CRITICAL ENERGY IN THE CYCLOTRON HEATING OF IONS IN AN ECR PLASMA SOURCE.*

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The problem of plasma cyclotron heating in ECR plasma sources, to sustain the discharge, remains important at present. There are two methods for the analysis of this problem. The first one is the one particle stochastic mechanism and the second one is related with the non-linear interaction of waves where the collective behaviour of particles becomes the most important. In this work, in the Hamilton formalism, the stochastic mechanism of the cyclotron heating is analyzed. It is considered the case of an ECR plasma source where a TE_{11} electromagnetic wave is applied. The critical energy for the resonance mixing is calculated an by the "section-of-surface" method the inhomogeneity of the external magnetic field is analyzed.

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1. AVERAGING METHOD

The equation of motion for one particle can be written as

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E} + \frac{e}{c}\left(\mathbf{v}\times\mathbf{B}\right), \tag{1}$$

where $\mathbf{p} = m\mathbf{v}$ is the momentum, e and m are the charge and the mass of the ions. The magnetic field $\mathbf{B}$ and the electric field $\mathbf{E}$ in eq. (1) are assumed as the sum of slowly varying in space fields plus rapidly varying fields in the form

$$\mathbf{E} = \mathbf{E}_0 + \tilde{\mathbf{E}}, \mathbf{B} = \mathbf{B}_0 + \tilde{\mathbf{B}}. \tag{2}$$

The electromagnetic wave propagating along the Oz axis is represented as

$$\begin{aligned}\tilde{\mathbf{E}} &= E_0\left\{\cos(\omega t - kz), \sin(\omega t - kz), 0\right\},\\ \tilde{\mathbf{B}} &= E_0\left\{-\sin(\omega t - kz), \cos(\omega t - kz), 0\right\},\end{aligned} \tag{3}$$

where ω is the frequency of the electromagnetic wave, and $k = \omega/c$ is the wave vector. The stationary magnetic field $\mathbf{B}_0$ until the first order terms in relation with the deviation from the axis z is represented as

$$\mathbf{B}_0 = \left\{-\frac{x}{2}\frac{dB(z)}{dz}, -\frac{y}{2}\frac{dB(z)}{dz}, B(z)\right\}. \tag{4}$$

Now, introducing the new dimensionless variables

$$\begin{aligned}&\tau = \omega t, X = kx, Y = ky, Z = kz,\\ &\mathbf{P} = \frac{\mathbf{p}}{mc}, \dot{X} = \frac{dx}{d\tau} = \frac{v_x}{c}, \rho = X + iY,\end{aligned} \tag{5}$$

into (1), and using expressions (3)-(5), the momentum components along OX and OY can be obtained

$$\begin{aligned}\frac{dP_x}{d\tau} &= \left(P_z - 1\right)g\cos(\tau - Z) - \left(P_y\Omega_0 + P_z\frac{Y}{2}\frac{d\Omega_0}{dZ}\right),\\ \frac{dP_y}{d\tau} &= \left(P_z - 1\right)g\sin(\tau - Z) - \left(P_x\Omega_0 + P_z\frac{X}{2}\frac{d\Omega_0}{dZ}\right),\end{aligned} \tag{6}$$

where $\Omega_0 = eB/mc\omega$, and $g = eE_0/mc\omega$ is the small parameter.

In order to use the Bogoliuvov's averaging method with resonances [1] it becomes important to transform the system of equations (6) into the standard form. This procedure is obtained by the introduction of the complex variable ρ :

$$\rho = \zeta\exp\left(\frac{i}{2}\int_0^{\tau}\Omega_0(\tau')d\tau'\right). \tag{7}$$

In this case, the equation of motion of the perpendicular motion is

$$\ddot{\zeta} + \omega_0^2\zeta = -g(1 - \dot{Z})\exp[i(\theta - Z)], \tag{8}$$

where $\omega_0 = \Omega_0/2$, and $\theta = \tau - \int_0^{\tau}\Omega_0(\tau')d\tau'$. The "surface-of-section" picture of equation (8) is shown in Fig. 1. From this figure, we can observe that as the amplitude of the wave increases the convergence to a point is more slowly. In the case of very high values of the amplitude we obtain high-order resonances (small circles). This picture illustrates that there exist critical values of the energy absorption in dependence of the power of the wave.

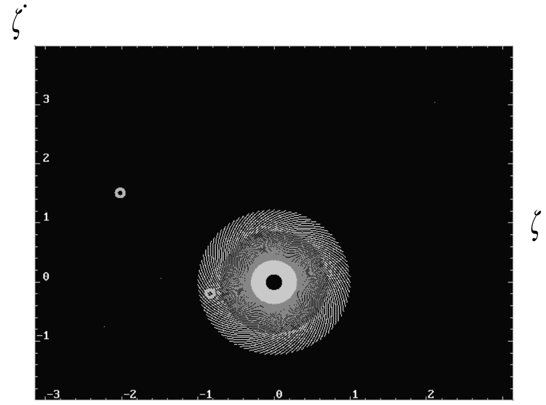

Fig. 1. Poincare map obtained from equation (8), for different powers of the electromagnetic field.

Near to the cycotron resonance,

$$\omega_0 \approx 1 - \Omega_0. \tag{9}$$

Thus, an easy way to obtain the standard form of equations (6) consist of introducing the new variables

$$\zeta_1 = a_1 \cos(\theta + \psi_1), \zeta_2 = a_2 \cos(\theta + \psi_2), \quad (10)$$

where $a_{1,2}$ and $\psi_{1,2}$ are unknown functions to be obtained, and ζ_1 is related with ζ_2 by $\zeta = \zeta_1 + \zeta_2$. Applying the averaging method [1] to equations (6) using variables (10), the system of equation for $a_{1,2}$ and $\psi_{1,2}$ are obtained in the limiting cases

$$g << 1, \frac{d\Omega_0}{dZ} << 1, |\Omega_0 - 1| << 1. \quad (11)$$

After a straightforward algebra we obtain the corresponding average energy

$$W_\perp = W_0 \left(\frac{P_x^2 + P_y^2}{2} \right) =$$

$$W_0 \left\{ \frac{\Omega_0(\tau)}{\Omega_0(0)} \frac{P_\perp^2}{2} - g P_\perp(0) \left[\frac{\Omega_0(\tau)}{\Omega_0^2(0)} \sin \chi_0 - \right. \right.$$

$$\left. \left. \frac{1 - \dot{Z}}{\sqrt{\Omega_0(0)\Omega_0(\tau)}} \sin \left(\chi_0 + Z + \int_0^\tau [\Omega_0(\tau') - 1] d\tau' \right) \right] \right\}$$

(12)

where χ_0 is the initial phase, $\Omega_0(0)$ is the initial value of $\Omega_0(\tau)$ and $W_0 = mc^2$ is the rest energy.

To study the energy absorption at the cyclotron resonance, we will consider the parameters of an electron cyclotron resonance plasma source reported in [3]. The magnetic field $B_z(z)$ is calculated using the method described in [4]. The parameters used in calculations are summarized in Table 1.

Table 1. Parameters for calculating the magnetic field and the perpendicular energy (12).

$J_1=J_2=50.0$A	$r_1=r_2=8.0$cm	$L_1=L_2=20$	$M_1=M_2=20$
$Z_1=-60-0$cm	$Z_2=60.0$cm	$\chi_0=0.0$	V(0)=2.65E8cm/s

Using these parameters, in Fig. 2 are shown two cases of the energy (12) in dependence of $\tau = \omega t$. The first one corresponds to the field density $E_0 = 2.62 \times 10^5$ V/cm, and the second one corresponds to the case of $E_0 = 2.62 \times 10^2$ V/cm. As can be seen from these charts, a maximum of the energy absorption is present. If the field density is increased, the energy absorption is stopped (the average is equal to zer). Here a big question arises, is there only one maximum in the energy absorption?

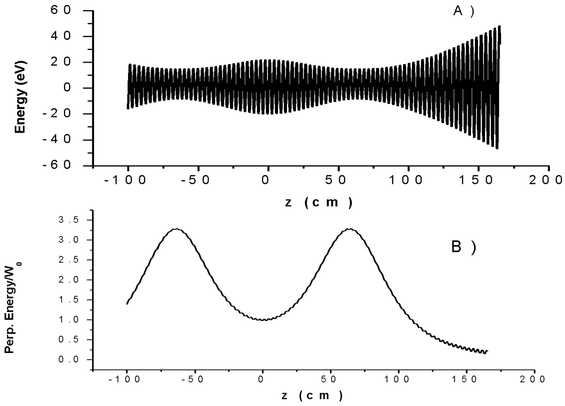

Fig. 2. Energy axial distribution for two values of the wave amplitude $E_0 = 2.62 \times 10^5$ (A) and $E_0 = 2.62 \times 10^2$ (B).

2. OBTANING OF THE HAMILTONIAN

Following the procedure reported in [5] to describe the motion of a charged particle in magnetic mirror systems in presence of an electromagnetic field, where the perturbation method applied to the Hamiltonian of the particle seems to be highly suitable in using canonical transformations and thus finding the cyclic coordinates (momentums and frequencies) of motion of the particle. An appropriated form of the Hamiltonian is obtained by replacing the cylindrical coordinates by orthogonal curvilinear coordinates, defined by the lines of force of the magnetic field. By applying the perturbation method of multi-periodic systems to the Hamiltonian in these coordinates, the Hamiltonian of the guiding centers is obtained from which the integrals of motion and the frequencies of motion can be determined. The magnetic geometry for a mirror system is defined by two equations $rot\mathbf{B} = 0, div\mathbf{B} = 0,$ and by the boundary conditions. The motion of a particle in this field is described by the Hamiltonian in the field coordinates [5]

$$H = H^0 + \Delta H, \quad (13)$$

where H^0 is the Hamiltonian which accounts for the external magnetic field, and ΔH is the Hamiltonian associated with the electromagnetic field. Introducing the magnetic field for a mirror system in the form

$$B_\varphi(\xi, \eta) = 0, B_z(\xi, \eta) = B(z)[1 + k\alpha(z)], \quad (14)$$

where $k = 1/B(z)$, and $B(z)$ is the magnetic field along the Oz axis.

Using the perturbative method described in [5] in relation with the small parameter k, and the initial and boundary conditions: if $k = 0$, $r = \xi$, $z = \eta$; $r(0,\eta) = 0, z(0,\eta) = 0$, as well as the components of the electric field for the case of a TE_{11} cylindrical wave-guide

$$\tilde{E}_r = -iC_0 Z_H \frac{k}{\gamma_{lq}^2}\frac{l}{r} J_l(\gamma_{lq} r)\sin l\varphi \times$$
$$\exp[i(kz - \omega t] + c.c.,$$
$$\tilde{E}_\varphi = -iC_0 Z_H \frac{k}{\gamma_{lq}} J_l'(\gamma_{lq} r)\cos l\varphi \times \quad (15)$$
$$\exp[i(kz - \omega t] + c.c.,$$
$$\tilde{E}_z = 0,$$

where $\gamma_{lq} = j'_{lq}/R$, j'_{lq} is the n-th root of equation $J'_l(x) = 0$, R is the internal wave-guide radius and Z_H is the impedance. In the limit of linear terms respecting the electric field we obtain an expression for the Hamiltonian

$$H = p_1\Omega_0\left(b_0 + kb_1 + k^2 b_2\right) + \frac{e}{2mc}\left(\tilde{b}_0 + k\tilde{b}_1 + k^2\tilde{b}_2\right)\times$$
$$\frac{E_{0r}}{\omega}\sin lq_2 \cos(k\eta - \omega t + \theta_0),$$

(16)

where b_i and $\tilde{b}_i$ (i=0, 1, 2) are functions of ($q_1, q_2, q_\eta, p_1, p_2, p_\eta$). Here we have transformed canonically by the introduction of variables [5]

$$\xi^2 = \frac{2}{m\Omega_0}\left[p_1 + p_2 + 2\sqrt{p_1 p_2}\sin(q_1 + q_2)\right],$$
$$p_\xi^2 \xi^2 = 2p_1 p_2 \cos^2(q_1 + q_2),$$
$$\varphi = \arctan\frac{\sqrt{2p_1}\cos q_1 + \sqrt{2p_2}\sin q_2}{\sqrt{2p_1}\sin q_1 + \sqrt{2p_2}\cos q_2},$$
$$p_\varphi = p_2 - p_1, \quad (17)$$

Where p_i $(i = 1,2,\eta)$ are the actions and q_i $(i = 1,2)$ are the angles having the meaning of the cyclotron and drift frequencies as is noticed in [5]. Expression (16) is obtained in the limit $\sqrt{p_1/p_2} << 1$.

3. STOCHASTIC ANALYSIS

From the expression (20) for H_0 we can determine η until zero-order terms with respect to k as follows

$$\eta = \frac{v_\eta}{p_1\Omega_0}t + \eta_0, \quad (18)$$

transforming the Hamiltonian into

$$H = p_1\Omega_0\left[\cos^2(q_1 + q_2) + \frac{p_\eta^2}{2mp_1\Omega_0}\right] +$$
$$\frac{eE_{0r}}{4mc\omega}\sqrt{2p_1 m\Omega_0}\sin lq_2 \times \quad (19)$$
$$\left[\cos\left(q_2 + \left(\Omega_0 - \bar{k}v_\eta + \omega\right)t - \theta_0\right) +\right.$$
$$\left.\cos\left(q_2 + \left(\Omega_0 - \bar{k}v_\eta + \omega\right)t + \theta_0\right)\right].$$

Here it was assumed that $q_1 = \Omega_0 t$, and $\bar{k} = k/p_1\Omega_0$. At exact resonances, when $\Omega_0 \pm \left(\bar{k}v_\eta - \omega\right) = 0$, from the last expression we obtain

$$H_{\text{res}} \approx p_1\Omega_0\left[\cos^2(q_1 + q_2) + \frac{p_\eta^2}{2mp_1\Omega_0}\right] +$$
$$\frac{eE_{0r}}{8mc\omega}\sqrt{2p_1 m\Omega_0}\left[\sin(l+1)q_2 + \right. \quad (20)$$
$$\left.\sin(l-1)q_2\right] = E = const.$$

The last result shows that no other constant of motion in addition to the energy exist, then we can be sure that the system is stochastic. This result was obtained in [6], where an electrostatic wave was considered. In order to analyze the merging of the stochasticity it becomes important the construction of "surface-of-section" pictures of Hamiltonian (20) in dependence of the wave amplitude value.

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