# LARGE AMPLITUDE SOLITARY HOLE IN RELATIVISTIC ELECTRON BEAM

V.I.Lapshin, V.I.Maslov, I.V.Tkachenko
NSC KIPT, Kharkov, Ukraine

## INTRODUCTION

The possibility to excite the coherent solitary wave structure like the electric potential hump of large amplitude in the accelerator electron beam has been considered. Its properties and dependencies of properties on amplitude have been investigated. The beam electron behavior determines the properties of solitary perturbation. This perturbation propagates in the rest frame of beam with velocity approximately equal to the thermal velocity of beam electrons. It represents a nonlinear perturbation on an electron - sound mode. The perturbation forms hole in the electron phase space. At small amplitude the width of perturbation decreases with amplitude growth so that area of the hole in phase space is not changed. At large amplitude the width of hole increases with amplitude growth. Therefore, part of beam electrons is trapped by electric field of perturbation and forms the vortex in the electron phase space which frames the hole. This hump of electric potential is the BGK perturbation. The presented analytical investigation of these solitary perturbation properties is initiated by the fact that similar perturbations have been observed in synchrotron experiments. Also similar perturbations have been formed in laboratory, magnetosphere plasmas and in numerical simulations.

## PROPERTIES OF SOLITARY PERTURBATION FOR SMALL AMPLITUDES

As a result of nonstationary and localized- in -space processes the perturbations of final width can be formed in the electron beam and propagate with characteristic velocity of a system [1].

The experiments on magnetized conductive cylindrical structures with electron beam have demonstrated existence of two kinds of solitary perturbations. One is formed on a mode of the magnetized conductive cylindrical structure with electron beam. In approach of small amplitudes it is described by the KdV equation and is the hollow of negative electrical potential. Second nonlinear perturbation, observed in magnetized conductive cylindrical structures with electron beam, is the solitary positive hump of electrical potential. It propagates relative to beam with velocity close to thermal velocity of beam electrons, $V_{th}$. Therefore, the resonant electrons can be trapped by such potential perturbation. Thus, it is possible to expect, that the kinetics of resonant electrons plays an important role in determination of properties and dynamics of this perturbation. As behaviour of resonant and nonresonant electrons is different in this perturbation field, electrons should be described within the framework of kinetic approach. Electrical potential hump, propagating relative to beam with velocity close to thermal velocity of the beam electrons, forms a hole in electron phase space, called electron hole.

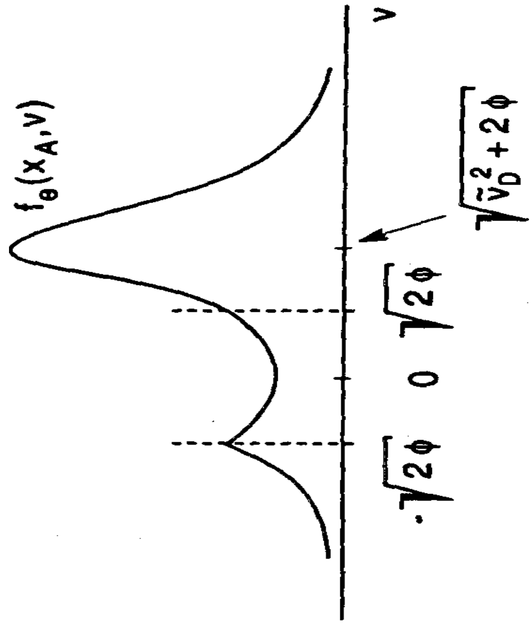


Fig. 1. Electron distribution function.

Other properties of an electron hole are found from experiments and numerical simulation:

For the electron hole cab appear the amplitude of external pulse should exceed the threshold value.

The amplitude of the electron hole decreases with a pressure increasing. First two properties are determined by the fact that resonant electrons are trapped by the hump of the electrical potential, their distribution function is mixed on phases. As a result the potential hump expends energy to increase the energy of resonant electrons.

The spatial size of an electron hole is less than the longitudinal size of potential hollow in the magnetized electron-filled conductive cylinder. As is shown below it is determined by that, the spatial size of the electron hole is determined by the electron Debye radius, and the spatial size of the potential hollow is determined by the transversal size of cylinder. The transversal size of cylinder is considerably more than the electron Debye radius.

The electron hole is stable. It is determined by that neglecting collisions of beam electrons there is no energy exchange of resonant electrons with symmetrical velocity distribution function and nonresonant electrons with nonsymmetrical distribution function.

Two colliding electron holes save their individuality after collision, if their relative velocity is rather high. They are merging when their relative velocity is low. During merging of two electron holes in one the sound packet can be radiated. The latter is determined by damping oscillations in the shape of integrated electron hole. These oscillations are determined by oscillations of resonant electrons in a field of integrated electron hole up to their full intermixing on phases.

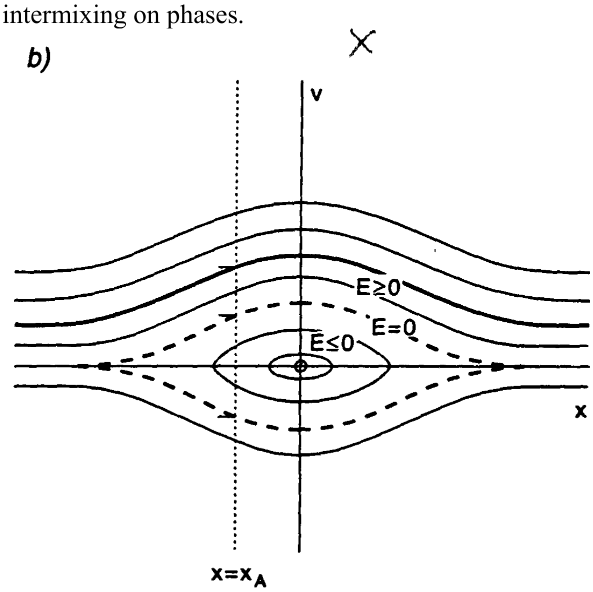


Fig. 2. Electron phase space.

Further we shall construct the analytical description of this stationary solitary potential perturbation (similar to [1, 2]), propagating relative to the beam with velocity close to thermal velocity of beam electrons. It ensures properties of this solitary perturbation: positive potential hump, velocity and hole in the electron phase space and dependence of properties on its amplitude.

Formation and existence of electrostatic structures: double layers, solitons and holes in the electron phase space are detected in many laboratory experiments and in numerical simulations [112, 142, 232 - 233].

These structures result from transformation of topology of a phase space and, therefore, lead to characteristic transformations of a nonlinear behaviour of the system.

An example of a modified behaviour in question is the excitation of solitary perturbation or the hole in the electron phase space [155, 156, 159].

Presence of the solitary perturbation leads to transformation of the electron distribution function to symmetrical form in the resonant area. Such condition is asymptotic one, arisen as a result of instability development and leading at nonlinear stage to trapping of resonant electrons by solitary perturbation with positive potential. In quasistationary approximation this perturbation represents BGK solution [235, 236].

With neglecting the interaction of an electron beam with cylinder this solitary perturbation is stationary. However, taking into account the interaction of an electron beam with cylinder it is necessary to expect the occurrence of slow increase in perturbation amplitude as a result of the instability development.

The purpose of the present paper is the derivation of the description of stationary soliton or hole in the electron phase space.

We describe the beam electron dynamics in one-dimensional approximation. We consider the solitary perturbation in the form of electric potential hump $\varphi$ $(x,t)$ with amplitude $\varphi_o$ and equal $\varphi(\pm\infty,t)=0$ at $x\rightarrow\pm\infty$, i.e.

$$0<\varphi(x,t)<\varphi_o$$

Longitudinal component of an electrical field $E_x$ is connected with an electrical potential $\varphi(x,t)$ as $E_x=\partial\varphi/\partial x$.

The description of wave modes, characteristic velocities of which in rest frame of particles are of order of particle thermal velocity, requires kinetic consideration. The expression for perturbation of electron distribution function of beam is followed from Vlasov equation for electrons. Integrating the latter by the velocity, in case of small amplitude of solitary perturbation $\varphi_o$, one can obtain the expression for perturbation of electron density, which in the second order on $\varphi_o(<<T_e/e = mV_{th}^2/e)$ equals

$$\begin{aligned} \delta n' &= \dot{\phi}[y + \\ &+ (1-2y^2)(1-R(y))/y] + \\ &+ \phi' R(y) + \phi\phi'[1-y^2 + \\ &+ (1.5-y^2)(R(y)-1)], \\ R(y) &= 1 + \\ &+ \frac{y}{\sqrt{\pi}}\int_{-\infty}^{\infty} dt \frac{\exp(-t^2)}{(t-y)}, \\ y &= V_o / V_{th}\sqrt{2} \end{aligned} \tag{1}$$

Here, the point means derivation on time, prime- spatial derivation. $V_o$, $\varphi$ are the velocity and potential of soliton. $\phi=e\varphi/T_e$.

Substituting (1) in Poisson's equation, one can obtain the evolution equation of KdV-type

$$\begin{aligned} &\dot{\phi}[y + (1-2y^2)(1-R(y))/y] + \\ &+ \phi\phi'[1-y^2 + \\ &+ (1.5-y^2)(R(y)-1)] + \\ &+ \phi' R(y) - \phi''' = 0 \end{aligned} \tag{2}$$

From (2) in the stationary approximation we obtain the equation describing spatial distribution of a potential:

$$\begin{aligned} (\phi')^2 &= \phi^2 R(y) - \\ &- [1 + (2y^2-3)R(y)]\phi^3/6 \end{aligned} \tag{3}$$

From (3) and $\phi'\big|_{\phi=\phi_o} = 0$ we have expression for $V_o$

$$V_o \approx 1.32\, V_{th}(1 - e\varphi_o/6T_e), \tag{4}$$

$V_o$ approximately equals $V_{th}$ and decreases with growth $\varphi_o$.

Let us determine approximately the width of soliton from (3):

$$\Delta x \approx (48T_e / e\varphi_o)^{1/2}. \tag{5}$$

The width of soliton decreases with the amplitude growth. The solitary perturbation is the electron phase hole. The area of this hole, approximately equal to $\Delta x(e\varphi_o/m)^{1/2}$, does not depend on $\varphi_o$.

## SOLITARY PERTURBATION FOR FINAL AMPLITUDES

In case of large amplitudes, $e\varphi_o/T_e>1$, from Vlasov and Poisson equations one can obtain without electrons, trapped by soliton field, the equation for the soliton shape

$$(\phi')^2 = -\phi + \\ + (2/\sqrt{\pi})^{1/2}\int_{-\infty}^{\infty} dt(t-y)^2 \exp(-t^2)\times \\ \times \{[1+\phi/(y-t)^2]^{1/2}-1\} \qquad (6)$$

From (6)

$$\Delta x \approx [2e\varphi_o/T_e(\sqrt{2}-1)]^{1/2} \qquad (7)$$

one can conclude that the soliton width increases with $\varphi_o$. Hence, the trapped electrons are necessary. Assuming for their density $n_{tr}(x) = n_2\exp[e\varphi(x)/T_{tr}]$, one can obtain that $\Delta x$ and $V_o$ grow with $\varphi_o$ (unlike the case of small amplitude).

Such soliton properties and their dependencies on amplitude have been observed in experiments and in numerical simulations.

So, the structure of nonlinear electrical solitary perturbation has been investigated here, namely, the hole in phase space of beam electrons. In a laboratory rest frame this hole, in the case of its small amplitudes, propagates with velocity, approximately equal to $V_o\approx V_b-1.32V_{th}$, here $V_b$ is the beam velocity; the width of solitary perturbation $\Delta x \approx (48T_e/e\varphi_o)^{1/2}/\gamma_b$ is inversely proportional to the relativistic factor of beam $\gamma_b$.

## REFERENCES


1. Maslov V.I. // Proc. of Conf. On Double Layers and Other Nonlinear Structures in Plasmas. Insbruck. Austria. 1992. P.82.
2. Maslov V.I. // Proc. of Conf. On Double Layers and Other Nonlinear Structures in Plasmas. Insbruck. Austria. 1992. P.381.
3. Shamel H. // Phys. Rep. 1986. V.140, N3. P.163.
4. O'Neil T.M., Winfrey J.H., Malberg J.H. // Phys. Fluids. 1971. V.14, N6. P.1204.
5. Raadu M.A. // Phys. Rep. 1989. V.178, N2. P.25.
6. Takeda Y. and Yamagiwa K. // Phys. Fluids. 1991. V. B3, N2. P.288.
7. Maslov V.I., Schamel H. // Phys. Lett. A. 1993. V.178, N1,2. P.171.
8. Schamel H., Maslov V.I. // Physica Scripta. 1994. V.T50. P.42.
9. Schamel H., Maslov V.I. // Spring College on Plasma Physics. Trieste. Italy. 1993. P.113.
10. Schamel H. // Plasma Phys. 1972. V.14, N10. P.905.
11. Bernstein I.B., Greene J.M. and Kruskal M.D. // Phys. Rev. 1957. V.108, N3. P.546.