EFFECTIVE GRAIN POTENTIAL IN A PLASMA WITH EXTERNAL SOURCES OF IONIZATION

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A new approach is proposed for analytical description of effective grain potentials in dusty plasmas. The basic idea is to describe absorption of electrons and ions by grain in terms of effective point sinks introduced into the equations of plasma dynamics. The proposed approach makes it possible to find explicit relations for the potential and particle densities distributions. The example of grain screening is considered for the case of weakly ionized plasma in the presence of the external sources of plasma ionization.
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1. INTRODUCTION

The problem of nonlinear grain screening still remains one of the most important issue of dusty plasma theory. It is of grate importance for description of such interesting phenomena as dusty crystal formation, excitation of dust acoustic waves, evolution of dust structures etc. In spite of the fact that the screened potentials have been studied during many years the problem of macroparticle screening still requires the further studies. First of all this concerns the grain screening in low-temperature plasma for which even the screening length is not determined in the general case [1]. The problem is that in the case of grain charging due the absorption of electrons and ions from plasma the effective potentials are crucially dependent on the processes in plasma background, in particular, on the details of plasma regeneration. In view of the importance of nonlinear effects and self-consistent description of charging processes the problem of grain (probe) screening usually is solving numerically [2-9]. Giving many details of the screening numerical results, however, can not be used for obtaining approximate analytical relations. At the same time it is very important to have such analytical approximations for the solution of the problems mentioned above. The purpose of the present paper is to work out analytical description of the screened potentials of grains charged by plasma current in low-temperature plasma.

We start from the formulation of the model (point-sink model) which provides the analytical solution of the problem, if the grain charge and the intensities of plasma particle fluxes are known (Section 2). Then we perform necessary analysis of the obtained relations and compare the results with the numerical solutions (Section 3). In Section 4 we apply the proposed model for treatment of screening in the collisionless Vlasov plasma.

2. POINT-SINK-MODEL. BASIC SET OF EQUATIONS FOR WEAKLY-IONIZED PLASMA

Let us consider single grain embedded into infinite weakly-ionized plasma. We assume that the grain absorbs all encountered electrons and ions. In the stationary case the grain charge is maintained by electron and ion fluxes which are equal to each other. In the case under consideration plasma dynamics can be described by the continuity equations which have the following form:

$$\operatorname{div}\Gamma_\sigma(r) = I_0 - \beta n_e(r) n_i(r)\,, \tag{1}$$

where

$$\Gamma_\sigma(r) = -e_\sigma \mu_\sigma n_\sigma(r)\nabla\Phi(r) - D_\sigma \nabla n_\sigma(r)\,, \tag{2}$$

μ_σ is the plasma particle mobility, D_σ is the diffusion coefficient, subscript σ ($\sigma=e$, i) labels plasma particle species, I_0 is the intensity of ionization sources, if present, β is the coefficient of the electron-ion recombination, the rest of notation is traditional.

The electric potential $\Phi(r)$ satisfies the Poisson equation

$$\Delta\Phi(r) = -4\pi\sum_\sigma e_\sigma n_\sigma(r)\,. \tag{3}$$

Eqs. (1) – (3) should be supplemented by the following boundary conditions

$$n_\sigma(r)\,|_{r=a} = 0, \qquad \frac{\partial\Phi(r)}{\partial r}\,\Big|_{r=a} = -\frac{q}{a^2}\,,$$
$$n_\sigma(r)\,|_{r\to\infty} = n_\sigma\,, \quad \Phi(r)\,|_{r\to\infty} = 0\,, \tag{4}$$

where q is the stationary grain charge which is determined by the equation

$$e_i\Gamma_i + e_e\Gamma_e\,|_{r=a} = 0\,, \tag{5}$$

a is the grain radius , n_σ is the unperturbed density of particles of σ species. The later quantities are determined by the conditions of plasma regeneration. They are given by the relation

$$n_e = n_i = \sqrt{I_0/\beta} \equiv n_0\,, \tag{6}$$

if external sources of ionization are present, or they are assumed to be given, if the sources of plasma regeneration are located at large distance from the grain.

In view of the nonlinearity of the basic set of equations it is usually solved numerically (see, for example, Refs. [2, 3, 5-9]). However, it is quite difficult to obtain analytical relations for the screened potentials and plasma particle distribution on the basis of the numerical solutions. Therefore, it would be highly desirable to have approximate analytical expressions for such quantities, at least for their asymptotic behavior. This problem can be solved taking into account that in many cases the nonlinearity produces considerable effects at small distances from the grain surface, only. Such conclusion follows from the analysis of the nonlinear numerical solutions for equilibrium screening (no particle fluxes through the grain surface [10]) and for the screening of grains charged by plasma currents in the collisionless plasmas [4]. Moreover, in the case of grains of small sizes the nonlinearity does not lead to considerable deviation from the linear solution even in the vicinity of the grain, but on the other hand, the plasma particle fluxes toward the grain contributes to the changes of the potential asymptotics [4]. Probably, it could be explained by the fact that in the case of grain absorbing plasma particles the electron and ion densities near the grain are small and thus the influence of nonlinearity is not well pronounced. The possibility to use linearized equation for the analytic description of the asymptotic behavior of the effective grain potentials in weakly-ionized plasma was also confirmed by the estimates presented in Ref. [11].

With regard to these arguments we propose to describe screened potentials on the basis of the linearized version of the point-sink model. In terms of such model it is assumed that the effects associated with plasma particle absorption by the finite-size grain can be satisfactory approximated by the effective point sinks, i.e. instead of Eq. (1) and the boundary conditions (4), (5) we propose to use the equation:

$$\mathrm{div}\,\Gamma_\sigma(r) = I_0 - \beta n_e(r) n_i(r) - S_\sigma \delta(r), \qquad (7)$$

where S_σ is the intensity of the point sink

$$S_\sigma = -\oint d\mathbf{s}\,\Gamma_\sigma . \qquad (8)$$

The linearized version of Eqs. (2) and (7) is the following:

$$-\frac{e_\sigma n_0}{T_\sigma}\Delta\Phi - \Delta\delta n_\sigma = -\frac{\beta_e n_0}{D_e}(\delta n_e + \delta n_i) - \frac{S_\sigma}{D_\sigma}\delta(r) \qquad (9)$$

$$\Delta\Phi = -4\pi(e_e\delta n_e + e_i\delta n_i) - 4\pi q\delta(r), \qquad (10)$$

where $\delta n_\sigma(r)$ is the density perturbation. We also take into account that in the absence of the grain the plasma density is given by Eq. (6) and the sink intensities for electrons and ions are equal to each other, i.e.

$$S_e = S_i = S . \qquad (11)$$

3. SCREENED POTENTIAL AND CHARGE DENSITY DISTRIBUTION

The solution of Eqs. (9), (10) can be easily obtained in the k-representation

$$\delta n_{ek}^{\circ} = \frac{1}{e\Delta}\left[-k^2(\tilde{S}_i + k_{Di}^2 q) - qk_0^4 - \tilde{S}_i k_{De}^2 - \tilde{S}_e k_{Di}^2\right], \qquad (12)$$

$$\delta n_{ik}^{\circ} = \frac{1}{e\Delta}\left[-k^2(\tilde{S}_e + k_{De}^2 q) + qk_0^4 - \tilde{S}_i k_{De}^2 - \tilde{S}_e k_{Di}^2\right], \qquad (13)$$

where

$$\begin{aligned} & e = |e_e| = e_i, \\ & \Delta = k^4 + k^2(k_D^2 + k_S^2) + 2k_0^4; \\ & k_0^4 = k_{Se}^2 k_{Di}^2 + k_{Si}^2 k_{Di}^2; k_S^2 = k_{Si}^2 + k_{Se}^2; \\ & k_{S\sigma}^2 = \frac{\beta n_0}{D_\sigma}, \qquad \tilde{S}_\sigma = \frac{eS}{D_\sigma}. \end{aligned} \qquad (14)$$

The potential Φ_k° is given by

$$\Phi_k^{\circ} = 4\pi\frac{q(k^2 + k_S^2) - k_D^2\tilde{S}}{k^4 + k^2(k_D^2 + k_S^2) + 2k_0^4}, \qquad (15)$$

where

$$\tilde{S} = \frac{eS}{k_D^2}\left(\frac{1}{D_i} - \frac{1}{D_e}\right). \qquad (16)$$

In its turn Eq. (15) can be rewritten as

$$\Phi_k^{\circ} = \frac{4\pi Q_1}{k^2 + k_1^2} + \frac{4\pi Q_2}{k^2 + k_2^2}, \qquad (17)$$

where

$$\begin{aligned} & Q_{1,2} = \frac{q(k_{1,2}^2 - k_S^2) + k_D^2\tilde{S}}{k_1^2 - k_2^2} \\ & k_{1,2}^2 = \frac{1}{2}\left(k_D^2 + k_S^2 \pm \sqrt{(k_D^2 + k_S^2)^2 - 8k_0^2}\right) \end{aligned} . \qquad (18)$$

In the coordinate representation

$$\Phi(r) = Q_1\frac{e^{-k_1 r}}{r} + Q_2\frac{e^{-k_2 r}}{r}, \qquad (19)$$

i.e. the effective potential is given by the superposition of two screened potentials with different screening lengths. Notice, that these screening lengths were obtained for the first time in Ref. [11]. At large distances the potential is determined by the term with the larger screening length. This result is in agreement with the numerical simulation [5-8].

The induced charge density is

$$\rho(r) = \tilde{Q}_1\frac{e^{-k_1 r}}{r} + \tilde{Q}_2\frac{e^{-k_2 r}}{r}. \qquad (20)$$

Here

$$\tilde{Q}_{1,2}=\frac{2qk_0^4-k_{1,2}^2k_D^2(q+\tilde{S})}{k_1^2-k_2^2}. \qquad (21)$$

If the plasma sources are located at large distance from the grain and recombination processes could be neglected ($\beta=0$)

$$\Phi(r)=(q+\tilde{S})\frac{e^{-k_Dr}}{r}-\frac{\tilde{S}}{r} \qquad (22)$$

and

$$\rho(r)=-k_D^2(q+\tilde{S})\frac{e^{-k_Dr}}{4\pi r}. \qquad (23)$$

As is seen, in the case under consideration the effective potential has the Coulomb-like asymptotics, but the induced charge is determined by the screened part. The quantity $\tilde{S}$ can be treated as an effective charge which generates the Coulomb part of the potential. It can not be found within the present theoretical treatment. Therefore, in what follows we take this quantity from the numerical solution (see Fig. 1), which is taken from the numerical data obtained in Ref. [6], assuming that $\tilde{S}$ is proportional to real grain charge, i.e. $\tilde{S}=-\alpha q$.

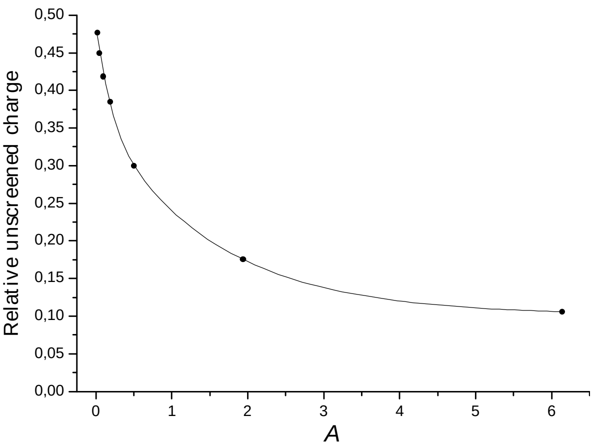

Fig.1. Dependence of dimensionless effective unscreened charge $-\tilde{S}/q$ on dimensionless grain radius $A=ak_D$

To compare the obtained results with the numerical simulation [6] it is convenient to use the following quantity

$$Q(r)=q+\int_0^r dr'\rho(r'), \qquad (24)$$

which is the total charge distributed in the sphere of the radius *r*. Substitution of Eq. (23) into Eq. (24) gives

$$Q(r)=\alpha q+q(1-\alpha)e^{-k_0r}(1+k_Dr). \qquad (25)$$

The dependences of the quantity *Q(r)* on the dimensionless distances $R_a=r/a$ for different grain sizes are presented in Fig. 2. They are in a good agreement with those obtained in Ref. [6]. The accuracy of the analytical estimates is seen from Fig. 3 in which the charge distributions *Q(r)* calculated analytically (thin curve) and numerically (bold curve) are presented. As is seen, in the case under consideration (β=0, *I*=0) at large distances *Q(r)* approaches some constant values which are nothing but the dimensionless effective charge α.

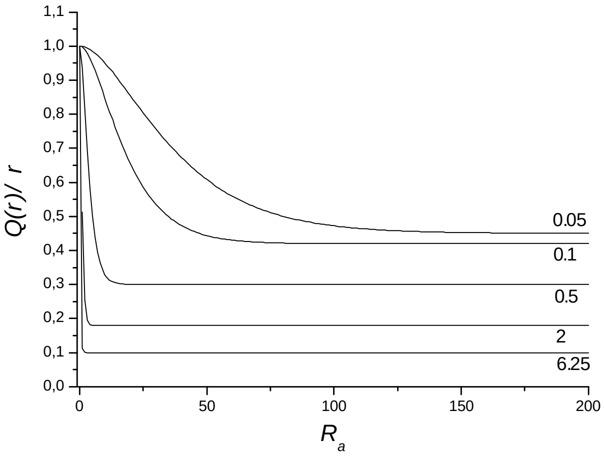

Fig.2 Relative charge distribution corresponding to Eq. (25), at various values of A (0.05, 0.1, 0.5, 2, 6.25)

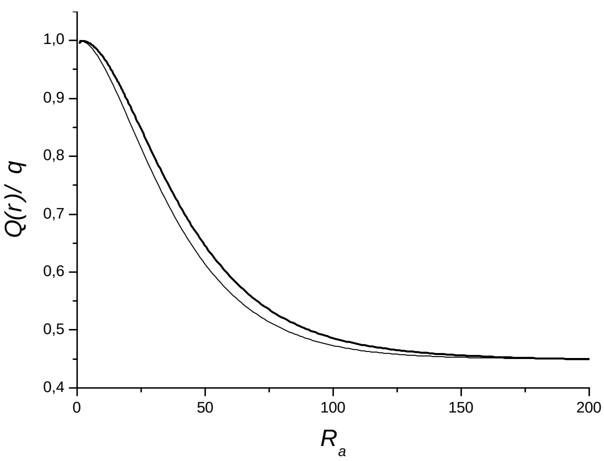

Fig.3. Relative charge distribution at A=0.05, bold line presents the data from [6], thin line corresponds to Eq. (25)

The situation is considerably changed, if the ionization sources are present. In this case

$$Q(r)=q+\frac{\tilde{Q}_1}{k_1^2}\left[1-e^{-k_ir}(1+k_ir)\right]-\frac{\tilde{Q}_1}{k_2^2}\left[1-e^{-k_Dr}(1+k_2r)\right]. \qquad (26)$$

Dependencies of this quantity on the dimensionless distance for the different values of ionization intensity are presented in Fig. 4. In this case the analytical estimates also are in a good agreement with the exact solution of the nonlinear equations (see, Fig. 5).

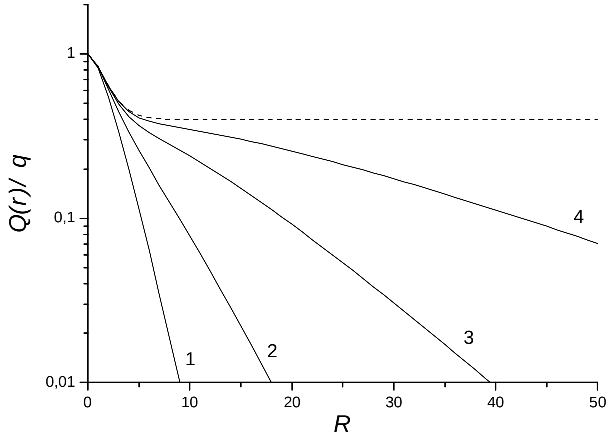

Fig.4. Relative charge distribution at A=0.158 and various values of $i_0 = Ia^5/D_i$ (1) $1.15\cdot10^{-2}$, (2) $2.5\cdot10^{-3}$, (3) $5\cdot10^{-4}$, (4) 10^{-4}, dashed line corresponds to the case of absence of bulk ionization and recombination

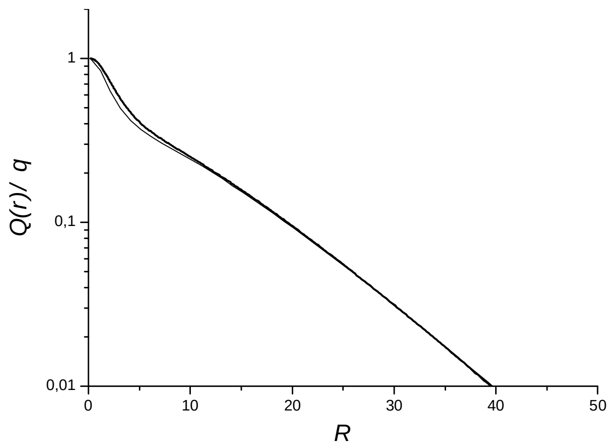

Fig.5. Relative charge distribution at A=0.158 and $i_0=5\cdot10^{-4}$, bold line presents the data from [6], thin line corresponds to Eq. (26)

4. POINT-SINK-MODEL FOR COLLISIONLESS PLASMA

In the case of collisionless plasma its dynamics is described by the Vlasov equation. With regard to the presence of the point-sink the stationary version of this equation has the form

$$\left(\vec v\frac{\partial}{\partial \vec r} - \frac{e_\sigma}{m_\sigma}\nabla\phi(\vec r)\frac{\partial}{\partial \vec v}\right) f_\sigma(\vec r,\vec v) = -\delta(\vec r)\sigma_\sigma(v) v f_\sigma(\vec r,\vec v), \qquad (27)$$

where $\sigma_\sigma(v)$ is the charging cross-section

$$\sigma_\sigma(v) = \pi a^2\left(1-\frac{2qe_\sigma}{mv^2a}\right)\theta\left(1-\frac{2qe_\sigma}{mv^2a}\right). \qquad (28)$$

Assuming that the sink produces small effect the equation for perturbation reduces to

$$\vec v\frac{\partial}{\partial \vec r}\delta f_\sigma(\vec r,\vec v) - \frac{e_\sigma}{m_\sigma}\nabla\phi(\vec r)\frac{\partial}{\partial \vec v} f_{0\sigma}(v) = -\delta(\vec r)\sigma_\sigma(v) v f_{0\sigma}(v) \qquad (28')$$

The potential $\Phi(r)$ satisfies the Poisson equation (10) with $\delta n_\sigma = n_\sigma \int d\vec v \delta f_\sigma(\vec v)$. Using k-representation it is easy to show that

$$\delta f_{\sigma\vec k}(\vec v) = -\frac{e_\sigma}{T_\sigma}\phi_{\vec k} f_{0\sigma}(v) + \frac{i\sigma_\sigma(v) v f_{0\sigma}(v)}{\vec k\vec v - i0}, \qquad (29)$$

$$\delta n_{\sigma\vec k} = -\frac{e_\sigma n_\sigma}{T_\sigma}\phi_{\vec k} - n_\sigma\frac{2\pi^2}{k}\times \int v^2\sigma_\sigma(v) f_{0\sigma}(v)dv \qquad (30)$$

and thus,

$$\phi(r) = \frac{q}{r}e^{-k_D r} - \frac{2\pi(C_i + C_e)}{rk_D} g(k_D r), \qquad (31)$$

where

$$g(x) = e^{-x}Ei(x) - e^{x}Ei(-x),$$
$$C_\sigma = e_\sigma n_\sigma \int v^2 \sigma_\sigma(v) f_{0\sigma}(v)dv. \qquad (32)$$

It follows from Eq. (31) that at $k_D r >> 1$

$$\phi(r) \approx -\frac{4\pi(C_i + C_e)}{(rk_D)^2}$$

in agreement with the result obtained by other methods [12].

CONCLUSIONS

The model of point-like sink is proposed to describe electric grain potential with regard to plasma particle absorption by grain. This makes possible to obtain analytical solution of the problem of grain screening and to find explicitly asymptotic behavior of the potential. With the appropriate choice of the parameters of the model (grain charge *q* and sink intensity *S*) the proposed approximation recovers numerical solutions of the consistent nonlinear problems.

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ЭФФЕКТИВНЫЙ ПОТЕНЦИАЛ ПЫЛЕВОЙ ЧАСТИЦЫ В ПЛАЗМЕ С ВНЕШНИМ ИСТОЧНИКОМ ИОНИЗАЦИИ

А.Г. Загородний, А.В. Филиппов, А.Ф. Паль, А.Н. Старостин, А.И. Момот

Предложен новый подход, позволяющий аналитически описывать эффективные потенциалы пылевых частиц в плазме. Предполагается, что учет поглощения электронов и ионов пылевой частицей может быть осуществлен с помощью эффективных точечных стоков в уравнениях, описывающих динамику плазмы. Предложенный подход позволяет получить явные выражения для эффективного потенциала и распределения заряда. Рассмотрен пример экранирования пылинки в плазме с внешним источником ионизации.

ЕФФЕКТИВНИЙ ПОТЕНЦІАЛ ПОРОШИНКИ У ПЛАЗМІ З ЗОВНІШНІМ ДЖЕРЕЛОМ ІОНІЗАЦІЇ

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Запропоновано новий підхід, який дозволяє описати аналітично ефективні потенціали порошинок в плазмі. Припускається, що урахування поглинання електронів та іонів порошинкою може здійснюватися за допомогою ефективних точкових стоків в рівняннях, що описують динаміку плазми. Запропонований підхід дозволяє одержати явні вирази для ефективного потенціалу та розподілу заряду. Розглянуто приклад екранування порошинки в плазмі з зовнішнім джерелом іонізації.