A brief narration about the history of those heated arguments and discussions around the nature of so-called parametric X-radiation, which were concluded by the recognition of the discovery the phenomenon of coherent polarization bremsstrahlung of relativistic charged particles in crystals. Some important information and comments, which stay over of notice of specialists till now are reported.

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1. INTRODUCTION

In 1983-1985 the two pioneer experiments with the goal to detect the coherent X-radiation of relativistic particles in crystals were carried out in Kharkov and Tomsk. For the first time the preliminary results of these experiments simultaneously were reported in Moscow at the Session of the Academy of Science of the USSR in January 1986 [1]. The results of the Tomsk experiment were reported by A.P. Potylitzin [2], [3], and the results of the Kharkov experiment were reported by me [4]. The discrepancy in the understanding and explanation the nature of observed coherent radiation arose during discussions at that Session.

The results of Tomsk experiment [2], [3] were interpreted as: "... the first experimental observation of a new mechanism for X-ray emission by relativistic charged particles in crystals: parametric Vavilov-Čerenkov X radiati". They approved that: "The reason for the effect is that the periodic arrangement of atoms (or nuclei) in a crystal causes the inequality \( n > 1 \) to hold even in the X-ray range if the frequency \( \omega \) and the momentum of the photon are near the edge of Brillouin zone" [3]. (It means, that \( n \) is the index of refraction of X-rays in the crystal media.)

During previous ten years the series of the theoretical works was done, where the so-called "parametric Vavilov-Čerenkov X-radiation" has been predicted: [5], [6], [7], [8], and the properties of such radiation have been calculated. My colleagues expressed their confidence about agreement between results of their experiment and those theoretical predictions.

I assumed that the coherent polarization bremsstrahlung had been observed in two our experiments. Besides, I established, that in both experiments the preliminary results were obtained: they had vital experimental imperfections (in both experiments \( \gamma \)-detectors were insufficiently shielded from \( \gamma \)-radiation emitted by accelerator devices), and therefore, we could not have the correct identification of the radiation mechanism. To my mind, my colleagues were in a hurry to make conclusions in their publications [2], [3]...

From that time the struggle for the priority in the discovery of the super intensive sources in the X-ray and \( \gamma \)-ray regions (based on supposed Vavilov-Čerenkov effect), which promised the financial support from military-industry complex\(^1\), began between two groups, which belonged to the two different ministries: "Education and Science" (Minsk and Tomsk) and so-called "Ministry of Middle Machine-Building" (Moscow, Kharkov). I did not take part in such competition.

I did not trust to the theoretical works, which had been mentioned above, because their results were in contradiction with Kharkov experiment. For example:

1. the coherent maximum existed in the much more wide region of angle deviation from the Bragg angle, that was in contrast with theoretically predicted very narrow region in the series of the works [5], [6], [7], [8];
2. the photon energy in the coherent maximum can be changed by the crystal rotation in the wide region, photon energy depends on relativistic electron energy, and its position on the energy scale is the same as for coherent bremsstrahlung.

All these properties were in the contradiction with theoretical predictions of [5], [6], [7], [8]. This and other contradictions between experimental results and existed theoretical predictions, have forced me to carry out my own theoretical work, where the new model was developed, which I called as the "...kinematical theory..." [10].

Studying the works of my predecessors and opponents, I have discovered that they deny the law of

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\(^1\)From March 1983 the word combination "starlit wars" [9] became often used by the politicians...
momentum-energy conservation not only using the phrase: "The phenomenon of radiation of electromagnetic waves by the charged particle, which moves straightforward and uniformly in the space heterogeneous medium". The denial of the fundamental law is in the essence of these works (!).

The conflict of interests was the hindrance for publication the experimental results obtained at Kharkov 2 GeV and 40 MeV accelerators. Only due to the support of academician Spartak Belyaev, though with delay, our first preliminary experimental results were published [4]. Our next more accurate experimental results were forbidden for publication?

Nevertheless, in June 8, 1989 we got the Author’s certificate of the invention of "The way (method) of generation of the monochromatic directed X-ray radiation" with priority from April 13 1987 [11], which is based on the process of coherent polarization bremsstrahlung of relativistic charged particles in crystals, and is described in [10] (Fig.1). It became possible because of the different properties of radiations, which were predicted within the framework of models of parametric Čerenkov radiation [5], [6], [7], [8] and in [10]. For the first time in [10] it was shown that expression of cross section, obtained within the framework of classical electrodynamics and perturbation (or kinematical) model agrees well with experiments, which were carried out in Kharkov.

Timely to remark, that in July 19, 1988 at the Scientific Counsel of Kharkov Institute of Physics and Technology academician Ja.B. Fainberg and professor N.A. Khiznjak raised a claim for their rights to the discovery [12]: "The phenomenon of radiation of electromagnetic waves by the charged particle, which moves straightforward and uniformly in the space heterogeneous medium", where they approved that Tomsk and Kharkov experiments had confirmed their theoretical predictions in [5]. Their suggestion about discovery was not admitted by scientific community because it was erroneous.

2. QUANTUM-MECHANICAL THEORY OF COHERENT X-RADIATION IN CRYSTALS

2.1. Coherent X-radiation in Crystals as a Result of Constructive Interference of Radiation Waves from Crystal Atoms

Thus, for the first time the physical sense of the process of the coherent X-radiation of relativistic charged particles in crystals correctly was described as a coherent polarization bremsstrahlung within the framework of classical electrodynamics in [10]. One can find the qualitative description of such radiation process in [10], (see paragraph: "About the Nature of X-Radiation of Relativistic Electrons in a Crystal"). The nature of the considered type of radiation can be clearly understood by the way of examination the

My reviewers-opponents required to call this type of radiation certainly as: "parametric Vavilov-Čerenkov X-radiation". In the struggle with them the type of X-radiation was called as: "parametric X-radiation" (abbreviation PXR), which does not express the nature of radiation.

In 1990 A.V. Shchagin, V.I. Pristupa and professor N.A. Khiznyak (N.A. Khiznyak was one of the pretenders to the discovery of "parametric Vavilov-Čerenkov X-radiation") published the results from [10] in Phys. Lett.A 148 (1990) p.485 without reference to the original work (!).
total bremsstrahlung of the fast charged particle in the collision with crystal.

The quantum-mechanical model of the total coherent X-radiation, which arises as a result of collision of relativistic charged particle and the crystal target, will be briefly described in this section.

Coherent X-radiation results from constructive interference of bremsstrahlung emitted as well by relativistic charged particle as by atomic electrons in crystal.

1. Thus, for a better theoretical understanding we will first consider the radiation caused by the interaction of relativistic charged particle with isolated atom only. Therefore let’s consider a system, composed of three parts: the projectile (i.e. relativistic charged particle), atom in rest, and the radiation field. The Coulomb gauge should be used for potentials of interaction.

Thus, one can represent the total Hamiltonian of the given system in the following form:

\[ \hat{H}_{\text{tot}} = \hat{H}_p + \hat{H}_a + \hat{H}_r + \hat{H}_{\text{int}}, \]

where \( \hat{H}_p, \hat{H}_a \) and \( \hat{H}_r \) are the unperturbed Hamiltonians of the particle, of the atom and of the radiation field, \( \hat{H}_{\text{int}} \) is the Hamiltonian of interaction.

The unperturbed state functions for the particle satisfy the stationary Dirac equation

\[ \hat{H}_p \psi_{\vec{p}, s}(\vec{r}) = \left\{ -i \hbar c \vec{\alpha} \vec{A}(\vec{r}) + mc^2 \beta \right\} \psi_{\vec{p}, s}(\vec{r}) = E(\vec{p}) \psi_{\vec{p}, s}(\vec{r}). \]

(2)

Solutions of Dirac equation are:

\[ \psi_{\vec{p}, s}(\vec{r}, t) = \psi_{\vec{p}, s}(\vec{r}) e^{-iEt/\hbar} = \sqrt{E + mc^2 \over 2E} \left( \frac{\hat{\gamma}_s}{mc^2 + E} \right) e^{i(\vec{p} \cdot \vec{r} - Et)/\hbar}, \]

(3)

where \( \vec{\alpha} \) are Pauli matrices and \( \vec{\gamma}_s \) are vectors of electron polarization.

The unperturbed state functions \( \varphi_n(\vec{r}) \) for the atom (i.e. atomic electrons in the field of the atomic nucleus and electrons) satisfy the stationary one-particle Schrödinger - Hartree equation:

\[ \hat{H}_a \varphi_n(\vec{r}) = e_n \varphi_n(\vec{r}), \]

(4)

where:

\[ \hat{H}_a = \left\{ \frac{-i \hbar \vec{\alpha}}{2m} + \frac{Ze^2}{\vec{r}} - e^2 \sum_{n' \neq n} \frac{|\varphi_{n'}(\vec{r}')|^2}{|\vec{r} - \vec{r}'|} \right\}. \]

Free radiation field should be possible to describe by vector potential \( \vec{A} \), which satisfies the wave equation and gauge condition simultaneously:

\[ \triangle \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = 0, \]

\[ \vec{\nabla} \cdot \vec{A} = 0. \]

(5)

The general solution of the equation 5 is the superposition of plane waves:

\[ \vec{A}(\vec{r}, t) = \sqrt{\frac{4\pi \epsilon_0}{c^2}} \sum_{k, \lambda} \vec{e}_{k, \lambda} (q_{k, \lambda} e^{i(k \cdot \vec{r} - \omega t)} + q_{k, \lambda}^* e^{-i(k \cdot \vec{r} - \omega t)}), \]

(6)

where \( \vec{e}_{k, \lambda} \) is the unit vector of photon polarization, and indexes \( \lambda = 1, 2 \) correspond to the two states of transverse polarizations. Vectors \( \vec{e}_{k, \lambda} \) satisfy conditions of orthogonality:

\[ \vec{e}_{k, 1} \cdot \vec{k} = 0, \quad \vec{e}_{k, 2} \cdot \vec{k} = 0, \quad \vec{e}_{k, 1} \cdot \vec{e}_{k, 2} = 0, \]

(7)

which are consequence of the Coulomb gauge. For convenience we choose the normalization multiplier in 6. Due to the boundary condition of the periodicity inside the cube volume \( V = L^3 \), wave vectors \( \vec{k} \) satisfy the equality

\[ \vec{k} = \frac{2\pi}{L} \vec{n}, \]

(8)

where \( n_i = 0, \pm 1, \pm 2, \ldots \). The frequency of the wave is connected with absolute value of the wave vector \( \vec{k} \) by the law of dispersion

\[ \omega(\vec{k}) = c |\vec{k}|. \]

(9)

The interaction Hamiltonian \( \hat{H}_{\text{int}} \) in 1 is given by:

\[ \hat{H}_{\text{int}} = \hat{H}_{p-n} + \hat{H}_{p-a} + \hat{H}_{p-r} + \hat{H}_{a-r}, \]

(10)

where \( \hat{H}_{p-n} \) describes the interaction between particle and atomic nucleus, \( \hat{H}_{p-a} \) describes the interaction between particle and atomic electrons, \( \hat{H}_{p-r} \) describes the interaction between particle and radiation field, \( \hat{H}_{a-r} \) describes the interaction between atom and radiation field (\( \hat{H}_{a-r} \) is neglected because of infinitely large nuclear mass). All these parts of interaction Hamiltonian should be described as:

\[ \hat{H}_{p-n} = \frac{Ze^2}{r}, \]

(11)

\[ \hat{H}_{p-a} = -e^2 \sum_n \int \frac{|\varphi_n(\vec{r})|^2}{|\vec{r} - \vec{r}'|} d^3 r', \]

(12)

\[ \hat{H}_{p-r} = -e \vec{A} \vec{A} \right), \]

(13)

\[ \hat{H}_{a-r} = \hat{H}_{a-r}^{(1)} + \hat{H}_{a-r}^{(2)} = \]

\[ = \frac{i}{\hbar} \sum_n \vec{A}(\vec{r}_n) \nabla_n + \frac{e^2}{2mc^2} \sum_n \vec{A}^2(\vec{r}_n). \]

(14)
Fig. 2. Feynman diagrams for electromagnetic radiation from electrons interacting with atom

The first four diagrams describe the emission of photon by the incident particle scattered at the nucleus (couple of diagrams "a") and atomic electron (couple of diagrams "b"). The last two diagrams (couple of diagrams "c") describe the emission of photon by atomic electrons due to the interaction with the incident particle field.

The differential cross section of the radiation process can be then written in units of $h = c = 1$ as

$$d^6\sigma = 2\pi |M_{st}|^2 \delta(E_i - E_f - \omega) \frac{d^3p_f d^3k}{(2\pi)^6}, \quad (15)$$

where $M_{st}$ is the atomic matrix element and $\vec{p}_i$, $E_i$ and $\vec{p}_f$, $E_f$ represent the initial and final momenta and energies of particle, respectively. Furthermore, $\vec{k}$ and $\vec{L}_\lambda$ are the momentum and the polarization vector of the radiated photon, and $\vec{q} = \vec{p}_i - \vec{p}_f - \vec{k}$ is the recoil momentum. Assuming, that the photon energies are much smaller than the energy of the incident particle, i.e. $\omega \ll E_i, E_f$, but exceed the energies $E_{n,m}$ for atomic transitions, i.e. $\omega \gg E_{n,m}$, the atomic polarizability can be approximated by the polarization of the free electrons (it means, that in the expression $H_{\omega,n}$ in formula Eq.15 the contribution from the $R^{(2)}_{\omega,n,m}$ is much larger than from $R^{(1)}_{\omega,n,m}$). Such problem was solved in: [13], [14], and in this case the square of the total matrix element of the radiation process for nonrelativistic atom has the form:

$$|M_{st}|^2 = |M^{BS} + M^{PR}|^2 =$$

$$= \frac{32\pi^3}{\omega} \left| \frac{e^2}{m_0} \frac{Z - F(\vec{q})}{\eta q^2} \frac{\vec{q}}{\omega - k\vec{q}} + \frac{e^2}{m} \frac{F(\vec{q})}{\omega} \frac{\vec{v}_\omega - \vec{q}}{(\vec{q} + \vec{k})^2 - \omega^2} \right|^2. \quad (16)$$

Here $\vec{v}$ is the particle velocity, $e_0$ and $m_0$ are charge and mass of the relativistic particle, while $e$ and $m$ are charge and mass of the electron, $F(\vec{q})$ is the atomic formfactor and $Z$ is the atomic number. Furthermore, $M^{BS}$ describes the bremsstrahlung of the relativistic charged particle, i.e. couples of diagrams "a" and "b" in Fig.2, and $M^{PR}$ results from polarization bremsstrahlung of the atomic electrons, i.e. couple of diagrams "c" in Fig.2. One can see that in the expressions for amplitudes of radiation 16: $M^{BS} \sim m_0^{-1}$ and $M^{PR} \sim m^{-1}$. It’s important to accentuate, that the presence of masses $m_0$ and $m$ in the denominators of the expressions for amplitudes $M^{BS}$ and $M^{PR}$ gives evidence, that both amplitudes have the same nature, i.e. both amplitudes correspond to the bremsstrahlung processes.

2. Now we are interested in calculating the probability of radiation of fast $(v \sim c)$ electrons (positrons) interacting with many atoms arranged in a crystal (i.e. in a thin crystal).

Let’s begin from estimations of the time characteristics of such process. The transit time $\tau$ of relativistic particle across the atom is order of $\tau \sim R_B Z^{-1/3} c^{-1} \approx 3 \times 10^{-20}$ s, where $R_B$ is a Bohr radius, $Z$ – atomic charge and $c$ – velocity of light. Characteristic vibration time for crystalline materials is order of $t \sim \omega^{-1} = \hbar/(k\Theta) \approx 10^{-13}$ s, where $k$ – Boltzmann constant, $\Theta$ – Debye temperature. One can see that $\tau \ll t$, and the particle transit length is order of $L \sim 10^{-3}$ cm during the time $t$.

Let’s assume that crystal thickness $L_c$ is so small, that the inequality $L_c \ll L$ is satisfied.

The cross section measurements of radiation process require the time $T \gg t$. So one can see that a single relativistic particle interacts with static accidental configuration of the crystal atoms, and different particles interact with different accidental configurations. Therefore we must calculate cross section of radiation of fast electron in an accidental static crystal configuration, and after that we must average across section over all atomic configurations, which depend on a crystal temperature.

So, following the procedure derived by Überall [15] in his perturbation theory, we can apply the result of Eq.16 not only to the single atom but to the thin crystal as well. For this purpose, in accordance with Huygens principle, it’s necessary to take into account the interference of amplitudes of radiation from all atoms of the crystal target. This method was successfully applied calculating the cross section of high-energy bremsstrahlung and electron pair production in the thin crystals by Überall [15] and Diambrini [16].

For each atom of the crystal one obtains the radiation amplitude $M_{st} e^{-i\vec{q}\vec{R}_L}$, where $\vec{R}_L$ are the relative coordinates of the crystal atoms. Thus, the cross section of the process for the entire crystal as it is described by the diagrams in Fig.2, differs from Eq.15 by the diffraction factor

$$\sum_L \exp(-i\vec{q}\vec{R}_L) \quad \quad (17)$$

The crystal diffraction factor, corresponding to one crystal cell and averaged over the thermal vibrations of the crystal atoms, has the form:

$$\left\langle \frac{1}{\omega_T} \sum L \exp(-i\vec{q}\vec{R}_L) \right\rangle^2 = D(\vec{q}) S^2(\vec{q}) e^{-\frac{q^2}{4} \frac{T^2}{\langle \omega_T \rangle}} + \nu \left[ 1 - e^{-\frac{q^2}{4} \frac{T^2}{\langle \omega_T \rangle}} \right]. \quad (18) \quad \quad \quad$$

where $\nu$ represents the number of atoms in the elementary crystal cell, $S(\vec{q})$ is the structure factor of the crys-
tal, $e^{-\frac{q^2}{\omega^2}}$ is Debye-Waller factor and $\omega^2/T$ is the mean square of the thermal vibrational amplitude $\bar{u}_T$ of the crystal atoms. Assuming the geometric shape of the crystal to be a thin plate and there is one of the crystal lattice basis vectors $\delta_1$ which forms a small angle with respect to the normal to the crystal surface and the other basis vectors $\delta_2$ and $\delta_3$. In this case the diffraction factor $D(q)$ for an ideal crystal with $N_1$ periods in the crystal direction $\vec{a}_1$ and with large numbers of periods $N_2, N_3 \to \infty$ in the perpendicular directions can be represented as:

$$D(q) = \lim_{N_2, N_3 \to \infty} \frac{3}{N_2} \sum_{j=1}^{N_2} \sin^2 \left( \frac{N_2}{2} \delta_1 j \right) = 4\pi^2 a^2 V \sum_{g} \delta^2 \left( \vec{q} - \vec{g} \right) \delta_{\perp \vec{a}_1} \sin^2 \left( \frac{N_2}{2} \delta_1 j \right),$$

(19)

where $V$ is the volume of the crystal elementary cell, $\vec{g}$ represents the reciprocal lattice vectors of the crystal, argument of the $\delta$-function is the component of the vector $(\vec{q} - \vec{g}) \parallel \vec{a}_1$, which is perpendicular to $\vec{a}_1$.

From Eq.18 (and Eq.15) it becomes apparent that the complete BS and PR bremsstrahlung in a crystal is divided into two parts:

$$d^6\sigma_{cr} = d^6\sigma_{coh} + d^6\sigma_{in}.$$

(20)

The first one is the coherent part and is proportional to $e^{-\frac{q^2}{\omega^2}}$ and is correlated to the part of the crystal atoms that contributes to the interference effect which results in the development of coherent X-radiation. In this case the crystal gets exactly a momentum $\vec{q} = \vec{g}$, as it can be seen from Eq.19 for $N_1 \to \infty$, and the recoil momentum is transferred to this part of crystal atoms.

The second term, which is proportional to $1 - e^{-\frac{q^2}{\omega^2}}$, is the incoherent part. It appears due to the thermal vibrations of crystal atoms and corresponds to the part of the atoms, that does not contribute to the coherent radiation.

### 2.2. Coherent Cross Section in a Thin Crystal

For the calculation of the coherent cross section we assume that the relativistic particle that causes the radiation is not detected. An integration of Eq.15 over $d^6p_f$ after multiplication with the coherent term of the diffraction factor (see Eq.18) will yield the cross section describing the production of coherent radiation. The integration can be performed by introducing new variables $p' = p_f - \vec{g}$, $E' = E_f - E$, and using $\vec{p}' \cdot \vec{v}' = E'$. After the replacement of $\delta(\vec{q} - \vec{g})\delta(E_0 - E_f - \omega)d^6p_f$ by $\delta(\vec{p}' - \vec{E} - \vec{g})\delta(E' - \omega)(\bar{u}_T \bar{a}_1^2)^{-1}d^4p_{\perp \vec{a}_1}dE'$ and summing over the two possible polarizations, denoted by $\lambda = 1, 2$, the integration yields for $N_1 \to \infty$ (see Eq.19) the total coherent spectral-angular cross section, normalized to the volume of a crystal cell:

$$\frac{\frac{d^4\sigma}{d\omega d\Omega}}{V} = \frac{8\pi \omega V}{(1 - \bar{\eta} \bar{\nu}_k)} \sum_{\vec{g}, \lambda} |M_{CR}|^2 S^2(\vec{g})e^{-\frac{q^2}{\omega^2}} \times$$

$$\times \delta \left( \omega - \frac{\bar{\nu} \bar{\omega} - \bar{\nu}}{1 - \bar{\nu} \bar{\nu}_k} \right),$$

(21)

where $\bar{\nu}_k = k/k$, and the sum includes all reciprocal lattice vectors $\vec{g}$ of the crystal and the photon polarization directions. Assuming for simplicity, that the charged particle is an electron, i.e. $e_0 = -|e|$ and $m_0 = m$, the square of the matrix element module in Eq.21 is of the form:

$$|M_{CR}|^2 = |M_{CBS} + M_{CPR}|^2 =$$

$$= \frac{e^6}{\omega^2 m^2} \sum_{\vec{g} \perp \vec{a}_1} \left( \frac{Z - F(\vec{g})}{\vec{g}^2} \frac{\omega^2}{\bar{\nu}} \left( \frac{\vec{g} \cdot \bar{\nu}_k}{\bar{\nu} - \bar{\nu}_k} \right) + \frac{F(\vec{g})}{\vec{g}^2} \left( \vec{v} \cdot \bar{\nu} - \vec{g} \right) \right)^2.$$

(22)

Like in Eq.16, the first part of the right hand side of Eq.22 corresponds to the coherent bremsstrahlung of the charged particle (CBS) and the second part corresponds to the coherent polarization radiation (CPR). Due to the condition $\vec{q} \approx \vec{g}$ for the momentum transferred the crystal, the X-radiation becomes almost monochromatic. This mechanism has been described in detail for coherent bremsstrahlung in [15, 16]. The second part, i.e. coherent polarization radiation in the sense of the coherent component of the bremsstrahlung produced by atomic electrons, exactly represents so-called PXR(B) [17].

If we discuss Eqs.21 and 22 it can be stated that both CBS and CPR (or PXR(B)) contribute to coherent X-radiation at the same photon energy, but they are emitted with different angular distributions. In the case of CBS all reciprocal lattice vectors contribute to the intensity emitted forward into the angle cone with an opening angle of $\approx 1/\gamma$. On the other hand, CPR (or PXR(B)) is emitted mainly into the cone of similar angular size but which is aligned to the direction of $\vec{v} \cdot \bar{\nu} - \vec{g}$.

Thus, for the different reciprocal lattice vectors CPR (or PXR(B)) can be obviously observed at different directions, and several individual maxima of coherent radiation appear. For high particle energies these maxima are well separated from each other. In contrast, for low energies, i.e. if $1/\gamma$ becomes comparable to the angles between two reciprocal lattice vectors, the angular cones corresponding to several reciprocal lattice vectors may partially overlap.  

5Such new variables represent momenta and energies of so-called pseudophotons of the field of relativistic charged particle.

6In [17] we introduced the abbreviation PXR(B), where the letter (B) is the first letter of the word "Bremsstrahlung", and indicates the nature of radiation.

7It is timely to remark, that in [10] in Fig.8 and Fig.9 the intersections of the reciprocal lattice plane with the "pancake" of recoil momenta, allowed by kinematics of the radiation process, are shown. And one can see, that Fig.8 corresponds to the case that only one reciprocal lattice bend is inside the "pancake" and Fig.8 corresponds to another case that the series of bends is inside the "pancake". In the case of CBS [15, 16] the first case (Fig.8) is called as "effect of point" and the second one (Fig.9) is called as "effect of row". In the case of CPR we have the same kinematical pictures. However, essential difference in angular distributions of radiation for CBS and CPR exists. For CBS all angular cones of partial radiations, which are caused by contributions from the series of the reciprocal lattice bends, are coaxial. Therefore in the case of CBS we have the effective interference of these partial radiations. This is the sense of the term of "effect of row". For CPR all angular cones of partial radiations, which are caused by contributions from the series of the reciprocal lattice bends, have different directions. Therefore the concept "effect of row", in contrast to CBS, can not be applied in the case of CPR.
It should be pointed out, furthermore, that in the two extreme cases Eqs.21 and 22 represent the known, independent representations of CBS and CPR (or PXR(B)). In the first case, for an observation angle θ = θemaker, the first part of Eq.22 becomes dominant, and thus Eqs.21 and 22 describe exactly CBS in the low photon energy approximation analogous to [15], [16]. On the other hand, if θ ≫ 1/γ, the contribution of CBS becomes small and it can be neglected. In this case Eqs.21 and 22 precisely represent the mathematical description of CPR (or PXR(B)) which has been published before in [10] and [17].

2.3. Influence of the Crystal Dielectric Properties

Up to here we have neglected the influence of the dielectric properties of the crystal, i.e., the radiation photons were treated as traveling through vacuum. However, the influence of the crystal media can be taken into account by changing the dispersion relation for the radiated photon to \( k^2 = \omega^2, \) where \( \varepsilon = 1 + \chi \) is the dielectric constant with \( \chi \) being the electric susceptibility of the crystal, which are the same as for amorphous media. Thus, using this dispersion relation in Eq.22 we obtain the expression of the square of the matrix element module:

\[
|M_{CR}|^2 = |M_{CBS} + M_{CPR}|^2 = \\
\frac{e^6}{\varepsilon \omega^2 m^2} \left( Z - F(\tilde{g}) \frac{\vec{j}}{\gamma g^2} \left( 1 - \varepsilon \frac{a}{\phi} \right) + \frac{F'(\tilde{g})(\sqrt{\varepsilon \omega} - \tilde{g})}{\omega^2/(1/\gamma^2) + 1 - \varepsilon} + \frac{\sqrt{\varepsilon \omega}(a_0 - \varepsilon)}{\tilde{g}^2 + \varepsilon} \right)^2.
\]

Comparing Eqs.22 and 23, it becomes apparent that the influence of the medium effects over the matrix element mainly at high particle energies, i.e. if \( 1/(\gamma^2) \ll |1 - \varepsilon| = |\chi| \). For low energies, i.e. if \( 1/(\gamma^2) \gg |\chi| \) the influence of the medium can be neglected and it turns out that the maximum of the cross section increases proportionally to \( \sim \gamma^2 \) [10], [17]. In contrast to this, at high energies the cross section is limited by \( \chi \) and the shape of the angular distribution becomes constant [18].

Timely to remark, that M.L. Ter-Mikhealyan in his book [6] concluded that his so-called "resonance radiation" (or it is the same as CPR) is characterized by the energy threshold (see formula (28.13) in [6]), and "resonance radiation" exists only if electron energy is larger, than the threshold energy. In our designations M.L. Ter-Mikhealyan’s condition is the same as the condition \( 1/(\gamma^2) \ll |1 - \varepsilon| = |\chi| \) in formula Eq.23. One can see, that considered here type of coherent radiation (i.e. CPR) does not have a threshold. Such condition can be called, for example, as "energy of saturation". Investigations of the properties of the coherent polarization X-radiation, which were made in [10] and [17] correspond to the case, when electron energies are less than so-called "threshold of resonance radiation", i.e. when \( 1/(\gamma^2) \ll |1 - \varepsilon| \) and we can assume, that in this case the dielectric constant of the crystal media can be assumed as \( \varepsilon_0 = 1. \) Such conditions were briefly discussed in [10] (see formula (34) in [10]).

V.G. Baryshevsky et al. in their work [20] made reference to the condition analogous to \( 1/(\gamma^2) \ll |1 - \varepsilon| = |\chi| \) and pointed out that: "... the PX intensity has no clearly marked threshold character" and later they confirmed that "Vavilov-Čerenkov condition is fulfilled only for" \( 1/(\gamma^2) \ll |1 - \varepsilon| \) \( \gg |\chi| \). Such "confirmations" respectively to the Vavilov-Čerenkov condition are erroneous.

Examining the coherent cross section Eq.21 one can see, that the photon energy in the coherent maximum for a specific reciprocal lattice vector \( \vec{q} \) can be obtained by turning the argument of \( \delta \)-function to zero. So we have:

\[
\omega = \frac{\vec{q}^2}{1 - \varepsilon \frac{a}{\phi} / \varepsilon^2},
\]

where \( c^2 = c/\sqrt{\varepsilon}. \)

Using \( \delta \)-function, the cross section Eq.21 can be simply integrated over the photon energy and the differential CPR yield per unit solid angle and per unit length can be obtained:

\[
\left( \frac{dN_\gamma}{d\Omega dL} \right) = \sigma(\vec{g}) \sum_\lambda \omega \left( \frac{\varepsilon_{\lambda}(\omega^2 - \vec{g}^2)}{(k + \vec{g})^2 - \omega^2 - \omega_f^2} \right)^2,
\]

where \( \sigma(\vec{g}) = (8\pi e^6)/(V m^2)S^2(\tilde{g}) \exp(-g^2\varepsilon^2)F^2(\tilde{g}), \) \( \omega_f \) is the plasma frequency, and the relation \( 1 - \omega_f^2/\omega^2 \) has been used.

Simply to show, that using \( (k + \vec{g})^2 - \omega^2 \equiv (\omega/\gamma)^2 + (\vec{K}_c^2)^2, \) where \( \beta = v/c \) and \( \vec{K} \neq 0, \) this value becomes identical to the one that describes by formula Eq.32 in [18] (or Eq.37 in [19]) for the case \( \varepsilon = 1. \)

3. THE RESULTS OF THE FIRST KHARKOV EXPERIMENT [21]

The next considerable work [21], in which the final results of the first Kharkov experiment had been presented, was published only in July 1993, after a prolonged struggle with the opponents. In this paper it was shown, that the cross section of the coherent polarization radiation of relativistic charged particles (see formulae 21-23 with contribution of only matrix element \( M_{\text{CPR}} \) from the sum 22 or 23) can be simply obtained using Weizsäcker-Williams method [22], and it is the same, as it was obtained within the framework of the classical kinematical theory [10], and these formulae are in good quantitative agreement with results of the Kharkov pioneer experiments.

The first Kharkov experiment was performed on the linear accelerator of electrons in Kharkov Physical Technical Institute Academy of Science of Ukraine - LINAC’2 GeV. The single crystal Si target had the form of plane-parallel plate, which was cut in the plane (110). Using 3-axes goniometer and the method of crystal orientation [23], crystal targets were placed on the trace of electron beam so, that the crystal plane (111) has been placed perpendicular to the plane of radiation and it created the angle \( \phi \) with the beam direction. The flux of photons, which were emitted under the angle \( \theta > \gamma^{-1} \) respectively to the electron beam direction, was measured. The photon beam was formed with the help of the circular collimators in the solid angle \( \Delta\Omega = \pi(\Delta\theta)^2, \) where the linear angle of collimation was \( \Delta\theta \ll \theta. \) After passing through the crystal target the electron beam and charged particles, created by initial electron beam in the crystal target, were turned by magnet into the beam absorber, where they were absorbed. Spectra of photons were measured with the help of semiconductor spectra-meter, placed at the end of the photon channel inside the shield, which protected \( \gamma \)-spectra-meter from electromagnetic background.
created by electron accelerator. The charge of the electron beam, which was passing through the crystal, was measured by a thin monitor of secondary-emission, placed behind the crystal target outside the photon channel, and the beam charge was integrated. Spectra of radiation were measured with electron beam current $I \leq 10^{-9} \, \text{A}$. The monitor of secondary-emission was calibrated using a standard Faraday cup. The calibration of $\gamma$-spectrometer was carried out with the help of standard $\gamma$-sources.

The scheme of measurements was described in Ref.[24] in detail. Schematic view of the experimental setup on the experimental area of the straight line beam exit of the $2 \, \text{GeV}$ Kharkov linear electron accelerator is shown in Fig.3. The most important parameters of the scheme of measurements on the accelerator LINAC − $2 \, \text{GeV}$ are represented in the table.

**Fig.3.** Schematic view of the experimental setup on the experimental area of the straight line beam exit of the $2 \, \text{GeV}$ Kharkov linear electron accelerator. $M_1$, $M_2$ − magnets of beam parallel dislocation; $M_3$ − deflection magnet; $M_4$, $M_5$ − cleaning magnets; $G_1$, $G_2$-goniometers; $K_1$, $K_2$ − electron collimators; $K_3$, $K_8$ − photon collimators; $J_1$, $J_2$, $J_3$ − electron beam current monitors (based on secondary-electron emission); $F$ − Faraday cup; $Q$ − quantameter; $S_1$-Ge(Li) − semiconductor gamma-spectrometer; $S_2$ − NaJ total absorbing gamma-spectrometer

<table>
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<tr>
<th>Parameters of the scheme of measurements on the accelerator LINAC − $2 , \text{GeV}$:</th>
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<tr>
<td>the energy of electrons in the maximum of the spectral distribution</td>
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<tr>
<td>the width of spectral distribution of the electron beam on the level $1/2$ of maximum current</td>
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<tr>
<td>the middle divergence of electron beam</td>
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<tr>
<td>the duration of the current pulse</td>
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<td>the frequency of the current pulses</td>
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<tr>
<td>the angle of photon registration $\theta$</td>
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<td>the accuracy of measurements of angles $\phi$, $\psi$</td>
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<td>the accuracy of measurements of angles $\alpha$</td>
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<tr>
<td>the material and orientation of the crystal target</td>
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<tr>
<td>the thickness of the crystal target</td>
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<td>the type of $\gamma$-spectrometer</td>
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<td>the energy resolution of $\gamma$-spectrometer</td>
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<tr>
<td>the efficiency of photon registration in the maximum of the coherent radiation</td>
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<tr>
<td>the frequency of counts of $\gamma$-spectrometer</td>
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<tr>
<td>the linear angle of photon channel acceptance $\Delta \theta$</td>
</tr>
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</table>

The spectra of electromagnetic radiation, which is generated by relativistic electrons with different energies in the single crystal Si with orientation such, that the crystal planes (111) placed perpendicular to the plane of radiation and under the angle $\phi$ respectively to the electron beam direction, are measured. One of them is shown in Fig.4. In Fig.4 we can observe one bright peak of CPR with energy $206 \, \text{keV}$ from the planes (111). "Small" peak with energy $412 \, \text{keV}$ is stipulated by the generation of two photons with energies $206 \, \text{keV}$ during one pulse of the accelerator current.

The energy dependence in the maximum CPR from the angle $\phi$ between electron velocity direction and the crystalline plane (111) is shown in Fig.5 for electron energy $1200 \, \text{MeV}$.
with the calculated yield using formula Eq.25, we excluded the continuous background from spectra (see Fig.4) and then each spectrum was integrated over the photon energy. So, the apparatus effect of the line broadening, caused by the final spectrometers resolution, was excluded.

Fig.4. The spectrum of coherent polarization radiation generated by the electrons with energy 1.2 GeV in 50 μm Si single crystal target under the angle $\theta = 17.88$ mrad. The crystal plane (111) has disposition perpendicular to the radiation plane and creates the angle $\phi = 8.63$ mrad respectively to the electron beam direction.

In Fig.6 it’s shown the dependence $dN_{\gamma}/d\Omega/L$ (where $L = aN$ is the crystal thickness) from the angle $\phi$ for electron energy 1200 MeV. The experimental values $dN_{\gamma}/d\Omega/L$ are given by points. Nearly the perfect agreement between experiment and the perturbative or kinematical theory, which is described in the previous section (and which results for CPR coincides with results obtained within the framework of classical electrodynamics in [10]) was observed.

It means, that using the perturbative or kinematical theory, which was described in the previous section and taking into account particle multiply scattering, photon absorption, crystal mosaicity, beam parameters, photon detector opening and efficiency, and other experimental conditions by Monte-Carlo manner [38] – is enough for exact calculation of properties of coherent polarization X-radiation of relativistic particles in crystals.

Fig.5. The energy in the maxima of spectra of coherent polarization radiation generated by electrons with energy 1200 MeV in 50 μm Si single crystal target under the angle $\theta = 0.3059$ rad as a function of the angle $\phi$ between crystal plane (111) and electron beam direction. Points-experiment; Dashed line-calculated function using formula 21 integrated over photon energy (i.e. Eq.25); solid line-calculated function using formula 21 integrated over photon energy with taking into account the following: particle multiply scattering, photon absorption, crystal mosaicity, beam parameters, photon detector opening and efficiency by Monte-Carlo manner.

Fig.6. The total number of photons in coherent maximum generated by the electrons with energy 1200 MeV in 50 μm Si single crystal target under the angle $\theta = 0.3059$ rad as a function of the angle $\phi$ between crystal plane (111) and electron beam direction. Points-experiment; Dashed line-calculated function using formula 21 integrated over photon energy (i.e. Eq.25); solid line-calculated function using formula 21 integrated over photon energy with taking into account the following: particle multiply scattering, photon absorption, crystal mosaicity, beam parameters, photon detector opening and efficiency by Monte-Carlo manner.

The solid line in Fig.5 is calculated using the formula Eq.24, where $\omega = E_{\gamma}/m c^2$. It is observed a good agreement between experimental dependence and the formula Eq.24.

For comparison experimental values of CPR yield

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For comparison experimental values of CPR yield
3.1. Discussion of the Results of [21]

1. The agreement of experimental energy dependence of CPR with dependence Eq.24 is the proof of the radiation coherency. The kinematics of CPR (or PXR(B)) is completely coincides with kinematics of coherent bremsstrahlung (see formula Eqs.21, 22 and [16]).

On the other hand, this dependence expresses the law of pseudo-photon refraction from the atomic planes of a crystal: \( \vec{k} = \vec{k}_0 + \vec{g} \), where \( \vec{k}_0 \vec{v} = \omega \), \( k = \omega/c^2 \), \( c^2 \) is the phase velocity of the free electromagnetic wave in a crystal. It is evident, that such dependence can be used for experimental determination of the refraction coefficient in the direction of radiation:

\[
n = \frac{c}{c^2} = \left( \frac{c}{v} - \frac{cg_\perp \sin \phi}{\omega} \right) / \cos \theta,
\]

so as at the right of (Eq.24) there are only values, which were measured in the experiment. So, it can be proved that \( n \approx 1 - 3 \cdot 10^{-6} \) for electron energy of \( 1200 \text{MeV} \). Therefore the properties of observed CPR are in the contradiction with radiation properties, predicted in [5]-[8], where \( n > 1 \) is the requesting condition.

2. The model of CPR was developed within the framework of perturbation theory, which had been used for development of: the model of resonance radiation [6]; X-ray transition radiation (XTR) [7]; and quasi-Čerenkov radiation, which is far from Bragg directions [25]. However CPR can not be identified as: a resonance radiation; XTR; and quasi-Čerenkov radiation, which is far from Bragg directions, because CPR is generated on the crystal planes, for which \( \vec{g} \cdot \vec{n} = 0 \) (where \( \vec{n} = \vec{a}_1/\alpha_1 \), see formula Eq.19), but for the other mentioned types of radiation it is necessary to be \( \vec{g} \cdot \vec{n} = g \) and \( g \neq 0 \). CPR is accompanied with recoil momentum received by a crystal with transverse component, which is much larger than longitudinal component, and it is not typical for Čerenkov type of radiation. The existing of large transverse recoil momentum leads to the result, that the effective dielectric constant, which was introduced in Ref.[25], is the following \( \epsilon_{eff} = \epsilon_0 \) (in the X-ray region of photon energy for the Si crystal \( \epsilon_0 \approx 1 - 5 \cdot 10^{-6} \)). The application of the perturbation theory in the CPR model is possible due to the condition of weak bond of incoming and scattered electromagnetic waves [10], which is also correct for the thick crystals. On the contrary, the models of: a resonance radiation, XTR [7], and quasi-Čerenkov radiation, which is far from Bragg directions [25] suppose the strong bond between incoming and scattered waves, and in this case the scattered wave can be small only for a thin crystal.

3. The spectral-angular distribution of radiation [10] (see Fig.6.) is proportional to the flux of virtual photons of the field of relativistic electron through the crystal atomic plane. Thus, the mechanism of radiation can be interpreted as a coherent scattering of virtual photons, associated with the relativistic charge, on the crystal electrons. The flux of virtual photons in the direction \( \vec{v} \) is equal zero and has the maximum on the cone with the generatrix, which is directed under the angle \( 0.5\gamma^{-1} \) respectively to \( \vec{v} \). The refraction of the virtual photons from atomic planes according to the law \( \vec{k} = \vec{k}_0 + \vec{g} \), where \( k_0 \vec{v} = \omega \), \( k = \omega/c \), is not mirror, so as \( v \neq \epsilon \). The minimum of the orientational dependence Fig.6. is in the angle \( \phi_3 = \theta/2 - 1/(2\theta^2) \), which is in accordance with our theoretical model [10]. The difference of the values in maxima of orientational dependence (see Fig.6., dashed line) is stipulated by different directions of generatrix of cone, where the flux of virtual photons (associated with relativistic charge) has its maximum. They are directed under the different angles \( \phi \) respectively to the atomic plane, and the coefficient of reflection depends on the angle \( \phi \).

Taking into account the multiple scattering of relativistic electrons in the crystal target, the divergence of electron beam and the angle of the acceptance of the photon channel, leads to the reduction the “height” of maxima and the “depth” between them (Fig.6, solid line). The agreement between theoretical and experimental orientational dependence gives a possibility to conclude, that formulae, which are presented in the theoretical part of the present paper, give the adequate description of the spectral-angular distribution of CPR.

4. The spectral-angular density of CPR, is proportional to \( \alpha^3 \), and it is nearly the same order as the spectral-angular density of channeling radiation, which is proportional to \( \alpha^2 \). It depends on small populations of the bound states of the fast electrons in crystal potential. CPR differs from channeling radiation by the possibility of changing the photon energy by changing the crystal orientation in the wide region; CPR is much more monochrome \( \Delta E_\gamma / E_\gamma \approx 7 \cdot 10^{-3} \), and its continuous background is two-order less than for channeling radiation. CPR has weak dependence from such parameters of electron beam as the width of its spectrum and angular divergence. Such properties of CPR make it possible to create the coherent sources of monochromatic directed X-rays, suitable in operation, which have continuous changing of the photon energy from units of \( keV \) to the units of \( MeV \) [11], and they can be applied for calibration of \( \gamma \)-spectrometers, selective raising the resonance systems, and others.

At that time we had two theoretical models, which described quite different physical processes, and which were used for the explanation the properties of the so-called PXR. The first of them is predicted in [5], [6], [7], [8] relates to the Vavilov-Čerenkov type of radiation, and the second one relates to the coherent polarization bremsstrahlung [10], [21].
4. THE RESULTS OF THE INTERNATIONAL COLLABORATION

The situation changed drastically at the beginning of 1993 when due to professor N. Shul’ga, who is an active propagandist of the achievements of Kharkov physicists7, and because of professor A. Richter who showed his interest to my work [10] (Professor A. Richter is the possessor of the encyclopedic knowledge and, besides, learned Russian). A. Richter initiated the translation of my paper [10] into English and placed it in the site of Darmstadt Kernphysik Institute (Germany), where he was a director. A. Richter invited me to the Darmstadt Kernphysik Institute for experimental investigation of PXR.9 Due to the great talent for organization of professor A. Richter and doctor H. Genz, the powerful international scientific group was created, and the program of experiments at the Darmstadt superconducting accelerator S-DALINAC was proposed [27].

Using high-energy electron beam from S-DALINAC, the most considerable experiments were carried out and properties of coherent polarization radiation were investigated in detail.

Let’s only briefly enumerate the main results, obtained within the framework of this international collaboration. Spectra of the CPR as functions from: electron beam parameters [17]; crystal parameters and crystal orientation and direction of radiation respectively to the electron beam [17]; shape and linewidth of coherent maxima [28]; absolute intensity and spectral density [29]; polarization properties of the CPR [30], [31], [32], [33]; interference between the CBS and the CPR [34], [35] were measured with high accuracy and they were compared with theoretical models.

Within the framework of this collaboration the following dissertations were defended: [36], [37], [38], [39], [40].

5. CONCLUSIONS

In the latest decade of the past century physicists intensively discussed the nature of co-called ”Parametric X-radiation of relativistic charged particles in crystals”. It was a unique situation, that the several experimental groups from the USSR (Tomsk, Yerevan, Kharkov) in their experimental works ”confirmed” incorrect predictions of the erroneous theories. At that time the physical phenomenon was described correctly only in one publication [10]. The Darmstadt group (wide international collaboration) carried out the most considerable and exact experiments and step by step disproved wrong confirmations.

Besides, professor Hideo Nitta (Tokyo Gakugei University) made an important contribution into the theoretical description of CPR (PXR(B)) in his works [18], [41], [42].

"Recently, a new coherent radiation process from crystals irradiated with relativistic electrons was discovered [3], [10], [17], [43], [44], which is called parametric X-ray radiation (PXR)". This statement was made by professor H. Nitta in the origin of his work "Theoretical notes on parametric X-radiation" [41] in 1996.

H. Nitta pointed to the five pioneer experimental works, where two of them [10], [17] were initiated and carried out by the author of this paper.

About 30 years have passed since that time when the first experiments had been started. My former colleagues from the deceased USSR, who were my opponents formerly, wrote the series reviews on PXR [45], [46] [47], [48], where they passed into silence the discussions on the nature of the PXR and about their participation in those discussions.

Here I use my right to comment some important information, which stays over of notice of specialists till now.

Our work within the framework of the wide international collaboration was supported by BMFT under Contract N.06DA64II, DFG Contract N.436UKR113-19 and a Max-Plank-Forschungspreis of Germany. Besides, our work was supported by the Ukrainian Academy of Science, The State Committee of Science and Technology of Ukraine (Projects "Semiclassical" and "Quanta"), and by Grants N. UA3000 and N. UA3200 from the International Science Foundation.

References


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7 N.F. Shul’ga reported our proposition to carry out experiments at S-DALINAC [26].
8 The possibility to visit Germany appeared with collapse of the USSR.
9 A.V. Shchagin, who permanently reports someone’s results as his own, even he walks off with new readers to the beginning of the discussion about the nature of PXR, instead of discussing the modern achievements.


КОГЕРЕНТНОЕ ПОЛЯРИЗАЦИОННОЕ ИЗЛУЧЕНИЕ РЕЛЕЙТИВИСТСКИХ ЭЛЕКТРОННЫХ В КРИСТАЛЛАХ

В. Л. Мороковский

Краткий рассказ об истории жарких споров и дискуссий вокруг природы так называемого параметрического рентгеновского излучения, которое завершилось признанием открытия явления когерентного поляризационного термоэлектронного излучения релативистских заряженных частиц в кристаллах. Изложены информация и комментарии, которые еще ранее находятся вне поля зрения специалистов.

КОГЕРЕНТНЕ ПОЛЯРИЗАЦІЙНЕ ВИПРОМИНУВАННЯ РЕЛЕЙТИВІСЬКИХ ЕЛЕКТРОНІВ В КРИСТАЛІХ

В. Л. Мороковський

Коротка розповідь про гарячі суперечки та дискусії відносно природи так званого параметричного рентгенівського випромінювання, які завершилися призnanням відкриття явища когерентного поляризаційного галамовського випромінювання релативістських заряджених часток в кристалах. Викладені інформація та коментарі, які все ще залишаються незагалом уваги спеціалістів.