість їх утворення. Спектр складається з сімейства нижньогібридних мод і сімейств «модифікованих» іонних циклотронних (МІЦ) мод. МІЦ-моди, які перетинаються з нижньогібридними модами, нестійкі в широкому діапазоні зміни полів і щільності плазми із-за анізотропії функції розподілу іонів.