RADIATIVE RELAXATION OF RELATIVISTIC ELECTRON BEAM IN HELICAL UNDULATOR

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A process of radiative relaxation of the monoenergetic relativistic electron beam, moving through a static spatially periodic helical magnetic field is investigated theoretically. Interaction of electron with random incoherent electromagnetic field of spontaneous radiation is shown to lead to spread in the electron-momentum. The conditions necessary for self-amplified spontaneous emission process realization in the short-wavelength region are derived.

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1. INTRODUCTION

Relativistic electron beam, moving through a periodic magnetic field (undulator), is known to be frequency tunable source of powerful short-wavelength electromagnetic radiation. Under certain conditions initial incoherent spontaneous radiation can be amplified by rather dense monoenergetic electron beam at single undulator passage (mode of self-amplified spontaneous emission (SASE)) [1-3]. Determination of real prospects of such process realization for the stimulated emission from an ultra-relativistic electron beams is of considerable interest in connection with the researches, directed on the development of X-ray free electron lasers (FEL).

In this work theoretical analysis of relativistic electron beam relaxation in the mode of incoherent spontaneous radiation in the undulator is presented. Interaction of electrons with incoherent field of spontaneous radiation is shown to lead to a growth of the electron momentum spread. The rate of change in the mean-square value of the longitudinal momentum is found for initially monoenergetic electron beam and conditions necessary for SASE process realization in the short-wavelength region are also formulated.

2. FORMULATION OF THE PROBLEM

Let us consider the beam of relativistic electrons with uniform particle density n_0, moving in the positive direction of z axis in the helical periodic magnetic field $\mathbf{H_u}$

$$\mathbf{H}_u = H_0\left[\mathbf{e}_x \cos(k_u z) + \mathbf{e}_y \sin(k_u z)\right],\ z>0, \tag{1}$$

where $k_u = 2\pi/\lambda_u$, H_0 and λ_u are the amplitude and period of magnetic field, $\mathbf{e}_x$, $\mathbf{e}_y$ are unit vectors along the x and y axes.

The electron beam is monoenergetic at z=0 the undulator entrance and unbounded on radius.

Taking into account a discrete structure of the beam, we will derive the mean-square longitudinal electron-momentum spread due to interaction of electrons with each other via incoherent electromagnetic field, produced by the same electrons moving through the undulator.

3. BASIC EQUATIONS

The evolution in time of mean-square value of electron-momentum spread one can be found directly by considering motion of some individual (test) electron in the undulator field and in the fields produced by all other electrons of beam, moving in the undulator. The equations of motion for i-th electron in the magnetic field of undulator and in the field, produced by other individual electrons of beam, can be written in the form:

$$\frac{d\,p_{zi}}{dt} = F_z\left[\mathbf{r}_i(t),t\right] = \sum_s F_z^{(s)}\left[\mathbf{r}_i(t),t;x_s\right], \tag{2}$$

$$\frac{d\mathbf{r}_i}{dt} = \frac{\mathbf{p}_i(t)}{m\gamma_i(t)}, \tag{3}$$

$$F_z^{(s)}(\mathbf{r},t;x_s) = e\left\{E_{zs}(\mathbf{r},t) + \frac{1}{c}\left[\mathbf{v}\left(\mathbf{H}_s(\mathbf{r},t)\right)\right]_z\right\}, \tag{4}$$

where $\mathbf{r}_i$, $\mathbf{p}_i$ are the position and momentum of i-th electron at time t; $F_z^{(s)}(\mathbf{r},t;x_s)$ is the longitudinal force of two electrons interaction via the fields of s-th electron, $x_s(t)=\{\mathbf{r}_s(t), \mathbf{p}_s(t)\}$ means the Cartesian coordinates and momentum of s-th electron, $\mathbf{E}_s(\mathbf{r},t)$, $\mathbf{H}_s(\mathbf{r},t)$ are the strength of electric and magnetic fields, produced by the s-th electron at time t in $\mathbf{r}$ coordinate; e, m are the charge and the mass of electron, $\mathbf{v} = d\mathbf{r}/dt$; $\gamma = \sqrt{1+p^2/m^2c^2}$.

The electric and magnetic fields are expressed via the retarded potentials of electromagnetic field for the individual electrons moving in an undulator [4]:

$$\mathbf{E}_i = e\left\{\frac{(\mathbf{n}'_i - \boldsymbol{\beta}'_i)(1-\beta'^2_i)}{R'^2_i(1-\mathbf{n}'_i\boldsymbol{\beta}'_i)^3} + \frac{\left[\mathbf{n}'_i\left[(\mathbf{n}'_i - \boldsymbol{\beta}'_i)\,\dot{\mathbf{v}}_i\right]\right]}{c^2 R'_i(1-\mathbf{n}'_i\boldsymbol{\beta}'_i)^3}\right\}, \tag{5}$$

$$\mathbf{H}_i = \left[\mathbf{n}'_i\mathbf{E}_i\right], \tag{6}$$

where $\mathbf{n}_i = \mathbf{R}_i/R_i$, $\mathbf{R}_i = \mathbf{r} - \mathbf{r}_i(t)$, $\boldsymbol{\beta} = \mathbf{v}/c$, $\dot{\mathbf{v}} = d\mathbf{v}/dt$, a prime denotes, that a value is taken in the retarded moment at time t′, determined by the following relation $t' = t - R_i(t')/c$.

The value of field at observation point $\mathbf{r}$ at time t is determined by only those electrons, initial coordinates of which satisfy condition:

$$t - t_{0i} > \left|\mathbf{r} - \mathbf{r}_i(t_{0i})\right|/c, \tag{7}$$

where $\mathbf{r}_{0i}=\{x_{0i}, y_{0i}, 0\}$ is the coordinate of the i-th electron at time t_{0i}.

Since coordinates of electron at the undulator entrance are random values, the electromagnetic field produced by these electrons at the coordinate $\mathbf{r}$ at time t, will be random too. This electromagnetic field gives rise to random deviation from mean value of electron momentum. The deviation of longitudinal momentum from the mean value Δp_{zi} for the i-th particle will be obtained by integrating Eq. (2):

$$\Delta p_{zi} = \int_{t_{0i}}^{t} \delta F_z\left[\mathbf{r}_i(t'), t'\right] dt' , \qquad (8)$$

where $\Delta p_{zi}=p_{zi}-\langle p_{zi}\rangle$; $\delta F_z = F_z - \langle F_z \rangle$; $\langle p_{zi}\rangle$, $\langle F_z\rangle$ are the mean value of longitudinal momentum and force.

Using Eqs. (2) and (8) of a test particle motion we obtain the expression for rate of change of mean-square spread in the longitudinal momentum of particles

$$\frac{d}{dt}\left\langle (\Delta p_{zi})^2 \right\rangle = 2\int_{t_{0i}}^{t} \left\langle \delta F_z\left[\mathbf{r}_i(t'),t'\right] \delta F_z\left[\mathbf{r}_i(t),t\right] \right\rangle dt' , \qquad (9)$$

where angular brackets indicate average values.

Averaging in the right-hand side of Eq. (9) is performed with the distribution function in phase space of coordinates and momentum of all particles, being in the undulator at time *t*. Using the principle of the phase volume conservation the averaging in Eq.(9) over the distribution at time *t* will be replaced by averaging over initial distribution. Take account that particles are identical and neglecting initial correlation between them it can be shown that the ensemble average of $\delta F_z(\mathbf{r},t)\delta F_z(\mathbf{r}',t')$ can be expressed as:

$$\left\langle \delta F_z(t)\, \delta F_z(t') \right\rangle = \int dx_{os} f_1(x_{os}) \times$$
$$\times F_z^{(s)}\left[t; x_s(t, x_{os})\right] F_z^{(s)}\left[t'; x_s(t', x_{os})\right] , \qquad (10)$$

where f_1 is the single-particle distribution function, $x_{0s}=x_s(0)$.

Choosing the time $\tau=t-t_{0i}$ as quite small in comparison with t_r time ($\tau<t_r$) in which essential electron movement change occurs, in the right-hand side of Eq. (9) $\mathbf{r}_i(t)$ can be replaced by unperturbed trajectory $\mathbf{r}_i^{(0)}(t)$ of electron in the undulator. Then the Eq. (9) for initially monoenergetic electron beam takes the form:

$$\frac{d}{dt}\left\langle (\Delta p_{zi})^2 \right\rangle = 2\int_{t_{0i}}^{t} dt' \int dq_{0s} n_0 F_z^{(s)}\left[\mathbf{r}_i^{(0)}(t), t; x_s(t, q_{0s})\right] \times$$
$$\times F_z^{(s)}\left[\mathbf{r}_i^{(0)}(t'), t'; x_s(t', q_{0s})\right] , \qquad (11)$$

$$\mathbf{r}_s^{(0)} = \mathbf{r}_{0s} - \mathbf{e}_x r_u \sin(k_u z_s) + \mathbf{e}_y r_u \cos(k_u z_s) + \mathbf{e}_z z_s ,$$

$$\mathrm{v}_s^{(0)} = -\mathbf{e}_x \mathrm{v}_\perp \cos(k_u z_s) - \mathbf{e}_y \mathrm{v}_\perp \sin(k_u z_s) + \mathbf{e}_z \mathrm{v}_{oz} ,$$

where $z_s(t) = \mathrm{v}_{0z}(t - t_{0s})$, $q_{0s}=\{x_{0s}, y_{0s}, \mathrm{v}_{0z}t_{0s}\}$, v_{0z} is the axial velocity of electrons at z=0, $r_u = cK/k_u \mathrm{v}_{0z}\gamma_0$, $\mathrm{v}_\perp = cK/\gamma_0$, $K = |e|H_0/mc^2 k_u$.

4. RESULTS AND DISCUSSION

Analytical expressions for the force, acting on the individual electron in the electromagnetic field, produced by other particle, is derived assuming that an undulator parameter is small: $K^2<<1$. Expanding expressions (5) and (6) in a power series of this parameter up to linear terms in K we substitute these expressions into Eq. (4). Considering interaction of electrons only via the fields of electromagnetic radiation the following expression for longitudinal components of the force, acting on the i-th electron in the field of the s-th one is obtained:

$$F_{zs}\left[\mathbf{r}_i^{(0)}(t), t; q_{0s}\right] = \left(eK k_u \gamma_0 \beta_0\right)^2 \Phi\left(\Delta z_{si}, \rho_{si}/\gamma\right) , \qquad (12)$$

$$\Phi = -\mathrm{Re}\left(\Phi_r e^{i\psi}\right) ,$$

$$\Phi_r(x,y) = \frac{1}{k_0 R_*}\left[\frac{1}{k_0 R_*}\left(\beta_0 + \frac{x}{R_*}\right) - \right.$$
$$\left. - i\left(\beta_0 + \frac{x}{R_*} - \frac{\beta_0}{k_0^2 R_*^2} - \frac{\beta_0 y^2}{2R_*^2}\right)\right] ,$$

where $\psi(x,y) = k_u \gamma_0^2 (x + \beta_0 R_*)$,

$$R_*(x,y) = \left(x^2 + y^2\right)^{1/2} , \ \Delta z_{si} = \mathrm{v}_{os}(t_{0s} - t_{0i}) ,$$

$$\rho_{si} = \left[(x_{os} - x_{oi})^2 + (y_{os} - y_{oi})^2\right]^{1/2} , \ k_0 = k_u \gamma_0^2 \beta_0 .$$

In the $K^2<<1$ approximation considered the inequality (7) can be written in the form of:

$$z \geq \gamma_0^2 (\Delta z_{si} + \beta_0 R_*) . \qquad (13)$$

Substituting expression (12) into Eq. (11), the following expression for the rate of change of mean-square longitudinal electron-momentum deviation can be obtained:

$$\frac{d}{dt}\left\langle (\Delta p_{zi})^2 \right\rangle = 2\left(eK k_u \gamma_0 \beta_0\right)^4 \times$$
$$\times \int_0^t dt' \int_{V_0(t')} \left[\Phi\left(\Delta z_{si}, \rho_{si}\right)\right]^2 n_0 d\mathbf{q}_{os} , \qquad (14)$$

where V_0 is the integration region determined by condition (13).

In approximation considered of a homogeneous and unbounded in radius electron beam the average beam density is independent on the initial transversal coordinates x_{0s}, y_{0s} and time t_{0s}. Therefore in the right-hand side of Eq. (14) integration over these coordinates will be replaced by integration over r′, θ, φ coordinates which is determined as: $x_{0s} - x_{0i} = \gamma_0 r' \sin\theta \cos\varphi$, $y_{0s} - y_{0i} = \gamma_0 r' \sin\theta \sin\varphi$, $\mathrm{v}_{0z}(t_{0s} - t_{0i}) = r' \cos\theta$.

Integrating right-hand side of Eq. (14) by taking into account the fields radiating in forward direction only ($z \geq z_s(t')$), the expression

$$\frac{d}{dz}\left\langle (\Delta p_z)^2 \right\rangle = \pi e^4 K^4 k_u^2 n_0 \beta_0^2 z^2 / c^2 \qquad (15)$$

can be obtained.

Here the distance z from the undulator entrance is chosen as the independent variable.

Integrating in Eq. (15) over z we obtain the following expression for the mean-square longitudinal electron-momentum deviation from mean value

$$\left[\left\langle (\Delta p_z)^2 \right\rangle\right]^{1/2} / p_{0z} = \frac{2}{\sqrt{3}} \frac{r_0}{\lambda_u} K^2 N^{1/2} \gamma_0 (k_u z)^{3/2} , \qquad (16)$$

where $N = n_0 \lambda_u^3 / 8\gamma^4$.

The formula (16) describes the mean square deviation from the mean value of longitudinal electron-momentum in the initially monoenergetic stream of the relativistic electrons, moving through the spatially periodic magnetic field (1) of undulator. From Eq. (16) follows, that longitudinal momentum spread increases as beam moving along z axis in undulator. Interaction of electrons via the incoherent electromagnetic field is shown to result in momentum spread, therefore on the whole a process can be considered as the initial before-kinetic stage of particles diffusion in momentum space at the incoherent spontaneous emission from the relativistic electron beam in undulator.

The expression for the momentum spread can be written in the form:

$$\left[\left\langle\left(\Delta p_z\right)^2\right\rangle\right]^{1/2} = \frac{z}{v_{0z}} F_R \sqrt{\frac{3}{2\pi} N_{coh}(z)}\ , \qquad (17)$$

where $N_{coh}=Nz/\lambda_u$, $F_R = (2/3)(r_0 H_0 \gamma_0)^2 \beta_0^3$ is the force of radiative deceleration for pointed electron, $r_0=e^2/mc^2$ is the classical electron radius.

It should be noted that the relativistic electron beams must be rather dense for self-amplification of spontaneous emission [5]: N>>1, therefore $N_{coh}>>1$ for such beams.

The condition for collective instability and exponential growth of electromagnetic radiation intensity at the self-amplified spontaneous emission is a small momentum spread in the electron beam [2,3,6]:

$$\Delta p_z / p_{0z} < \rho\ ,$$

where $\rho = \left(K^2 n_0 r_0 \lambda_u^2 / 16\pi\right)^{1/3} / \gamma_0$.

Instability saturation takes place at the distance L_{sat} [2,3,6]: $L_{sat}=\lambda_u/\rho$.

Consequently, the mean-square momentum spread due to the effect of radiative relaxation of electrons in incoherent electromagnetic field produced by them, is possible to be neglect when

$$\rho >> 8\pi^2 K\left(\frac{r_0}{3\lambda_u}\gamma_0\right)^{1/2}. \qquad (18)$$

If a condition opposite (18) is fulfilled and momentum spread brought in by the incoherent spontaneous undulator radiation will be considerably larger than parameter ρ, the mode of the intensive amplification of incoherent spontaneous radiation will not be able to be realized.

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REFERENCES

1. R. Bonifachio, C. Pellegrini., L.M. Narducci. Collective instabilities and high-gain regime in a free-electron laser // *Opt. Commun.* 1984, v.50, №6, p.373-379.
2. C. Pellegrini. Summary of the Working Group on FEL theory // *Nucl. Instr. and Meth. A*. 1985, v.239, №1, p.127-129.
3. K.J. Kim. An analysis of self-amplified spontaneous emission // *Nucl. Instr. and Meth. A.* 1986, v.250, №1,2, p.396-403.
4. L.D. Landau, E.M. Lifshits. *Theory of field.* M.: "Nauka", 1967, p.460 (in Russian).
5. V.I. Kurilko, V.V. Ognivenko. Dynamics of monoenergetic stream of point electrons in spiral undulator // *Physics of plasma*. 1994, v.20, p.634-639 (in Russian).
6. P. Sprangle, R.A. Smith. Theory of free-electron lasers // *Phys. Rev. A*. 1980, v.21, №1, p.293-301.

РАДИАЦИОННАЯ РЕЛАКСАЦИЯ РЕЛЯТИВИСТСКОГО ЭЛЕКТРОННОГО ПУЧКА В СПИРАЛЬНОМ ОНДУЛЯТОРЕ

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Теоретически исследован процесс радиационной релаксации моноэнергетического релятивистского электронного пучка, движущегося в пространственно периодическом спиральном магнитном поле ондулятора. Показано, что взаимодействие электронов со случайным некогерентным электромагнитным полем спонтанного излучения приводит к разбросу по импульсам электронов. Сформулированы условия, необходимые для реализации процесса самопроизвольного усиления спонтанного излучения в ультракоротковолновой области.

РАДІАЦІЙНА РЕЛАКСАЦІЯ РЕЛЯТИВІСТСЬКОГО ЕЛЕКТРОННОГО ПУЧКА В СПІРАЛЬНОМУ ОНДУЛЯТОРІ

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Теоретично досліджено процес радіаційної релаксації моноенергетичного релятивістського електронного пучка, що рухається в просторово-періодичному спіральному магнітному полі ондулятора. Показано, що взаємодія електронів з випадковим некогерентним електромагнітним полем спонтанного випромінювання призводить до розкиду по імпульсах електронів. Сформульовано умови, необхідні для реалізації процесу самочинного підсилення спонтанного випромінювання в ультракороткохвильовий області.