SEARCHING FOR OPTIMAL PARAMETERS WHILE DEVELOPING CAD SYSTEM FOR STELLARATOR MAGNETIC TRAPS

V.P. Vorobyova, V.P. Lukyanova, S.A. Martynov, M.A. Khazhmuradov

National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine
e-mail: khazhm@kipt.kharkov.ua

This article is about development of mathematical model, algorithms and software for determining optimal parameters of magnetic field of toroidal stellarator magnetic systems.
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1. INTRODUCTION

Toroidal stellarator magnetic trap is based on plasma confinement in magnetic field with rotational transformation of force lines. Such field has some feature, and the main one is magnetic surface, created with single non-closed magnetic helical force line. Topologically magnetic surface can be only toroidal. Enclosed in each other toroidal surfaces are used for practical purposes (Fig. 1) [1]. Such surfaces have two extreme values. On the one hand transversal dimension tending to zero gives us conception of magnetic axis, which is closed plane or space curve. On the other hand it gives us conception of maximal boundary surface, which is called separatrix and characterize maximum holding volume when configuration of outer (with respect to plasma) conductors with current is given [2].

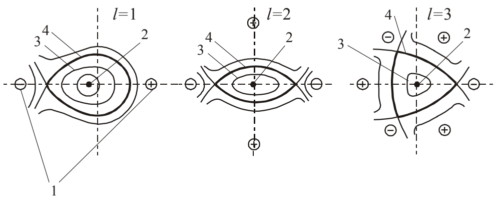

Fig. 1. *Transversal sections of magnetic fields in stellarators: 1 – current filaments traces (plus or minus sign shows current direction); 2 – magnetic axis; 3 – set of enclosed magnetic surfaces; 4 – separatrix*

Stellarator magnetic configuration has several important characteristics besides maximum holding volume. They are: helical force line pitch L; rotational transformation angle $t = \Pi / L$ (Π – magnetic axis perimeter); toroidal and poloidal fluxes Φ and χ (Φ – magnetic field flux through magnetic surface transversal section, and χ – flux through surface bounded with magnetic axis and similar line on the magnetic surface). Volume V and poloidal flux χ can be considered and function of toroidal flux. In this case $t = d\chi / d\Phi$, and

$$dV / d\Phi = V' = L_S / \langle B \rangle , \tag{1}$$

where $\langle B \rangle$ – average magnetic field, L_S – length of force line at one full diversion round toroid. Transformation angle t depends on average surface distance from axis r. Value

$$\frac{dt}{d\,\mathrm{r}} \cdot \frac{\mathrm{r}^2}{R_1} = S \tag{2}$$

is called shear. Here R_1 is average radius of curvature of magnetic axis.

On the magnetic surfaces $t(\mathrm{r})$ is irrational. Inverse to $t(\mathrm{r})$ value, $1/t(\mathrm{r}) = q$, is called stability factor.

Magnetic hole and magnetic field strength modulation down the force line are also the characteristics of magnetic configuration. First conception characterizes possibility of plasma stabilization in magnetic trap. Effect of creation of configuration with magnetic field increasing towards outer plasma border is used for this purposes. Quantitatively, this value is characterized with fluctuating of integral

$$U = \int \frac{d\ell}{B} , \tag{3}$$

taken down force line in the direction from inner magnetic surfaces to outer ones. Second conception characterizes field spiral heterogeneity and is determined by formula

$$\zeta_h = \frac{B_h}{B_0} , \tag{4}$$

where B_h – spiral harmonics magnetic field amplitude, B_0 – toroidal field average value.

Stellarator magnetic field can be determined in different ways [1]. Its magnetic characteristics, form and dimension depend on studied stellarator system type.

The aim of this article is to determine values of the above mentioned parameters which provide optimum choice from engineering, technological and physical point of view.

2. PROBLEM FORMALIZATION

Problem of searching of magnetic configuration optimal characteristics belongs to set of multicriteria optimization tasks [3]. While its solving it is necessary to take into account several features:

- large overall dimensions of stellarator system magnetic configuration;
- complex spatial form;
- magnetic system configuration consists from similar parts, where magnetic field characteristics are the same;
- similarity of magnetic configuration geometric characteristics and vacuum chamber ones (vacuum chamber is one the most complex units of stellarator system;

➢ usage of infinity thin current filaments as magnetic winding model at the first designing stages.

Let's consider stellarator magnetic system [1], which provides minimum costs while its making and exploitation and simultaneously high holding features of magnetic field. Then problem of optimal design choosing cam be formulated in the following way. It is necessary to determine such spatial location of current filaments that provide the best magnetic characteristics of created field. The best characteristics are interpreted as determining maximum volume of bounded with separatrix stellarator magnetic configuration at maximum angle of rotational transformation, presence of shear, magnetic hole and favorable field intensity modulation down force line. This characteristics are determined with directed parameters. They are:

➢ geometric configuration of torus surface (Fig. 2), its surface and dimensions in selected coordinate system. Let's denote them as vector

$$X_1 = f(R, a, a_1, b_1, c, d, e, g, h), \quad (5)$$

where R – longer radius of torus, a – its shorter radius, a_1 and b_1 – longer and shorter ellipse axes (if meridian section is ellipse), c and d – edges lengths (if meridian section is triangle or rectangle);

➢ values from the law of current filaments winding at the torus surface;

$$\varphi = \frac{1}{m_h}(\vartheta - \alpha \sin\vartheta - \beta \sin 2\vartheta) + \frac{2\pi}{\ell}k, \quad (6)$$

where φ – location angle of meridian section through point M on the winding line, α, β – modulation coefficients, m_h – number of spiral conductor pitches, ℓ – number of spiral conductors sets, $k = 0, 1, \ldots, m-1$ – pole number;

➢ values from the Biot-Savart law

$$B = \frac{J}{c}\oint\frac{|d\ell \cdot S|}{S^3}, \quad (7)$$

where $S = (r_1 - r_0)$ – radius-vector form current element Jdl to point of observation, c – electrodynamic constant, J – current in conductor.

Thus, set of constraints can be considered as follows

$$X_1 = f_1(R, a, a_1, b_1, c, d, e, g, h) \quad (8)$$

$$X_2 = f_2(\vartheta, \alpha, \beta, m_h, l, k) \quad (9)$$

$$X_3 = f_3(J, dl, S) \quad (10)$$

Expression (8) is geometric constraints, (9) – is parameters that determines the law of spiral conductors placing at the torus surface, (10) – parameters that determine direction and value of magnetic field in considered point belonging to magnetic surface. Later on expressions (8-10) will be called directed parameters.

So formulated task can be reduced to the task of finding of optimal parameters $X(X = X(X_1, X_2, X_3))$, which provide maximum value of criterion functional [3]

$$\max_{X \in D} Q(X), \quad (11)$$

where D – parameters X admitted region.

Thus, solving of optimal design problem reduces to solving of criterion functional determining problem (11). This task is nonlinear programming one, because optimization criterion (separatrix) and set of constraints are nonlinear functions of parameter *X*. Let's use direct search methods to solve this task. Algorithm for this task can be given in the following way:

➢ spatial location of separatrix, its form and dimensions are determined;

➢ field characteristics are calculated using numerical methods;

➢ parameters satisfying criterion functional are chosen from the set of allowed values of directed parameters;

➢ rotational transformation angle, shear, magnetic hole value, magnetic field strength modulation down the force line is calculated using known mathematical formulas.

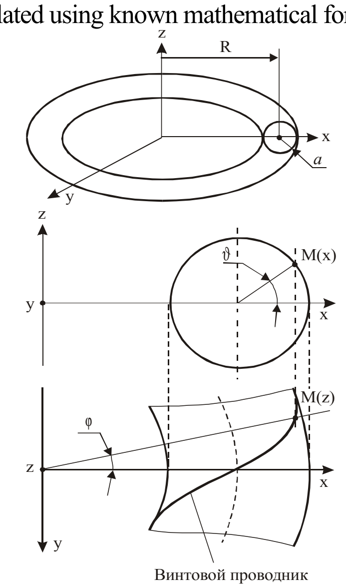

Fig. 2. *Torus surface model: XYZ – rectangular coordinate system; M – point on spiral conductor;* ϑ, φ *– toroidal coordinates of point M*

3. SOFTWARE AND REALIZATION ALGORITHMS

Computer program for obtaining of separatrix 3D-model was developed using MS Visual C++ 6.0 and mathematical models of algorithms. This program provides obtaining of necessary data for technological and engineering development of vacuum chamber. User interface meets both general requirements and specific ones, which are results of kinematic modeling method usage. First of all system have to be interactive graphical one, allowing modeling main designer operations connected with surface magnetic configuration design. Methods of graphical interface organization are well studied now. Let's dwell on the requirements to which special attention was paid.

1. User must see on the screen all the necessary objects and easily orient among them.

2. Number of steps, necessary for execution of each operation, must be minimal.

3. User must work mainly this 2D locator.

4. All necessary information must be on the screen.

Existing program libraries were widely used when developing program. It made possible to reduce considerably time for its writing and debugging. In our case library MFC 6.0. was used under platform MS Windows. Stinqray Objective TooPkit 6.0 library was used to improve program interface. TGS Open Inventor V2.62 library and its enhancement TGS 3D Mastersuite V3.62. was used for obtaining of 3D image. Both libraries are high-level and build on top of Open GL.

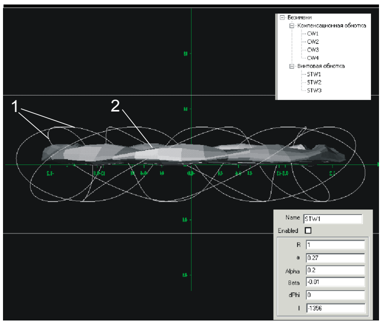

Fig. 3. *Separatrix front projection*

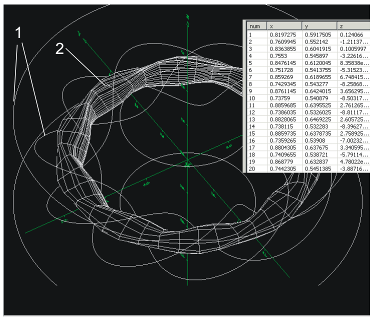

Fig. 4. *Separatrix raster image. Characteristic point coordinates are given in the table*

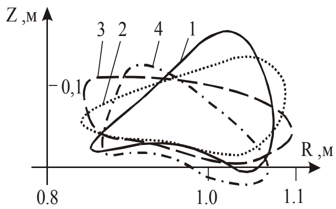

Fig. 5. *Geometry of separatrix meridian sections set (pos. 1, 2, 3, 4)*

Next methods can be considered among algorithms:

- ➢ methods of determining of magnetic field force line trajectory;
- ➢ method of kinematic modeling, as most suitable for our case among other methods of corset tightening [4];
- ➢ usage of Bezier spline determining of points coordinates when points are located arbitrarily at modulated surface [5];
- ➢ raster presentation of modeled surface [6].

Results of such algorithms realization are given on Fig. 3-5. Separatrix front projection using penumbra effects are shown on Fig. 3.

Separatrix raster presentation is shown on Fig. 4 characteristic point coordinates of magnetic force line at meridian section are given in the table.

Set of separatrix meridian sections is shown on Fig 5. This set makes possible to implement technology of vacuum chamber producing.

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ПОИСК ОПТИМАЛЬНЫХ ПАРАМЕТРОВ ПРИ СОЗДАНИИ САПР СТЕЛЛАРАТОРНЫХ МАГНИТНЫХ ЛОВУШЕК

В.П. Воробьева, В.П. Лукьянова, С.А. Мартынов, М.А. Хажмурадов

Приведено описание функции цели при решении задач многокритериальной оптимизации магнитных характеристик стеллараторных систем. Сформулированы исходные данные и ограничения на расчетные параметры. Получена 3D-модель граничной магнитной поверхности, для чего разработаны соответствующие математические модели, алгоритмы и компьютерные программы.

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В.П. Воробйова, В.П. Лук'янова, С.О. Мартинов, М.А. Хажмурадов

Приведено опис функції цілі при вирішенні задач багатокритеріальної оптимізації магнітних характеристик стелараторних систем. Сформульовано вихідні дані та обмеження на розрахункові параметри. Отримано 3D-модель граничної магнітної поверхні, для чого було розроблено відповідні математичні моделі, алгоритми та комп'ютерні програми.