ESTIMATION OF DIMENSIONS OF INTERACTION SPACE NECESSARY FOR PARTICLE ACCELERATION IN STOCHASTIC FIELD

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(Received Month 24, 2014)

It is considered electron acceleration in the field having time dependence, which differs from sinusoidal one with phase jumps introduction. The connection is found between electron energy and characteristic dimension of device, in which such energy may be achieved with considered acceleration method or with collision heating. It is shown that the acceleration in the field with jumps of phase requires much greater dimension of space for electron motion than the heating to the same energy in sinusoidal field in presence of elastic collisions.

PACS: 29.17.+w, 52.80.Pi

1. INTRODUCTION

One of the methods of electron energy increase in high frequency low-pressure gas discharge is so called collisionless heating, which recently attracts some attention ([1], [2]). In fact, in this method, electrons are subjected to the force, which time dependence is complicated, but determined completely, regardless of electron motion, and another forces are absent (in particular, collisions are absent). So, electrons are accelerated in a given field, as it takes place in accelerators, and the question arises naturally about the use of such devices as charged particle accelerators. In the present work, it is made the estimation of device dimension necessary for electron acceleration up to a given energy in the field with phase jumps similar to one described in [1], [2]. As the effectiveness of such field use is substantiated there with aid of analogy with collision heating, the relevant dimension is also estimated here in the case of collision heating, and for the case of phase jumps the statistical characteristics of the jumps are taken according to the different variants of ones of collisions.

2. MOTION IN THE FIELD WITH FULLY ACCIDENTAL PHASE JUMPS

In this case, the values of phase before and after jump are assumed to be independent: at $t \in (0, t_1)$ the electron velocity time dependence is

$$v(t) = v_0 + u\cos(\omega t + \varphi_0), \qquad (1)$$

at once after t_1 the dependence is

$$v(t) = v_1 + u\cos[\omega(t - t_1) + \varphi_1], \qquad (2)$$

and the values of φ_0 and φ_1 are independent. Due to continuity of velocity at $t = t_1$, the equality

$$v_1 = v_0 + u\cos(\omega t_1 + \varphi_0) - u\cos\varphi_1 \qquad (3)$$

takes place. With integration of velocity over time interval $(0, t_1)$, one comes to the equality

$$z_1 = z_0 + v_0 t_1 + (u/\omega)[\sin(\omega t_1 + \varphi_0) - \sin\varphi_0], \qquad (4)$$

which connects the coordinates z_0 and z_1 at time instants 0 and t_1. From the equalities (3) and (4), averaging the products and squares of their different parts over φ_0 and φ_1 independently, denoting such averages by angle brackets with index φ, and taking into account the equality $2\langle\sin(\alpha + \varphi_0)\sin\varphi_0\rangle_\varphi = \cos\alpha$ (for a fixed α), one gets the equalities
$\langle v_1^2\rangle_\varphi = v_0^2 + u^2$,
$\langle z_1 v_1\rangle_\varphi = (z_0 + v_0 t_1)v_0 + u^2(2\omega)^{-1}\sin(\omega t_1)$,
$\langle z_1^2\rangle_\varphi = (z_0 + v_0 t_1)^2 + (u/\omega)^2[1 - \cos(\omega t_1)]$.
The next averaging is carried out over the values of t_1, with use of statistical characteristics of Poisson process having frequency ν. Taking into account the equality $\int_0^\infty \nu\exp(-\nu t + i\omega t)\,dt = \nu(\nu - i\omega)^{-1}$, one comes to the equalities
$\langle z_1 v_1\rangle = z_0 v_0 + v_0^2\nu^{-1} + u^2\nu\left[2\left(\nu^2 + \omega^2\right)\right]^{-1}$,
$\langle z_1^2\rangle = z_0^2 + 2z_0 v_0\nu^{-1} + 2v_0^2\nu^{-2} + u^2\left(\nu^2 + \omega^2\right)^{-1}$
(angle bracket without indexes means the last averaging). For the velocity and coordinate after the great number N of phase jumps, when full velocity becomes much greater than oscillation velocity, $\langle v_N^2\rangle \gg u^2$, one gets the relationships $\langle v_{N+1}^2\rangle - \langle v_N^2\rangle = u^2$,
$\langle z_{N+1}v_{N+1}\rangle - \langle z_N v_N\rangle \approx \nu^{-1}\langle v_N^2\rangle$,
$\langle z_{N+1}^2\rangle - \langle z_N^2\rangle \approx 2\nu^{-1}\langle z_N v_N\rangle$.
Taking the sums over jumps, one obtains

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$$\left\langle v_N^2\right\rangle \approx u^2 N, \left\langle z_N v_N\right\rangle \approx (2\nu)^{-1} u^2 N^2,$$

$$\left\langle z_N^2\right\rangle \approx \left(3\nu^2\right)^{-1} u^2 N^3 \approx \left(3\nu^2 u^4\right)^{-1} \left\langle v_N^2\right\rangle^3. \qquad (5)$$

3. MOTION IN SINUSOIDAL FIELD WITH ELASTIC COLLISIONS

In this case, the phases in (1) and (2) are not independent, but connected with the equality

$$\varphi_1 = \omega t_1 + \varphi_0. \qquad (6)$$

As collisions are elastic, at the instant of collision the absolute value of velocity is not changed, but the motion direction is changed, so that the equality $v_1 + u\cos\varphi_1 = -v_0 - u\cos(\omega t_1 + \varphi_0)$ is kept. Taking into account the equality (6), from the equalities $v_1 = -v_0 - 2u\cos(\omega t_1 + \varphi_0)$ and (4), with taking of products and squares of their different parts and averaging over φ_0, one gets the equalities $\left\langle v_1^2\right\rangle_\varphi = v_0^2 + 2u^2$, $\left\langle z_1 v_1\right\rangle_\varphi = -(z_0 + v_0 t_1) v_0 - u^2\omega^{-1}\sin(\omega t_1)$, $\left\langle z_1^2\right\rangle_\varphi = (z_0 + v_0 t_1)^2 + (u/\omega)^2 [1 - \cos(\omega t_1)]$.
The next averaging, over the value of t_1, according to Poisson process with the frequency ν_0, yields $\left\langle z_1 v_1\right\rangle = -z_0 v_0 - v_0^2\nu_0^{-1} - u^2\nu_0\left(\nu_0^2 + \omega^2\right)^{-1}$, $\left\langle z_1^2\right\rangle = z_0^2 + 2z_0 v_0\nu_0^{-1} + 2v_0^2\nu_0^{-2} + u^2\left(\nu_0^2 + \omega^2\right)^{-1}$.
For the next collision, keeping the possibility of frequency change in Poisson process, with indexes change one comes to the equalities
$\left\langle z_2 v_2\right\rangle = -z_1 v_1 - v_1^2\nu_1^{-1} - u^2\nu_1\left(\nu_1^2 + \omega^2\right)^{-1}$,
$\left\langle z_2^2\right\rangle = z_1^2 + 2z_1 v_1\nu_1^{-1} + 2v_1^2\nu_1^{-2} + u^2\left(\nu_1^2 + \omega^2\right)^{-1}$.
For the averaged result of two successive collisions the equalities

$$z_2 v_2 - z_0 v_0 = v_0^2\nu_0^{-1} - v_1^2\nu_1^{-1} + (\nu_1 - \nu_0) \times$$
$$\times u^2\left(\nu_0\nu_1 - \omega^2\right)\left[\left(\nu_0^2 + \omega^2\right)\left(\nu_1^2 + \omega^2\right)\right]^{-1},$$
$$z_2^2 - z_0^2 = 2v_1^2\nu_1^{-2} + (\nu_1 - \nu_0)\nu_1^{-1} \times$$
$$\times\Big\{2z_0 v_0\nu_0^{-1} + 2v_0^2\nu_0^{-2} + \left[2\omega^2 + \nu_1(\nu_1 - \nu_0)\right] \times$$
$$\times u^2\left[\left(\nu_0^2 + \omega^2\right)\left(\nu_1^2 + \omega^2\right)\right]^{-1}\Big\}$$

take place (averaging symbol in them is not written).

It is assumed that collision frequency is much less than oscillation frequency and the full velocity is much greater than the oscillation velocity. The dependence of the collision frequency on the electron energy may influence on the rate of increase of the averaged squares of coordinate and velocity.

Let us consider two cases, in which from electron energy it is independent either mean free path λ, or collision frequency ν, respectively.

If $\lambda = \text{const}$, then for the velocity and coordinate after the great number N of collisions, taking into account the relationship $\left\langle \nu_N\right\rangle \approx \left\langle v_N^2\right\rangle^{1/2}\lambda^{-1}$, one gets $\left\langle v_{N+1}^2\right\rangle - \left\langle v_N^2\right\rangle = 2u^2$, $\left\langle v_N^2\right\rangle \approx 2u^2 N$,

$$\left\langle z_{N+2} v_{N+2}\right\rangle - \left\langle z_N v_N\right\rangle \approx$$
$$\approx\Big(\left\langle \nu_N\right\rangle^{-1}\left\langle v_N^2\right\rangle - \left\langle \nu_{N+2}\right\rangle^{-1}\left\langle v_{N+2}^2\right\rangle\Big)/2,$$

$$\left\langle z_N v_N\right\rangle \approx -\left\langle \nu_N\right\rangle^{-1}\left\langle v_N^2\right\rangle/2, \left\langle z_{N+2}^2\right\rangle - \left\langle z_N^2\right\rangle \approx 2\lambda^2,$$
$$\left\langle z_N^2\right\rangle \approx \lambda^2 N \approx \left(2u^2\right)^{-1}\lambda^2\left\langle v_N^2\right\rangle. \qquad (7)$$

But if $\nu = \text{const}$ then the relevant relationships have the following form:

$$\left\langle v_N^2\right\rangle \approx 2u^2 N,$$
$$\left\langle z_{N+2}^2\right\rangle - \left\langle z_N^2\right\rangle \approx 2\nu^{-2}\left\langle v_N^2\right\rangle,$$
$$\left\langle z_N^2\right\rangle \approx (u/\nu)^2 N^2 \approx \left(4u^2\nu^2\right)^{-1}\left\langle v_N^2\right\rangle^2. \qquad (8)$$

4. MOTION IN THE FIELD WITH CORRELATED PHASE JUMPS

In this case, it is assumed that the phase at instant of jump is changed so, that field strength projection on the electron motion direction at any time is the same, as one in the considered process with elastic collisions in the sinusoidal field, and difference is only in absence of change of motion direction at instant of phase jump. In such case, at once after time instant t_1 the velocity is changed according to the time dependence $v(t) = v_1 - u\cos[\omega(t - t_1) + \varphi_1]$, and the equalities (6) and
$v_1 - u\cos\varphi_1 = v_0 + u\cos(\omega t_1 + \varphi_0)$ are kept, to ensure the continuity of velocity at the instant of jump and the correspondence of the values of acceleration projection on the electron motion direction to ones in elastic collisions. From the equalities $v_1 = v_0 + 2u\cos(\omega t_1 + \varphi_0)$, and (4), with taking of products and squares of their different parts and averaging by φ_0, one comes to the equalities $\left\langle v_1^2\right\rangle_\varphi = v_0^2 + 2u^2$, $\left\langle z_1 v_1\right\rangle_\varphi = (z_0 + v_0 t_1) v_0 + u^2\omega^{-1}\sin(\omega t_1)$, $\left\langle z_1^2\right\rangle_\varphi = (z_0 + v_0 t_1)^2 + (u/\omega)^2 [1 - \cos(\omega t_1)]$.
The next averaging, over t_1, according to Poisson process with frequency ν_0, gives the equalities $\left\langle z_1 v_1\right\rangle = z_0 v_0 + v_0^2\nu_0^{-1} + u^2\nu_0\left(\nu_0^2 + \omega^2\right)^{-1}$, $\left\langle z_1^2\right\rangle = z_0^2 + 2z_0 v_0\nu_0^{-1} + 2v_0^2\nu_0^{-2} + u^2\left(\nu_0^2 + \omega^2\right)^{-1}$.
For the velocity and coordinate after the great number N of phase jumps one comes to the relationships $\left\langle v_{N+1}^2\right\rangle - \left\langle v_N^2\right\rangle = 2u^2$, $\left\langle v_N^2\right\rangle \approx 2u^2 N$, $\left\langle z_{N+1} v_{N+1}\right\rangle - \left\langle z_N v_N\right\rangle \approx \left\langle \nu_N\right\rangle^{-1}\left\langle v_N^2\right\rangle$, $\left\langle z_{N+1}^2\right\rangle - \left\langle z_N^2\right\rangle \approx 2\left\langle \nu_N\right\rangle^{-1}\left\langle z_N v_N\right\rangle$.

If the frequency of phase jumps is changed so, that $\left\langle \nu_N\right\rangle^{-1}\left\langle v_N^2\right\rangle^{1/2} \approx \lambda = \text{const}$, then one gets $\left\langle z_{N+1} v_{N+1}\right\rangle - \left\langle z_N v_N\right\rangle \approx \lambda u(2N)^{1/2}$, $\left\langle z_N v_N\right\rangle \approx \lambda u(2N)^{3/2}/3$, $\left\langle z_{N+1}^2\right\rangle - \left\langle z_N^2\right\rangle \approx 4\lambda^2 N/3$,

$$\left\langle z_N^2\right\rangle \approx 2\lambda^2 N^2/3 \approx \left(6u^4\right)^{-1}\lambda^2\left\langle v_N^2\right\rangle^2. \qquad (9)$$

But if the frequency of phase jumps is constant, $\nu = \text{const}$, then the corresponding relationships have the following form: $\left\langle z_N v_N\right\rangle \approx u^2\nu^{-1}N^2$,

$$\left\langle z_N^2\right\rangle \approx 2(u/\nu)^2 N^3/3 \approx \left(12u^4\nu^2\right)^{-1}\left\langle v_N^2\right\rangle^3. \qquad (10)$$

5. CONCLUSIONS

The relationships (5), (7–10), for the different methods of electron energy increase, give the square of the characteristic dimension of the device, in which a given value of square of velocity may be achieved. The relationships (7), (8) deal with collision heating, in the different assumptions, and the relationships (5), (9), (10) deal with electron motion without collisions in the field with phase jumps. In the case (5), the values of phase before and after jump are independent, but in the cases (9) and (10) they are connected in such a way as ones in elastic collisions. Comparing of the relationships (5) and (10) with (8) shows that the similarity between electron energy increase due to the phase jumps and due to the elastic collisions is not full. If the frequency of phase jumps is close to the frequency of elastic collisions then in all cases the energy gained by electron is proportional to the number of phase jumps or elastic collisions. But the square of dimension of the motion space necessary to achieve a given energy in the case of the acceleration in the field with phase jumps is proportional to the third power of the energy, whereas in the case of the collision heating it is proportional to the second power of the energy, so, the collision heating requires much less device dimensions than the collisionless heating. The main cause is change of the electron motion direction in collisions, in contrast to the case of motion in the field with phase jumps, where only the acceleration value is changed at instant of the phase jump, and an electron continues to move in the same direction as before the phase jump.

As for comparing of the acceleration in the field with phase jumps and the acceleration in the constant field, it is obvious that any decrease of the field strength from the maximum attainable value (and so much the greater, the change of the strength direction) leads to decrease of the energy passed from field to electron during the same displacement, to decrease of acceleration rate, and to increase of necessary device dimensions.

It may be paid attention to the fact that when electron energy increases due to elastic collisions the relationship between the square of characteristic dimension of the space of electron motion and the square of energy gained by electron is the same as it is when electron is accelerated in the constant field. However, it is obvious that collision heating is suitable for electron energy increase only up to energy values, at which the cross-sections of non-elastic processes are relatively small.

References

1. V.I. Karas', Ja.B. Fainberg, A.F. Alisov, et al. Interaction with plasma and gases of microwave radiation with stochastically jumping phase // *Plasma Physics Reports.* 2005, v. 31, p. 748-760.

2. V.I. Karas', A.F. Alisov, O.V. Bolotov, et al. Optical radiation special features from plasma of low-pressure discharge initiated by microwave radiation with stochastic jumping phase // *PAST. Series: Plasma Electronics and New Methods of Acceleration.* 2013, N4(86), p. 183-188.

ОЦЕНКА РАЗМЕРОВ ОБЛАСТИ ВЗАИМОДЕЙСТВИЯ, НЕОБХОДИМОЙ ДЛЯ УСКОРЕНИЯ ЧАСТИЦ В СТОХАСТИЧЕСКОМ ПОЛЕ

В. Остроушко

Рассмотрено ускорение электрона в поле с зависимостью от времени, отличающейся от синусоидальной введением скачков фазы. Установлена связь между энергией электронов и характерным размером устройства, в котором такая энергия может быть достигнута, при рассмотренном способе ускорения и при столкновительном нагреве. Показано, что ускорение в поле со скачками фазы требует значительно большего размера области движения электронов, чем нагрев до такой же энергии в синусоидальном поле при наличии упругих столкновений.

ОЦІНКА РОЗМІРІВ ОБЛАСТІ ВЗАЄМОДІЇ, НЕОБХІДНОЇ ДЛЯ ПРИСКОРЕННЯ ЧАСТИНОК У СТОХАСТИЧНОМУ ПОЛІ

В. Остроушко

Розглянуто прискорення електрона у полі із залежністю від часу, яка відрізняється від синусоїдальної введенням стрибків фази. Встановлено зв'язок між енергією електронів та характерним розміром пристрою, у якому така енергія може бути досягнута, при розглянутому способі прискорення та при зіткненному нагріванні. Показано, що прискорення у полі зі стрибками фази потребує значно більшого розміру області руху електронів, ніж нагрівання до такої саме енергії у синусоїдальному полі за наявності пружних зіткнень.