WEC PROGRAM FOR CALCULATION OF SYSTEMS OF ACCELERATING VOLTAGE PULSES FORMATION ON LINES WITH DISTRIBUTED PARAMETERS

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INTRODUCTION

For some years at VNIIEF there have been developed iron-deprived linear induction accelerators (LIA) [1-3], applying, as basic elements of accelerating system, inductors with water insulation on the base of lines with distributed parameters. When creating such facilities one of the most important tasks is development of systems of forming high-voltage pulses of accelerating voltage with required electric characteristics. On the basis of solving the Maxwell equations system an algorithm of numerical simulation of non-stationary electromagnet processes in axially symmetrical structures consisting of elements with different electric properties (dielectric constant and magnetic permeability, conductivity) is developed. The form of the calculation area is supposed to be rectangular, limits of subareas (elements) separation may be random. There is given a description of applied difference scheme on regular inhomogeneous grid and their realization in the computer program. There is given an example of the program application for calculating electromagnet processes in STRAUS-2 accelerator [4] and their comparison with experimental results.

1. SETTLEMENT OF THE PROBLEM

Electric and magnetic fields in the systems with distributed parameters satisfy the non-stationary Maxwell equations:

$$\mathrm{rot}\,\vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j}\,, \tag{1}$$

$$\mathrm{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}\,, \tag{2}$$

$$\mathrm{div}\,\vec{D} = \rho\,, \tag{3}$$

$$\mathrm{div}\,\vec{B} = 0\,, \tag{4}$$

where $\vec{E}$ and $\vec{D}$ - electric field strength and induction; $\vec{H}$ - magnetic field strength; $\vec{B}$ - magnetic induction; $\vec{j}$ - current density, moreover $j = j_{ext} + j_c$, where j_c is calculated in the media with known conductivity, $\vec{j}_{ext}$ in other cases. Such a presentation of current density was made for convenience of calculation program realization. Fields and currents of conductivity are connected by the following additional ratios:

$$\vec{D} = \varepsilon\varepsilon_0\vec{E}\,,\ \vec{B} = \mu\mu_0\vec{H}\,,\ \vec{j} = \sigma\vec{E}\,,$$

where ε_o, μ_o - dielectric constant and magnetic permeability of vacuum, ε, μ - relative dielectric constant and magnetic permeability of material media.

Due to axial geometry symmetry in the system there exist electromagnet waves of TE and TM type, being not connected to each other. In our case Maxwell equations can be solved only for TM modes [5]. In the coordinate form they are recorded in the following way:

$$\frac{\partial B_\theta}{\partial t} = \frac{\partial E_Z}{\partial r} - \frac{\partial E_r}{\partial z}\,, \tag{5}$$

$$\frac{\partial D_r}{\partial t} = -\frac{\partial H_\theta}{\partial z} - \sigma E_r - j_r\,, \tag{6}$$

$$\frac{\partial D_Z}{\partial t} = \frac{1}{r}\frac{\partial (rH_\theta)}{\partial r} - \sigma E_Z - j_Z\,. \tag{7}$$

The equations system (5-7) is solved with the following boundary conditions [5-7]:

-metal surface: $E\tau=0$;

-boundary of media separation with different electric and magnetic properties, while free charges and currents are absent on the separation surface:

$$\begin{cases} E_{1\tau} = E_{2\tau}\,, \\ D_{1n} = D_{2n}\,, \\ H_{1\tau} = H_{2\tau}\,, \end{cases}$$

where indexes n and τ mean normal and tangent components of the field;

-symmetry axis:

$$\begin{cases} E_r = 0, \\ H_\theta = 0; \end{cases}$$

-open boundary with radiation constraint (input and output TM wave):

$$\sqrt{\varepsilon\varepsilon_0}E_\tau \pm \sqrt{\mu\mu_0}H_\theta = 0\,.$$

At the initial time moment (t=0) the electric fields and magnetic field $H_\theta=0$ are supposed to be given in the calculation area. The second constraint is applicable only for devices with capacity energy accumulation. At non-significant updating the program can be also used for devices with inductive energy accumulation. Distribution of the initial electric field is found through solution of Laplace [8] equation with corresponding boundary conditions:

$$\Delta\phi = 0\,,$$

where ϕ - potential of electric field.

2. DIFFERENCE SCHEME OF MAXWELL EQUATION SYSTEM SOLUTION

The equation system (5-7) is solved on the rectangular irregular grid circuit. For solving there is used a difference scheme with overstepping of the accuracy $O\left(\Delta t^2 + \Delta r^2 + \Delta z^2\right)$. The equations are approximated by central differences in the following way:

$$\left(\mu\frac{\Delta H_\theta}{\Delta t}\right)^n = \left(\frac{\Delta E_z^n}{\Delta r} - \frac{\Delta E_r^n}{\Delta z}\right), \tag{8}$$

$$\left(\varepsilon \frac{\Delta E_r}{\Delta t}\right)^{n+0.5} = -\frac{\Delta H_\theta^{n+0.5}}{\Delta z} - \sigma E_r^{n+0.5} - j_r^{n+0.5}, \qquad (9)$$

$$\left(\varepsilon \frac{\Delta E_z}{\Delta t}\right)^{n+0.5} = \frac{1}{r}\frac{\Delta\left(rH_\theta^{n+0.5}\right)}{\Delta r} - \sigma E_z^{n+0.5} - j_z^{n+0.5}, \qquad (10)$$

where

$$\left(\mu \frac{\Delta H_\theta}{\Delta t}\right)^{n} = \mu_{i+0.5,j+0.5} \frac{\left(H_\theta\right)^{n+0.5}_{i+0.5,j+0.5} - \left(H_\theta\right)^{n+0.5}_{i+0.5,j+0.5}}{\Delta t},$$

$$\frac{\Delta E_z^n}{\Delta r} = \frac{\left(E_z\right)^n_{i+0.5,j+1} - \left(E_z\right)^n_{i+0.5,j}}{r_{j+1} - r_j},$$

$$\frac{\Delta E_r^n}{\Delta z} = \frac{\left(E_r\right)^n_{i+1,j+0.5} - \left(E_r\right)^n_{i,j+0.5}}{z_{i+1} - z_i},$$

$$\left(\varepsilon \frac{\Delta E_r}{\Delta t}\right)^{n+0.5} = \varepsilon_{i,j+0.5} \frac{\left(E_r\right)^{n+1}_{i,j+0.5} - \left(E_r\right)^n_{i,j+0.5}}{\Delta t},$$

$$\frac{\Delta H_\theta^{n+0.5}}{\Delta z} = \frac{\left(H_\theta\right)^{n+0.5}_{i+0.5,j+0.5} - \left(H_\theta\right)^{n+0.5}_{i-0.5,j+0.5}}{z_{i+0.5} - z_{i-0.5}},$$

$$\sigma E_r^{n+0.5} = \sigma_{i,j+0.5}\left(\left(E_r\right)^{n+1}_{i,j+0.5} + \left(E_r\right)^n_{i,j+0.5}\right)/2,$$

$$\left(\varepsilon \frac{\Delta E_z}{\Delta t}\right)^{n+0.5} = \varepsilon_{i+0.5,j} \frac{\left(E_z\right)^{n+1}_{i+0.5,j} - \left(E_z\right)^n_{i+0.5,j}}{\Delta t},$$

$$\frac{1}{r}\frac{\Delta\left(rH_\theta^{n+0.5}\right)}{\Delta r} = \frac{r_{j+0.5}\left(H_\theta\right)^{n+0.5}_{i+0.5,j+0.5} - r_{j-0.5}\left(H_\theta\right)^{n+0.5}_{i+0.5,j-0.5}}{r_j\left(r_{j+0.5} - r_{j-0.5}\right)},$$

$$\sigma E_z^{n+0.5} = \sigma_{i+0.5,j}\left(\left(E_z\right)^{n+1}_{i+0.5,j} + \left(E_z\right)^n_{i+0.5,j}\right)/2.$$

Here Δt – time step, n index - number of time layer, i and j indexes correspond to the numbers of the grid points by z and r, $t^{n+0.5} = 0.5(t^{n+1} + t^n)$, half-integer indexes i+0.5 and j+0.5 correspond to the center of cells by r and z. In this scheme there occurs averaging of magnitudes ε, μ and σ by cells areas. The function data are defined in the cell center, and averaging scheme is specified by the formula:

$$f_{i+0.5,j} = \frac{r_{j+0.5}^2 - r_j^2}{r_{j+0.5}^2 - r_{j-0.5}^2} f_{i+0.5,j+0.5} + \frac{r_j^2 - r_{j-0.5}^2}{r_{j+0.5}^2 - r_{j-0.5}^2} f_{i+0.5,j-0.5},$$

$$f_{i,j+0.5} = \frac{z_{i+0.5} - z_i}{z_{i+0.5} - z_{i-0.5}} f_{i+0.5,j+0.5} + \frac{z_i - z_{i-0.5}}{z_{i+0.5} - z_{i-0.5}} f_{i-0.5,j+0.5}.$$

Scheme (8-10) is steady when performing inequality for integration step Δt:

$$\Delta t < \tau_{min} = \min(\tau_c),$$

where τ_c magnitude is determined from the equation [7]:

$$(1 - \sigma\tau_{яч}) = \sqrt{\varepsilon\mu}\,\tau_{mesh}^2\left(h_r^{-2} + h_z^{-2}\right),$$

here h_r and h_z - cell sizes by r and z, correspondingly.

The difference scheme (8-10) was implemented in the WEC program, written in Fortran language.

3. CALCULATION RESULTS

With the aid of WEC program there were conducted calculations of transient electromagnet processes in STRAUS-2 pulsed electron accelerator [4], whose calculation geometry is given in Fig.1. The accelerator consists of the system of high-voltage pulse formation 1 and accelerating tube with a diode unit. The forming system was implemented on the basis of a double-step forming line (DSFL).

DSFL consists of 5 sections of similar lines of approximately equal electric lengths of 18 ns with impedances, optimized for obtaining maximal voltage transformation factor. DSFL switching is performed through a multichannel switch, formed by 20 gas-filled switch of trigatron type 2, uniformly distributed by azimuth. The first positive voltage pulse is cut off by the pre-pulse switch 3. The second working pulse goes onto the diode and, due to this process, the electron beam current pulse is formed.

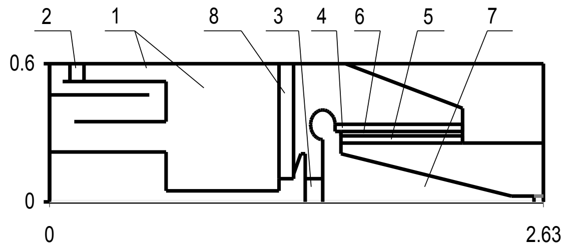


Fig.1. Calculation geometry of STRAUS-2 accelerator

The accelerating tube is formed by a polyethylene tube 4 and a section insulator 5, the electrolyte layer 6 is placed between them and serves for equalling electric potential distribution by insulator length. In the inner vacuum volume of accelerating tube there is located a cathode 7. The diode unit is separated from the forming system by a polyethylene diaphragm 8, the volume between it and the accelerating tube is filled by transformator oil.

At the initial time moment (t=0) DSFL, whose high-voltage electrode is charged up to 630 kV voltage, begins to discharge through a ring switch. The fields distribution at such a switching represents a superposition of waves spreading from individual spark channels. In this case consideration of all the fields requires solution of the three-dimensional problem.

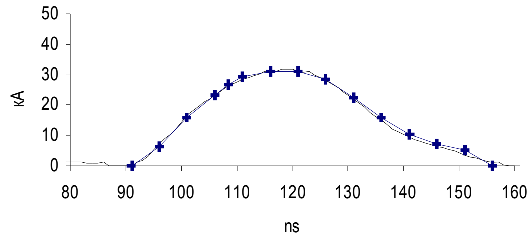
The numerical method, accepted in the paper, requires such a description of the switch, in order to provide a possibility to bring a three-dimensional waves propagation process description to a two-dimensional process with averaging by azimuth coordinate; this can be realized through a change of the multichannel switch for a completely ring switch in the calculation model. In paper [9] it is shown that the multichannel switch with accuracy, being sufficient for practice, can be described by the axially symmetric ring switch with inductance, which is constant in time.

When switching the pre-pulse switch there appear no waves, which are inhomogeneous in azimuth direction, that is why the given switch is completely inscribed in the calculation diagram and it is described by inductance.

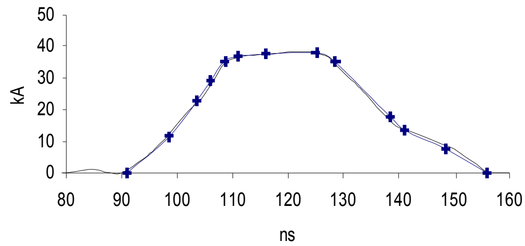
Total current in the sub-area occupied by the switch, is calculated by the formula:

$$I = \frac{1}{L}\int_0^t U d\tau$$

where U - voltage between the switch electrodes, L - its inductance. From the calculated I value there is determined the current density in the area occupied by the switch, which is substituted in the equations, in order to calculate electric fields (9, 10).



a)



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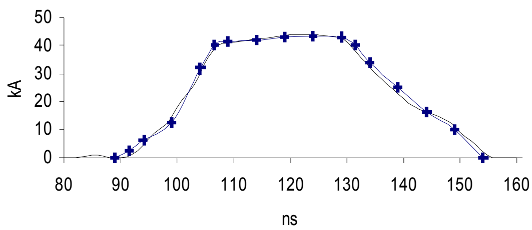

c)

Fig.2. Calculation and experimental (+) current pulses in the diode at electrolyte load impedance 29 Ohm (a), 100 Ohm (b), 196 Ohm (c).

In the calculation model the cathode-anode gap of diode is represented by impedance, which is cut in at negative (accelerating) diode voltage.

When calculating STRAUS-2 accelerator the ring switch inductance is 2.7 nH, pre-pulse switch - 70 nH and diode impedance - 75 Ohm.

There were conducted calculations of one of operation modes of STRAUS-2 accelerator for values of electrolyte resistance 29,100,196 Ohm. Calculation diode current pulses were compared to current pulses measured experimentally. From the comparison it is clear that the difference of time-amplitude parameters of calculation pulse from those of experimental current pulse do not exceed 10%, what is within the accuracy of measurements.

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