

QUASILINEAR DESCRIPTION OF ELECTRON CLOUD DYNAMICS IN THE SOLAR CORONA PLASMA

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In this study, we consider dynamics of maxwellian electron cloud with the temperature significantly exceeding that of surrounding plasma. We investigate the transition from initially free electron propagation to a quasilinear one. We show qualitatively and quantitatively that only a small part of the cloud propagates into a plasma, whereas the main part of electrons is concentrated in the place of injection. These electrons turn out to be locked by the Langmuir turbulence generated by the electrons left that region.

1. Introduction

The propagation of the electron stream through plasma has been widely discussed [1-10]. The interest to this problem is stimulated by different astrophysical applications (activity induced by particle beams in the Earth ionosphere, in the solar corona, pulsar magnetospheres, etc.). The difficulty of the problem originates from the fact that electrons generate Langmuir waves, which influence on electron propagation [9,10]. If the consideration is carried out in the framework of weak turbulence theory, when the level of Langmuir waves generated is quite low $W/nT < (k\lambda_D)^2$ (n , T are the density and the temperature of surrounding plasma, λ_D is Debye radius, k is a wave number), then analytical results as well as numerical solutions of the kinetic equations point out that electrons can propagate at large distances being a source of Langmuir turbulence. An analytical solution of the problem when in the injection place electron distribution function had $\partial f / \partial v > 0$ was found in [11]. Later in [14] analytical and numerical solutions of the initial problem for $\partial f / \partial v > 0$ was derived. In both cases the solution is a beam-plasma structure. It consists of electrons and plasmons and moves at a constant velocity. The method used in [11,14] for finding the solution based on the smallness of quasilinear relaxation time in comparison with the time of electron propagation. For the first time Ryutov and Sagdeev [1] proposed this method at consideration of the dynamics of maxwellian electron cloud propagation. They supposed that electron propagation would be self-similar at large distances and big times. There were some doubts [9,10,12] because at the initial moment $\partial f / \partial v < 0$ and at first electrons propagate freely. Only in some time when fast electrons overtake slow ones a positive derivative $\partial f / \partial v$ is appeared and quasilinear relaxation begins. So free propagation of electrons defines the next stage.

In this study numerically and analytically dynamics of electron maxwellian cloud are considered beginning from the free propagation. It was found that there were some electrons groups that propagate in plasma in different regimes. Among them we recognize electrons with

low density that propagate in the form of beam-plasma structure and the main part of electrons "locked" by Langmuir turbulence generated with the left electrons.

2. Free and quasilinear electron propagation

Let the fast cloud electrons have the maxwellian distribution function at the initial moment $t = 0$

$$f(v, x, t = 0) = \frac{n_0}{\sqrt{\pi}v_0} \exp\left(-\frac{x^2}{d^2}\right) \exp\left(-\frac{v^2}{v_0^2}\right), \quad (1)$$

where d is the cloud size, n_0 , v_0 are the density and the thermal velocity of the fast electrons, and $v_0 \gg v_{Te}$ is the thermal velocity of plasma electrons.

For estimations and numerical simulations we take the following parameters: $n = 6 \times 10^8 \text{ cm}^{-3}$, $T_e = 10^6 \text{ K}$,

($v_{Te} = 4 \times 10^8 \text{ cm/s}$), $n_0 = 2000 \text{ cm}^{-3}$, $v_0 = 10^{10} \text{ cm/s}$;

$d = 3 \times 10^9 \text{ cm}$. In the scope of the weak turbulence theory, the one-dimensional electron distribution function, $f(v, x, t)$, and the one-dimensional spectral energy density of Langmuir waves, $W(v, x, t)$, are described by the system of the kinetic equations [13]

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} = \frac{4\pi^2 e^2}{m} \frac{\partial W}{\partial v} \frac{\partial f}{\partial v}, \quad (2)$$

$$\frac{\partial W}{\partial t} = \frac{\pi \omega_p}{n} v^2 W \frac{\partial f}{\partial v}, \quad \omega_p = kv. \quad (3)$$

Since the initial electron distribution is stable as to generation of plasma waves, then electrons initially propagate freely and their distribution evolve following

$$f(v, x, t = 0) = \frac{n_0}{\sqrt{\pi}V_0} \exp\left(-\frac{(x-vt)^2}{d^2}\right) \exp\left(-\frac{v^2}{v_0^2}\right). \quad (4)$$

For $x > 0$, there is a positive derivative, $\partial f / \partial v$, at the electron distribution function (Fig. 1), because fast

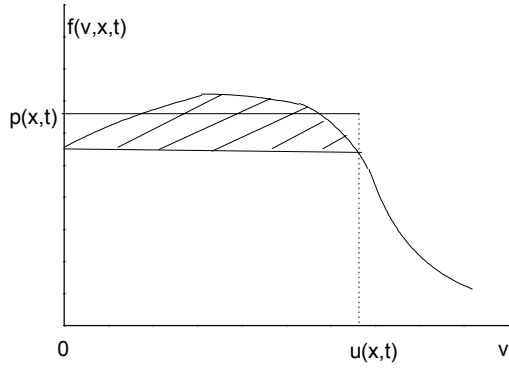


Fig. 1 Unstable electron distribution function.

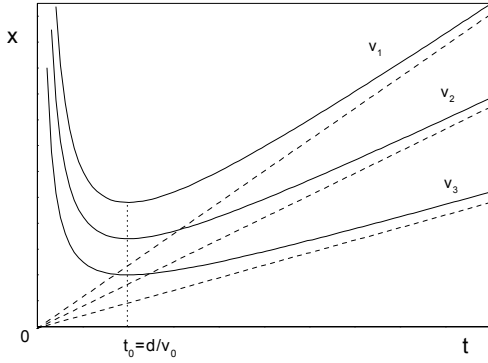


Fig.2 The curves in (x, t) plane for the three different velocities $v_1 > v_2 > v_3$, where $\partial f / \partial v = 0$ are below the corresponding curves. electrons overtake slow ones. One derives the equation of the curve separating regions with $\partial f / \partial v > 0$ and $\partial f / \partial v < 0$ [12]

$$x = v \left(t + \frac{d^2}{v_0^2 t} \right) \quad (5)$$

from $\partial f / \partial v = 0$ and (4). Note, that positive derivative can be only for $x > 0$, and therefore, propagation of electrons is free in the region $x \leq 0$. In Fig. 2 curves are presented for three different velocities $v_1 > v_2 > v_3$. It is obvious that the positive derivative at a given point x initially appears at small velocities. The maximum velocity $v^* = v_0 x / 2d$, when $\partial f / \partial v > 0$, is reached at the moment $t_0 = d / v_0$. A positive derivative means that the condition for the quasilinear relaxation exists.

Since only a small part of all the particles takes part in the relaxation, then plateau is said to be formed up to the velocity $v = u(x, t)$,

$$u(x, t) \approx \frac{2xtv_0^2}{d^2 + v_0^2 t^2}, \quad (6)$$

which is derived from the approximate equality (see Fig. 1)

$$f(0, x, t) \approx f(u(x, t), x, t). \quad (7)$$

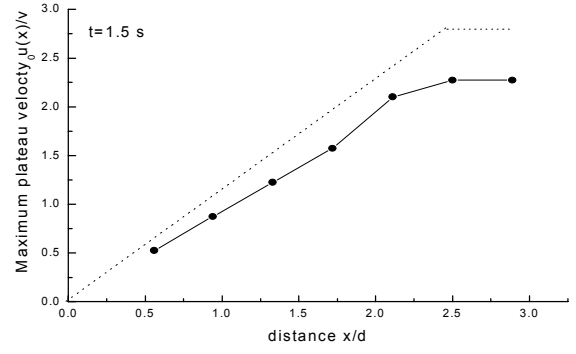


Fig. 3. Maximum velocity of the plateau electrons $u(x, t)$. Dashed and solid lines are analytical and numerical results, correspondingly.

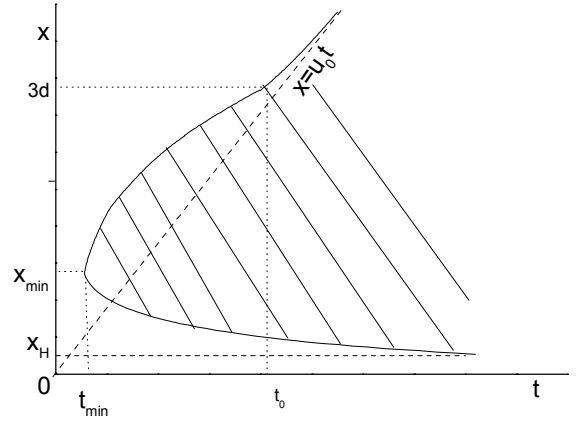


Fig. 4 The curve (11) in (x, t) plane separating regions where electrons propagate freely (not striped) and under relaxation (striped)

Maximum of $u(x, t)$ (at the moment $t_0 = d / v_0$) linearly grows with distance $u(x) = v_0 x / d$ (Fig. 3). Taking into account (6), the number of relaxing particles can be estimated as

$$n' = \int_0^{u(x, t)} (f(v, x, t) - f(0, x, t)) dv \quad (8)$$

Consequently, the quasilinear relaxation time $\tau = \left(\omega_{pe} \frac{n'}{n} \right)^{-1}$ follows substituting the result of integration from (8)

$$\tau(x, t) = \tau_0 \frac{\sqrt{d^2 + v_0^2 t^2}}{\sqrt{2}d} \exp\left(\frac{x^2}{d^2}\right) \left[\exp\left(\frac{x^2 v_0^2 t^2}{d^2 (d^2 + v_0^2 t^2)}\right) - 1 \right]^{-1}, \quad (9)$$

where $\tau_0 = n / n_0 \omega_{pe}$. It is natural to consider that the quasilinear relaxation influences the dynamics of electron propagation only if $\tau(x, t)$ becomes equal to the electron propagation time, t ,

$$\tau(x, t) = t. \quad (10)$$

The curve $x(t)$, where the condition (10) takes place, is shown in Fig. 4. The long time lengths of the quasilinear relaxation far and near the injection site are connected with the fact that in the first case the number of fast ($v \gg v_0$) particles is vanishing, and in the

second case the number of slow ($v < v_0$) particles is big. As a result, in both cases, $\partial f / \partial v$ as well as n' turn out to be small.

For large x the curve $x(t)$ has an asymptotic $x = u_0 t$ with $u_0 = v_0 \ln\left(\frac{\sqrt{2}t_0}{\tau_0}\right)$, and for the parameters chosen $u_0 \approx 3v_0$. Therefore, particles with velocities $v > u_0$ mainly do not participate in the quasilinear relaxation, i.e. they propagate freely. At small x

$$x < x_{low} \approx d\sqrt{\tau_0 v_0 / d} \quad (11)$$

electrons move freely also. At the same time, for electrons with $v \leq v_0$ interaction with Langmuir waves is displayed at the distances comparable with the size of electron cloud d , at the moment $t_{min} \equiv (\tau_0 d^2 / v_0^2)^{1/3} \approx 0.04s$, which corresponds to $x_{min} = x(t_{min}) \approx d$.

Let us consider the most interesting region $x_{low} < x < 3d$, where electron propagation is ruled by Langmuir waves. After the first stage of free spread, plateau is formed at the electron distribution function from $v = 0$ to $v = u(x, t_{min})$ due to quasilinear relaxation, since $t \approx t_{min}$. Further, electrons arrived to a given point x have a positive derivative $\partial f / \partial v$ for the greater velocities and as a result plateau becomes wider. As it was mentioned above, its maximum width $u(x) = v_0 x / d$ will be at $t_0 = d / v_0$.

In order to understand what occurs after the moment $t_0 = d / v_0$ we need to consider the distribution of Langmuir turbulence in this region. Since the plateau height is

$$p(x, t_0) = \frac{1}{u(x, t_0)} \int_0^{u(x, t_0)} f(v, x, t_0) dv \equiv \frac{n_0}{v_0} \exp\left(-\frac{x^2}{d^2}\right) \quad (12)$$

the spectral energy density follows

$$W(v, x, t_0) = \frac{mv^3}{\omega_{pe}} \int_0^v \left[\frac{1}{u(x, t_0)} \int_0^{u(x, t_0)} f(v, x, t_0) dv - f(v, x, t_0) \right] dv \approx \frac{mn_0 v^4}{\omega_{pe}} \exp\left(-\frac{x^2}{d^2}\right) \left[1 - \frac{1}{2\sqrt{2}} \operatorname{erf}(v/v_0) \right] \quad (13)$$

where $\operatorname{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x \exp(-x^2) dx$ is the probability integral. Taking into account (13) one derives the spatial distribution of wave energy

$$E_w \equiv \int W dk = \omega_{pe} \int_0^{u(x, t_0)} W(v, x, t_0) / v^2 dv \approx$$

$$\approx 0.1mn_0 v_0^2 \left(\frac{x}{d}\right)^3 \exp(-x^2/d^2), \quad (14)$$

and its maximum is close to $1.7d$. In comparison to the distribution of particle energy

$$E_e \equiv \frac{m}{2} \int_0^{u(x, t_0)} f(v, x, t_0) v^2 dv \approx \approx 0.2mn_0 v_0^2 \left(\frac{x}{d}\right)^3 \exp(-x^2/d^2), \quad (15)$$

one concludes that Langmuir turbulence is placed deeper in a plasma.

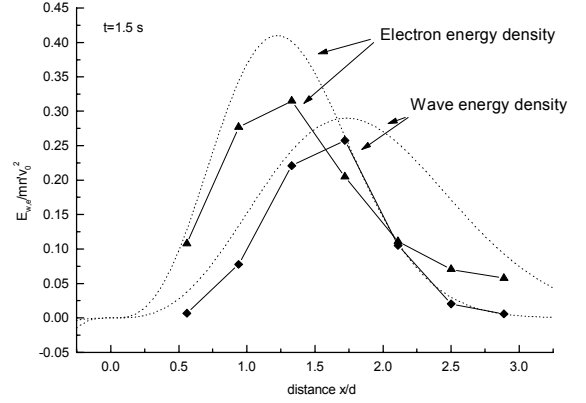


Fig. 5 The energy density of Langmuir waves (14) and electrons (15). Dashes (solid) lines correspond to theoretical (numerical) results.

The turbulence excited near $1.7d$ drastically influences the electron spread. Two groups of electrons are formed: the particles, whose propagation is ruled by Langmuir turbulence, and those that pass through this region without essential interaction with the turbulence. Electrons with velocities greater than maximum plateau velocity at this point $v > u(x = 1.7d) = 1.7v_0$, are related to the latter group. Generally, this value should be considered to be the low limit, because the high level of Langmuir turbulence exists for $x > 1.7d$, where $u(x)$ is greater, and therefore the real velocity must be slightly more. This fact is also supported by the numerical solution (Fig. 5).

The density of particles with velocity $v > 1.7v_0$ can be estimated as

$$n^* \approx \int_{1.7v_0}^{\infty} f(v, x = 0, t = 0) dv \approx 0.05n_0 \quad (23)$$

and consequently only after the moment $t = n/n^* \omega_{pe}$ these particles begin to relax, but it happens at distances $x > 1.7d$. Later electrons with greater velocities (up to $v = u_0$) are added to these particles. Plateau from $v \approx 0$ to $v \approx u_0$ is formed at their distribution function. The electrons are accompanied with Langmuir waves and together they form beam-plasma structure [11,14,15].

What happens with the electrons whose velocity is

less than $1.7v_0$? If there were not Langmuir waves, those electrons could propagate freely due to $\partial f / \partial v \approx 0$. Again, the fast electrons would overtake the slow ones and the positive derivative $\partial f / \partial v$ would appear. But the derivative is small and therefore free propagation continues till it will become great enough to start quasilinear relaxation. However, Langmuir turbulence presence in this region completely changes the dynamics. As soon as $\partial f / \partial v > 0$, process of relaxation starts ($\tau \propto W^{-1}$) and the electron distribution function becomes flatter, i.e. electrons slow down immediately. This means that Langmuir turbulence “locks” electrons within the region, $x < 1.7d$.

If processes leading to a decrease in the level of Langmuir turbulence are taken into consideration, the conditions for separation of the next small part of electrons appears. Since the maximum velocity of the electrons locked is $v_{\max} \leq 1.7v_0$, then it is clear that the velocity of the next group is less than that of the first one. As a result beam-plasma structures propagate into a plasma one by one with declining velocity.

3. Numerical solution of quasilinear equations

The numerical solution [18] of the kinetic equations (2, 3) has been conducted for the case when at the initial moment a cloud of maxwellian electrons is determined by (1), and the spectral energy density is uniform and equals

$$W_T(v, x, t = 0) = 10^{-5} \frac{n_0 T v_{Te}}{\omega_{pe}} \quad (16)$$

We performed our calculations for the parameters presented in the previous section. The results of numerical simulations are presented in fig. 3, 5-8. Generally, they support the conclusions of the qualitative consideration in the previous section. Indeed, at the moment $t_0 = d/v_0 \approx 1s$ a group of electrons separates from the main part (Fig. 8). The full separation is considered to be at the moment $t = 1.5s$ (Fig. 5). For $x < 3d$ (Fig. 6) plateau is formed at the electron distribution function as a result of electron interaction with Langmuir waves. The plateau height rapidly declines with x , but the plateau maximum velocity linearly grows with distance (Fig. 3), that was predicted by the qualitative consideration. Congruent agreement is found for the energy density distribution of electrons and plasma waves (Fig. 5). During the numerical simulations, a fact of “locking” takes place after $t = 1.0s$ and the distribution of electrons near the initial site seems to be steady till the end of the simulations $t = 10s$.

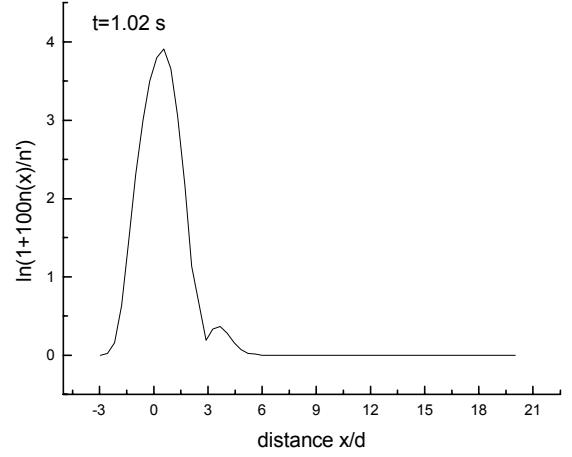


Fig. 6 The electron density at $t = 1.02s$

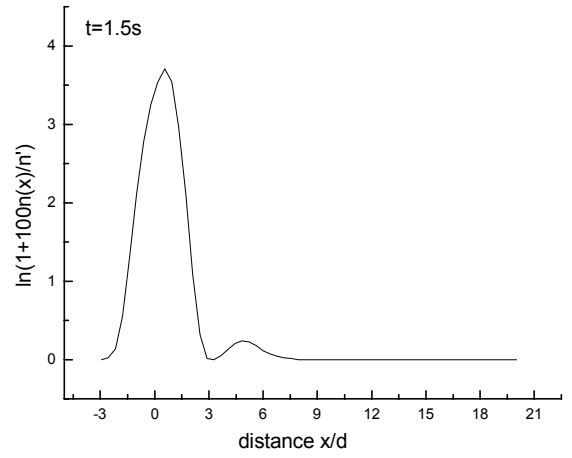


Fig.7 The electron density at $t = 1.5s$.

4. Conclusion

So we see that four groups of electrons can be recognized in the initially maxwellian cloud. Electrons with velocity

$v > 3v_0$ propagate in plasma without quasilinear relaxation because of smallness of their density that yields nonequality $\tau > t$. The next group of electrons with velocity $1.7v_0 < v < 3v_0$ moves in plasma in the form of beam-plasma structure. For them equality $\tau \leq t$ is fulfilled. A further group is concentrated in the site $x_{low} < x < 3d$. These electrons are “locked” by Langmuir turbulence generated by the previous group. At every point plateau is formed at electron distribution function because of interaction with Langmuir waves. At last there are electrons (at $x < x_{low}$) that has negative derivative $\partial f / \partial v$ and for this reason they move freely. Thus our results show that self-similar solution [1] is not realized.

According to observations of sporadic solar radio emission fast electrons propagate out of flare site as isolated beams. Problem of their formation is very important [9] for understanding both beam radiation and

flare phenomenon. In the scope of our consideration we see that for appearance of beams it is enough there was cloud of hot electrons somewhere. In the time equals $3d / v_0$ a small group of electrons with velocity $\approx 2v_0$ separates from the cloud and moves into plasma. Because this group of electrons is accompanied by Langmuir waves there is an opportunity to radiate with plasma mechanism.

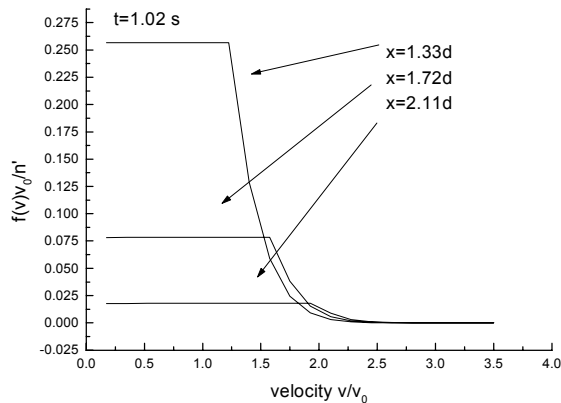


Fig. 8 The electron distribution function at $t = 1.5s$ at different x .

Authors are pleased to thank Supercomputing Systems AG (Switzerland) for the free access to the supercomputer Gigabooster. One of the authors (E.P.K.) would like to thank the Ukrainian Office of International Science Foundation for financial support in the framework of grant PSU082125.

References

- [1] D.D. Ryutov, and R.Z Sagdeev, 1970//*JETP*, **58**, 739
- [2] V.V. Zaitsev, N.A. Mityakov, and V.O. Papoport, //*Solar Physics*, 1972, **24**, 444.
- [3] R.J.-M. Grogard, //*Australian Journal of Physics*, 1975, **28**, 731.
- [4] G.R. Magelssen, D.F. Smith, //*Solar Physics*, 1977, **55**, 211.
- [5] R.J.-M. Grogard //*In. Radio Physics of the Sun*, ed. M.R. Kundu, T.E. Gergely, D. Reidel Publishing Company, 1980, 303.
- [6] R.J.-M. Grogard //*Solar Radiophysics ed. McLean, N.J., Labrum, N.R., Cambridge Univ. Press*, 1985, 289.
- [7] T. Takakura, and H. Shibahashi//*Solar Physics*, 1976, **46**, 323.
- [8] T. Takakura //*Solar Physics*, 1982, **78**, 141.
- [9] D.B. Melrose //*Solar Physics*, 1990, **130**, 3.
- [10] L. Muschietti, //*Solar Physics*, 1990, **130**, 201.
- [11] V.N. Mel'nik //*Plasma Physics Reports*, 1995, **21**, 94.
- [12] V.N. Mel'nik, and Kontar, E.P. //*New Astronomy*, 1999, **4**, 41.
- [13] A.A. Vedenov Ryutov D.D. //*Voprosi teorii plasmu ed. Leontovich L.I., Moscow: Atomizdat*, 1972, **N6**, 3.
- [14] E.P. Kontar, V.I. Lapshin, V.N. Mel'nik, *Plasma Physics Reports*, 1998, **24**, 772.
- [15] V.N. Mel'nik, V.I. Lapshin, and E.P. Kontar, //*Ukrainian Journal of Physics*, 1999, **44**, 951.
- [16] A. Benz, A. Magun, W. Stelling, and H. Su //*Solar Physics*, 1992, **141**, 335.
- [17] N.J. McLean Solar Bursts Observations at meter and decameter wavelengths //*Solar Radiophysics, ed. McLean N.J., Labrum N.R., Cambridge University Press*, 1985, 240.
- [18] A.A. Samarski Theory of difference schemes, Moscow: Science, 1989, 616.