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PARAMETRIC INSTABILITIES EXCITED BY ECR-RESONANCE HEATING IN A MIRROR MACHINE

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The process of phase randomization during electron cyclotron resonance heating has been studied intensely in different aspects with single particle approximations. Studies of parametric instabilities in plasmas introduce another system of interacting oscillators, namely plasma waves. The appearance of collective motion implies a different mechanism of phase randomization with time scales close to the inverse growth rate of the modulational instability shorter than the electron bounce frequency in the mirror trap. Systematic experiments on ECR-heating show the presence of both a spectral broadening of the pump wave as well as low frequency noise close to the lower hybrid frequency, which roughly corresponds to the growth rate of the instability. The necessity of considering potential noise, the plasma eigenmodes respectively, possibly leads to a change of the existing model of phase randomization based on the single particle motion.

1. Introduction

The problem of phase randomization during electron cyclotron resonance heating has been of great interest beginning from [1]. Subsequently a different approach was proposed explaining the phase mixing by non-linear resonances appearing if particle-wave interactions are taken into account [2]. Here, phase randomization will occur, when the external electric field exceeds a certain threshold determined by the condition of overlapping resonances ([3],[4],[5] and references cited there). Starting with particles drifting in the energy space, a diffusion-like equation for the electron distribution function is derived. Most plasma properties such as collective processes appeared to be beyond the scope of that consideration [3]. In particular, the electrostatic short-wave perturbation common for any plasma were not taken into account since the initial Hamiltonian contained only curled components of the electric field. Thus, one deals with a single particle approximation, valid only for sufficiently low plasma densities.

On the other hand, the introduction and development of quasilinear theory [6] allows to describe the ECR based on a collective approach. For a linear mirror machine as described in section 3 this was done in [7] to examine the development of the hot electron population in such a discharge. Here, the existence of plasma-induced phase-mixing process was assumed. The main problem of this approach, namely the phase randomization, calls for a distinct mechanism of the plasma itself, which will be responsible for the process. Since the electron cyclotron frequency is sufficiently close to the electrostatic eigenmodes of the plasma, several parametric instabilities ([8],[9]) can occur, thus bringing up a spectral broadening of the pump wave as well as both high-frequency and low-frequency electrostatic waves. Earlier experiments ([7], [10]) in an ECR-heated linear magnetic mirror proved the existence of low frequency noises with a maximum frequency close to the ion plasma frequency ω_{pi} or to the lower hybrid frequency ω_{LH}. Recent systematic measurements are presented in chapter 3. It is supposed, that a system of three non-linear oscillators (the initial electromagnetic wave and two electrostatic ones as a consequence of a decay process for example) becomes a random phase system after a time of the order of the inverse growth rate of the parametric instability [11].

Hence, at first hand the effective spectral broadening of the pump wave is due to non-linear resonance overlapping of a single particle. This process is characterized by the electron bounce frequency ω_B, the frequency of the particle's oscillation between the mirrors of the magnetic trap. Since the given pump wave is monochromatic, the process of phase randomization is inevitable for heating the electrons. At second hand one deals with the collective behaviour of the plasma described with its eigenmodes. In this approach we deal with the growth rate of a parametric instability, which can be larger than the lower hybrid frequency or the ion plasma frequency [12].

2. Parametric instabilities

In many cases dealing with wave packets, the spectral composition of the pump wave can be broadened due to the interaction of the pump wave with plasma eigenmodes excited by the incident wave itself. In this section parametric instabilities are considered, which can be responsible for this spectral broadening of the incident wave.

If the frequency Ω of the electromagnetic wave is close to one of the short wavelength electrostatic eigenmodes of the plasma one can use the traditional approach for describing non-linear parametric processes, when the external electric field depends on time only:

$$\vec{E} = \vec{E}_0 \sin \Omega t \tag{1}$$

The standard method [8] yields

$$-\phi_n = \sum_{p,s} \delta\varepsilon_e\left[\omega - (s-p)\Omega\right]\cdot J_p(a) J_{n+p-s}(a)\,\phi_s + \ldots \quad (2)$$
$$\ldots + \delta\varepsilon_e(\omega - n\Omega)\phi_n$$

Where ϕ_n is the n-th harmonic of the potential perturbation, $a = \frac{eE_0}{m\Omega^2}k_x$, ω_{pe} and ω_{pi} are the electron and ion plasma frequency respectively,

$$\delta\varepsilon_\alpha(\omega) = \frac{\omega_{p\alpha}}{k^2}\int \frac{\frac{\vec{k}}{n_0}\frac{\partial f_0^e}{\partial \vec{v}_0}}{\omega - \vec{k}\cdot\vec{v}}d\vec{v}_0 \quad (3)$$

are the contributions of electrons and ions respectively to the plasma permittivity. Equation (2) is valid in a general manner, including the cases of a stationary magnetic field or inhomogeneous plasmas, etc. To analyse a certain configuration, one has to use corresponding values for the $\delta\varepsilon_\alpha(\omega)$, obtained in the linear theory of the electrostatic eigenmodes. It was shown [8], that for $a << 1$ and $\omega/\Omega << 1$ only three equations of (2) (namely $n = 0, \pm 1$) may be used, which reduces the system to the well known expression

$$\frac{1}{1+\delta\varepsilon_e(\omega)} + \frac{1}{\delta\varepsilon_i(\omega)} + \frac{a^2}{4}\left[\frac{1}{\varepsilon(\omega+\Omega)} + \frac{1}{\varepsilon(\omega-\Omega)}\right] = 0 \quad (4)$$

with $\varepsilon(\omega) = 1 + \delta\varepsilon_e(\omega) + \delta\varepsilon_i(\omega)$

Here we consider the case when the frequency Ω of the pump wave is close to the plasma wave frequency. Two situations can be realized:

1) the pump wave frequency Ω is somewhat higher than ω_{pe} (decay instability);
2) $\Omega = \omega_{pe}$ and lower than the frequency of plasma eigenmodes.

The latter case corresponds to the modulation instability which is the first step of Langmuir wave collapse, which is preceded by a weak turbulence stage, where most of the wave energy is confined in a region of small wave vectors (close to $k = 0$).

The same situation occurs, when the external pump wave frequency equals ω_{pe} and its wavelength is sufficiently large.

Evaluating equation (4) for the supersonic regime, when the growth rate γ of the parametric instability is larger than the ion sound frequency $\omega_s = \omega_{pi}k\lambda_D$, the latter can be neglected from the beginning to derive

$$\omega^2\cdot\left(\omega^2 - \delta^2\right) + \tfrac{a^2}{4}\omega_{pe}^2\omega_{pi}\delta = 0 \quad (5)$$

where $\delta = \Omega - \omega_{pe}\left(1 + \frac{3}{2}k^2\lambda_d^2\right) = -\frac{3}{2}k^2\lambda_d^2\omega_{pe}$ is the frequency mismatch. Here both modulational instability ($\delta < 0$) and the so called modified decay ($\delta > 0$) can be studied analytically [9]. The maximum growth rate γ_m is obtained with

$$\gamma_m \sim |\delta_m| \sim \left(a^2\omega_{pe}\omega_{pi}^2\right)^{\frac{1}{3}} \sim \omega_{pi}\left(a^2\sqrt{\tfrac{M}{m}}\right)^{\frac{1}{3}} \quad (6)$$
$$\sim \omega_{pi}\left(k^2\lambda_D^2\,\tfrac{W}{n_0T_e}\sqrt{\tfrac{M}{m}}\right)^{\frac{1}{3}}$$

(for the modulation instability when $k_m\lambda_D \sim \sqrt[4]{\frac{W}{n_0T_e}\frac{m}{M}}$),

where $W = E_0^2/4\pi$. If k_m achieves a value close to a fraction of the inverse Debye length, the electron distribution function begins to change since electrons with superthermal energy begin to absorb the short length Langmuir waves. This is a different way to describe Langmuir collapse.

The same approach permits to consider the case for magnetically confined plasmas, where Bernstein modes can by excited. The electric field of these modes is directed perpendicular to the magnetic field and their frequency is close to the electron cyclotron frequency.

The following expression gives the electron part of the permittivity:

$$\delta\varepsilon_e(\omega) = -2\cdot\sum_{n=1}^{\infty}\frac{n^2\omega_{ce}^2}{\omega^2 - n^2\omega_{ce}^2}\cdot\Phi_n \quad (7)$$

where $\Phi_n = \dfrac{I_n\left(\frac{k^2v_{th}^2}{2\omega_{ce}^2}\right)\cdot e^{-\left(k^2v_{th}^2/2\omega_{ce}^2\right)}}{k^2\lambda_D^2}$ with I_n as Bessel function with imaginary argument was introduced.

The dispersion relation $1 + \delta\varepsilon_e(\omega) = 0$ yields the eigenfrequencies for the Bernstein modes:

$$\omega_{en}(k) = n\omega_{ce}(1 + \Phi_n(k)). \quad (8)$$

This result is valid for $\rho_e < \lambda_D$ which we suppose to be fulfilled. If $\Phi_n(k) << 1$ the harmonics of the pump wave are in resonance simultaneously with a number of Bernstein modes. Finally the dispersion equation for parametric excitation of Bernstein modes is derived [12]:

$$1 - \frac{\omega_{pi}^2}{\gamma^2 + \omega_{pi}^2}\cdot\sum_{n=1}^{\infty}\frac{2n^2\omega_{ce}\left(\omega_{ce}\Phi_n - \Delta\right)\Phi_n}{\gamma^2 + n^2\left(\Delta - \omega_{ce}\Phi_n\right)^2}\cdot J_n^2(a) = 0 \quad (9)$$

with $\Delta = \Omega - \omega_{ce}$.

It was shown that simultaneous excitation of several Bernstein modes occurs depending on the frequency mismatch Δ with growth rates exceeding the ion Langmuir frequency. The threshold is suggested to be determined by the geometry of the system, since it is negligible in the homogeneous case. The modulation instability appears to be dominant for both longitudinal and transversal launching of the pump wave. The excitation of electrostatic modes having longitudinal wave vector components was studied earlier [11].

At first glance the excitation of electrostatic waves seems to be inessential for the stochasticity. However, the interaction of the incident wave with excited electrostatic modes can cause a spectral broadening of the pump wave by the parametric instability growth rate [11].

If this broadening occurs the resonances with different n will overlap under the following condition: The growth rate of the parametric instability is comparable or larger than the electron bounce frequency in the mirror trap.

Since the growth rate if of the order of the ion plasma frequency (or more precisely the lower hybrid frequency), the non-linear collective oscillations become more important than the phase mixing due to the non-linear motion of a single electron. The quasilinear equation will take a different form, the resonances on the various harmonics of the bounce frequency will disappear. Moreover, if one takes into account the dependence of the incident wave amplitude on z, which is natural since the wave is damped, another parameter, namely $k_z v_{\|0}$ can cause the overlap of resonances. Thus, if the random phase approximation takes place due to parametric instabilities, the form of the quasilinear equation will change drastically. The presumed criterion for the separation of collective and single-particle approach is determined as $\gamma > \omega_B$, where γ is of the order of the lower hybrid frequency.

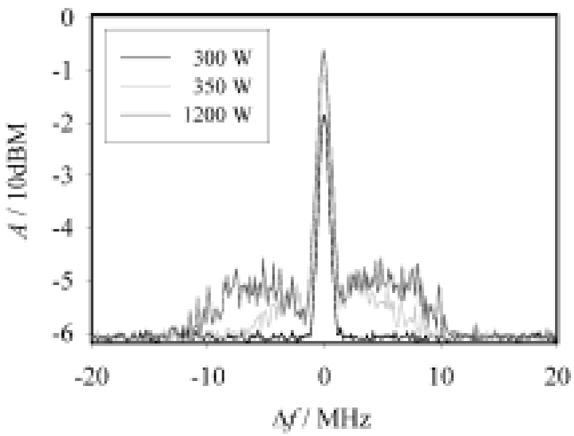

Fig 2. X-band emission spectra of the ECR-plasma around the pump frequency (f_0 = 10.115 GHz) for different incoming microwave power P_{in}

3. Experiments

The experiments were performed on an ECR discharge in is simple magnetic mirror (see Fig.1). The plasma runs in a cylindrically-symmetric copper resonator of 24×10^{-3} m^{-3} mounted in a high vacuum chamber.

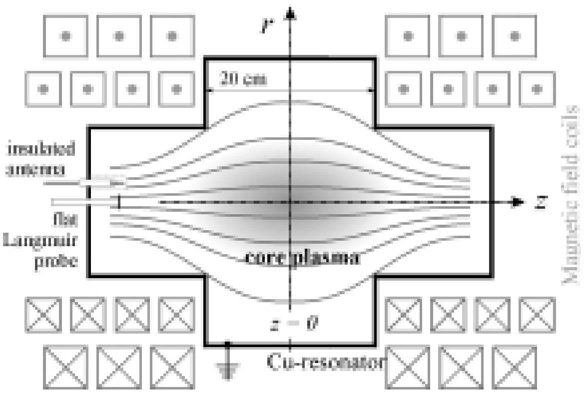

Fig 1. Schematic drawing of the ECR-discharge

For the shown experiments the discharge was run in pure nitrogen at a constant gas pressure of 4.4×10^{-5} Tore. The confinement is obtained by two sets of independently fed coils generating an axisymmetric magnetic mirror field of adjustable mirror ratio $R_M = B_{\max} / B_{\min}$ = 3-5. The discharge is ignited by radially launching an extraordinary wave at z=0 from the low-B-field side into the resonator. ($f_0 = \Omega/2\pi$ = 10.115 GHz, P_{in} = 0…1750 W cw). Incoming and reflected microwave power are measured with bolometric sensors. For measuring the rf- and X-Band emission spectra [7,10] of the ECR-plasma, two different probes mounted axially into the high-B-field side (mirror throat) were used. One probe (antenna) is surrounded by a ceramic housing to electrically insulate it from the plasma, the other one consists of a flat Langmuir probe (diameter 5.2mm) together with a small floating reference probe. The latter system can also be used to determine the local electron density of the plasma as described below. The signals of the floating probes were recorded with a spectrum analyser (Hewlett Packard HP 8560E).

In figure 2 some typical emission spectra centred around the pump frequency f_0 (i.e. Δf =0 MHz) are shown. At low incoming microwave power (P_{in} < 300 W), the spectrum only shows the pump wave signal. Above a sharp power threshold sidebands around the pump frequency appear. These sidebands are almost symmetric to the centre frequency and spread from $\Delta f = \pm$ 5 MHz just above the threshold up to $\Delta f = \pm$ 15 MHz for the maximum input power.

Here P_{in} denotes the power launched from the X-band waveguide into the resonator of which a part is reflected back und thus not absorbed by the plasma.

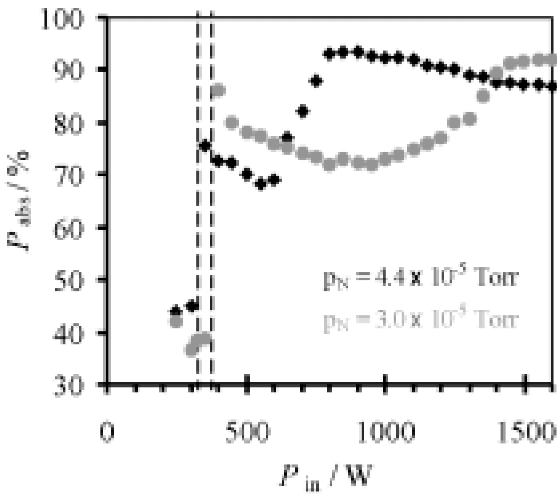

Fig 3. Absorbed microwave power versus incoming power. The higher pressure case (black squares) corresponds to the shown emission spectra

Figure 3 shows that the observed power threshold for the occurrence of the sidebands comes along with a sharp non-linear increase of absorbed microwave power.
Figures 4 and 5 show emission spectra simultaneously recorded in the radio frequency region.

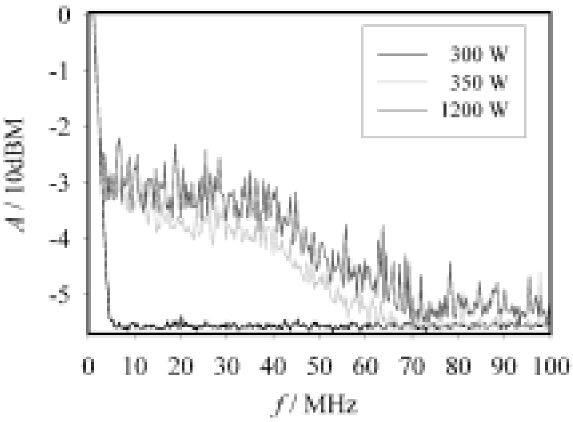

Fig 4. Radio frequency emission spectra of the ECR-plasma detected with the electrostatic Langmuir probe

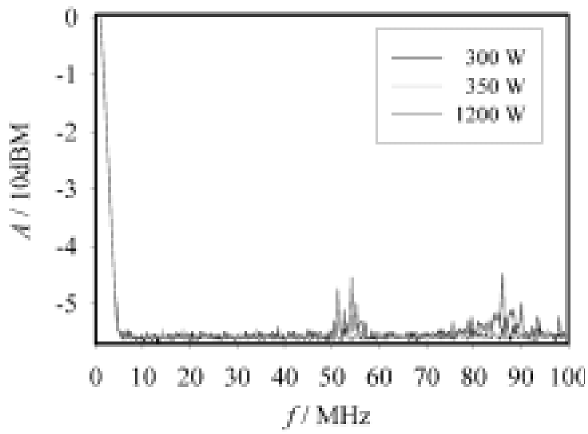

Fig 5. Radio frequency emission spectra of the ECR-plasma detected with the insulated antenna

In analogy to the X-Band spectra, the Langmuir probe detects oscillations setting in above the same threshold. The maximum frequency of this noise changes from 60 MHz just above threshold up to 80 MHz for maximal P_{in}.
The insulated antenna, on the contrary, does not record those pronounced noises. Here some weak peaks are detected, which set in above a second threshold (see figure 4) around P_{in} = 700 W.
Beside the power threshold, the configuration of the magnetic mirror field is an important parameter for the occurrence of the above-shown spectra. In Figure 6 and 7 spectra are shown for fixed input power of P_{in}=1200W and varying magnetic field $|B|$ (the mirror ratio almost keeps constant). The sidebands to the pump wave as well as the radio frequency noise appear within a very small range of the magnetic field setting. A detuning of $|B|$ by ±1mT in the centre of the discharge ($B_0 \approx 180$mT) is sufficient to suppress the effect. Calculations of the

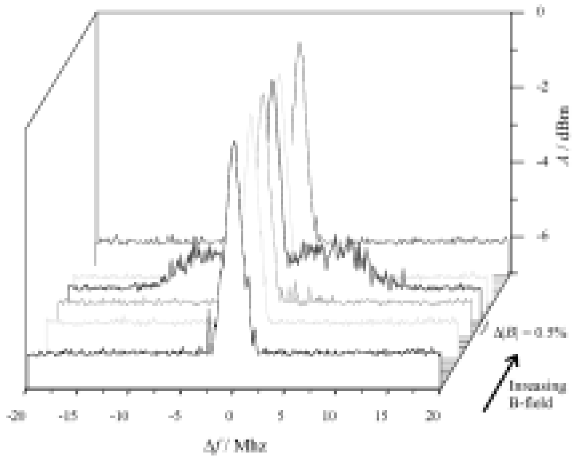

Fig 6. X-band spectra for different magnetic field strength. The total change of field is 20%. The centre frequency corresponds to the pump wave

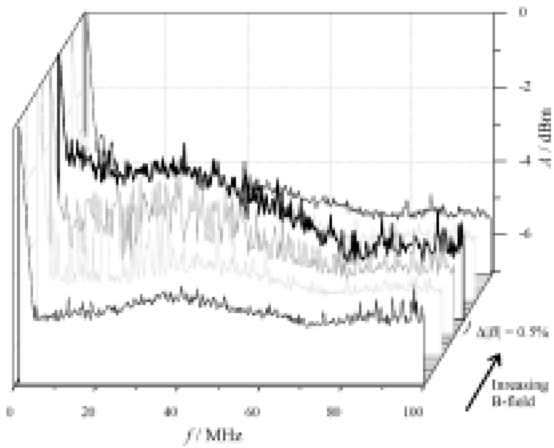

Fig 7. Radio frequency spectra for different magnetic field strength. The total change of field is 20%

vacuum magnetic field (without taking into account diamagnetic effects due to hot relativistic electrons) show that this unique configuration is characterized by the osculation of the first harmonics of the electron cyclotron resonance contours in the centre of the magnetic trap.
The system of the flat Langmuir probe with the floating reference probe can be used simultaneously to determine plasma parameters in a magnetized plasma, namely the electron density and temperature ([13],[14]) from the second derivative of the probe characteristics $I(U)$ ([15],[16]):

$$\frac{\mathrm{d}^2 I}{\mathrm{d}U^2} = \frac{2\pi e^3}{m_e^2} A_{probe} \cdot f_e(v), \qquad (10)$$

where $f_e(v)$ is the electron distribution function [17].

In figure 8 electron densities measured with the probe system according to the experimental parameters described above can be seen.

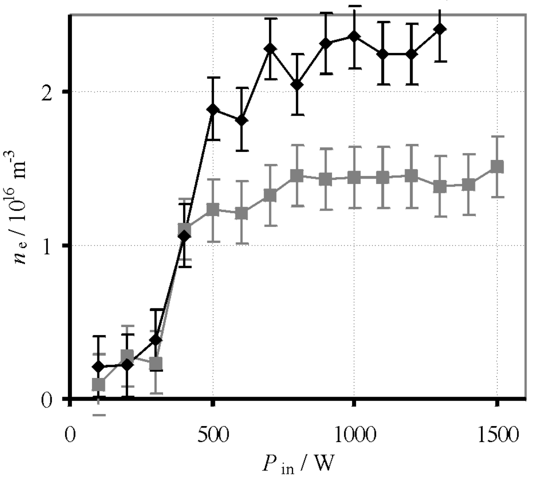

Fig 8. Electron densities versus incoming microwave power. Grey squares: configuration with lowest B-field ($I_{outer\ coil}$ = 400A, see front spectra in fig. 7). Black squares: configuration, where noise and sidebands are detected ($I_{outer\ coil}$ = 501A, see corresponding spectra in figs7)

The above mentioned power threshold between 300-350 W corresponds to an increase in plasma density (at least locally in the mirror throat of the discharge) by a factor of at least 3 to 5. Moreover, if the magnetic field is tuned to the configuration where the noise is detected, the density is again increased. This latter density increase, in contrast to the general increase at the threshold, is coupled with the occurrence of the noise.

4. Summary

We have studied the possible role and mechanism of parametric instabilities concerning the ECR heating. In the experiment a threshold for the incoming microwave power is observed, at which the plasma absorbs more power and at which the electron density grows significantly. Simultaneously to these effects electrostatic noises in the frequency range of ion-plasma- or lower hybrid frequencies occur together with a broadening of the pump microwave frequency hence suggesting a change in the entire heating scenario as described in section 2, where the necessity of considering potential noise corresponding to the plasma eigenvalues was emphasized and collective processes were included in the model of ECR-heating.

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References

1. M. Seidl // Plasma Phys., 1964, vol.6, p.597.
2. G.M.Zaslavsky, B.V.Chirikov //"Soviet Uspekhi", 1971, vol. 105, p.3.
3. F.Jaeger, A.J.Lichtenberg, M.A.Liberman //Plasma Phys., 1973, vol.15, p.125.
4. M.A.Liberman, A.J.Lichtenberg // Plasma Phys. 1973, vol.15, p.125.
5. A.V.Timofeev, Cyclotron oscillations of an equilibrium plasma //Reviews of Plasma Physics edited by B. B. Kadomtsev, New York-London, Consulting Bureau, 1989, 14, p.63.
6. A.A.Vedenov, E.P.Velikhov, R.Z.Sagdeev// Nuclear Fusion, 1961, vol.1, p.82.
7. K.Bernhardi, R.P.Brinkmann, Stochastische Beschleunigung von Elektronen im Bereich der Elektron-Zyklotron-Resonanz im inhomogenen Magnetfeld// SFB-Report 88-05-178, 1988, (in Germany)
8. V.P.Silin The parametric influence of the irradiation of large power on plasmas. M.: "Nauka", (in Russian) (1973). A.A.Ivanov, Interaction of high frequency fields with plasmas //Reviews of Plasma Physics edited by M.A.Leontovich, New York-London, Consulting Bureau, 1972, vol.6, p.70.
9. V.D.Shapiro, V.I.Shevchenko Fundaments of Plasma Physics M.: "Energoatomizdat", 1984
10. D.Meyer, D.Bolshukhin, K.Wiesemann // Rev.Sci.Instr., 1998, vol.69, N2, p.1126.
11. A.A.Ivanov, V.V.Parail // Soviet Phys. JETP, 1971, vol.33, N6, p.1138. A.A.Ivanov, Physics of the strongly non equilibrium plasmas. –M.: "Atomizdat", 1977, p.282, (in Russian)
12. A.A.Ivanov, K.S.Serebrennikov, V.Yu.Fedotov, M.Bacal, J.-M.Buzzi. Multi-mode Parametric Excitation of Bernstein Waves under Electron Cyclotron Heating report on Maastricht Conference on Plasma Physics, Maastricht, 1999.
13. P.C. Stangeby// Plasma Phys. Control. Fusion, 1995, vol.37, p.1337.
14. D.Bolshukhin, D.Meyer,F.Petriconi, I.Vinogradov, K.Wiesemann// Contrib. Plasma Phys., 1996, vol.36, p.95.
15. J.D.Swift, M.J.Schwar, *Electric probes for plasma diagnostics* (London: Elsevier) p. 76-80.
16. U.Flender, K.Wiesemann// Plasma Chem. Plasma Process, 1995,vol.15, p.123.
17. M.J.Druyvesteyn// Z.Phys., 1930, vol.64, p.781.