DEVELOPMENT OF TRANSPORT THRESHOLD MODEL OF NEOCLASSICAL TEARING MODES

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1. The topic of neoclassical tearing modes (NTMs) takes an important place in recent investigations on fusion-oriented magnetizied plasma physics. In a great degree, the interest to these modes is explained by the fact that, according to numerous experiments on many devices [1-10], they can limit the ultimate plasma pressure in the designed thermonuclear reactor ITER [11].

The primary theory of NTMs has been formulated in the works [12, 13]. The physical essence of this theory is prediction that generation of magnetic islands is possible even in a tokamak with favorable radial distribution of the equilibrium current, i.e. for $\Delta' < 0$, where Δ' is the standard tearing mode theory parameter [14]. Mathematically, the essence of [12, 13] is generalization of the Rutherford equation [15] for the magnetic island width evolution by incorporating the term responsible for the bootstrap drive of the islands.

In addition to confirming the possibility of generation of magnetic islands for $\Delta' < 0$, predicted by [12, 13], the above experiments [1-10] have shown that such generation takes place only if plasma pressure exceeds some critical value, or in other words, that the onset of NTMs has a threshold with respect to the beta parameter. Meanwhile, though the theory [12, 13] evidences clearly a beta dependence of NTMs, it did not point out any their beta threshold. In order to obtain theoretically such a threshold in the scope of the generalized Rutherford equation, it should be augmented by incorporating additional terms responsible for different physical effects. Depending on incorporated effects, one can arrive at different versions of more complicated generalized Rutherford equation. One of such versions has been called in [11] the transport threshold model of NTMs. This model is obtained as a modification of the bootstrap drive by incorporating anomalous perpendicular transport [16].

Originally, such a model was constructed in the approximation of nonrotating magnetic islands and in neglecting the anomalous perpendicular viscosity [16, 17]. Recently this model has been generalized by allowing for both the island rotation [18] and the anomalous perpendicular viscosity [19, 20]. The goal of the present work is to overview the results of [18-20].

2. The authors of [18-20] were interested in the island width evolution equation of the form

$$\frac{\tau_s}{r_s}\frac{dW}{dt} = r_s\left(\Delta' + 4\Delta_{bs}\right). \tag{1}$$

Here W is the island width, r_s is the radial coordinate of the rational magnetic surface in whose vicinity the island chain is localized, τ_s is the resistive time calculated for $r = r_s$. The value Δ_{bs} describes the bootstrap drive of NTMs and is given by

$$\Delta_{bs} = -\frac{2^{5/2}Rq}{csWB_0}\sum_{\sigma_x}\int_{-1}^{\infty} d\Omega \overline{J}_{bs}\oint\frac{\cos\xi d\xi}{(\Omega+\cos\xi)^{1/2}}. \tag{2}$$

Here $\overline{J}_{bs} \equiv \left\langle \hat{J}_{bs}\right\rangle$, $\hat{J}_{bs}$ is the perturbed bootstrap current, $\langle\cdots\rangle$ is the averaging over the island magnetic surfaces (for other definitions see [18]).

3. The paper [18] started from the model expression for the total bootstrap current density

$$J_{bs} = -\epsilon^{1/2}\frac{cn_0}{B_\theta}\frac{\partial T}{\partial x}, \tag{3}$$

where $\epsilon = r_s/R$ is the local inverse aspect ratio, n_0 is the equilibrium plasma number density, B_θ is the equilibrium poloidal magnetic field for $r = r_s$, and T is the total plasma temperature. For calculation of $\hat{J}_{bs}$ it is necessary to know the perturbed plasma temperature. It was looked for in [18] by means of the heat conductivity equation of the form

$$n_0\frac{dT}{dt} = -\frac{2}{3}\boldsymbol{\nabla}\cdot\mathbf{q}. \tag{4}$$

Here $\mathbf{q} = -n_0\left(\chi_\parallel\boldsymbol{\nabla}_\parallel + \chi_\perp\boldsymbol{\nabla}_\perp\right)T$ is the heat flux, $d/dt = \partial/\partial t + \mathbf{V}\cdot\boldsymbol{\nabla}$, $\mathbf{V}$ is the plasma velocity, $\chi_\parallel$ and $\chi_\perp$ are the parallel and perpendicular heat-conductivity coefficients, respectively.

In addition to (4), in order to allow for the convective transport [16], the authors of [18] considered the collisionless regime described by the drift kinetic equation. As a result, they have found that Eq. (1) reduces to

$$\frac{\tau_s}{r_s}\frac{dW}{dt} = r_s\Delta' + \beta_p\frac{r_sC_{bs}W}{W^2+W_*^2}. \tag{5}$$

Here β_p is the poloidal beta for $r = r_s$, $C_{bs} = c_{bs}\epsilon^{1/2}r_s/sL_T$, $c_{bs} = 1.58$, $L_T = -T_0/T_0'$ is the characteristic scale length of the equilibrium temperature gradient, W_* is the critical magnetic island width defined by

$$\frac{1}{W_*^2} = \frac{1}{c_{bs}}\left(\frac{1}{W_R^2} + \frac{1}{W_{COL}^2} + \frac{1}{W_{CONV}^2}\right). \quad (6)$$

The values W_R, W_{COL} and W_{CONV} are given by

$$W_R = 10.21\,(\chi_\perp/\omega)^{1/2}, \quad (7)$$

$$W_{COL} = 12.00\left(\frac{L_s^2}{k_y^2}\frac{\chi_\perp}{\chi_\parallel}\right)^{1/4}, \quad (8)$$

$$W_{CONV} = 3.91\left[\chi_\perp^{(e)}L_s/(v_T k_y)\right]^{1/3}, \quad (9)$$

ω is the island rotation frequency in the plasma rest frame, v_T is the electron thermal velocity, $\chi_\perp^{(e)}$ is the electron perpendicular heat conductivity.

Equation (5) generalizes the following three threshold models of NTMs. The first is the rotation - transport threshold model suggested in [18] (the term with the subscript "R" in (6) is kept only). The second is the collisional (or the standard or the Fitzpatrick's) transport threshold model suggested in [16] (the term with the subscript "COL" in (6) is kept only). At last, the third is the convective - transport threshold model going back to [16] (the term with the subscript "CONV" in (6) is kept only).

Taking $W = W_*$ one can find from (5) that generation of NTMs is possible only if

$$\beta_p \geq \beta_p^{crit} = 2\,(-\Delta')\,W_*/C_{bs}. \quad (10)$$

If one knows $\chi_\perp$, one can use the relations (6), (10) as a basis for elucidation whether the generalized transport threshold model of NTMs is compatible with the experimental results [1-10] and, thus, is suitable for ITER designing.

4. The effect of perpendicular viscosity can be incorporated into the expression for Δ_{bs} by means of two approaches: MHD and kinetic ones. The work [19] followed the MHD approach, while [20] - the kinetic one.

The effect of interest can not be studied if one appeals to a model MHD expression for J_{bs} like (3) since (3) is a particular case of a more complicated expression for J_{bs} which can be obtained as follows.

It is convenient to start from the parallel projection of the electron motion equation (the parallel Ohm's law) of the form [21]

$$0 = e_e n_0 E_\parallel - \nabla_\parallel p_e - \mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_\parallel^e + \frac{e_i n_0}{\sigma_\parallel}j_\parallel. \quad (11)$$

Here $\boldsymbol{\pi}_\parallel^e$ is the electron parallel viscosity tensor and the remaining definitions are standard. It hence follows that the bootstrap current averaged over magnetic island surface, $\langle J_{bs}\rangle$, is given by

$$\langle J_{bs}\rangle = \frac{\sigma_\parallel}{en_0}\left\langle\left\langle\mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_\parallel^e\right\rangle_\theta\right\rangle, \quad (12)$$

where $\langle ...\rangle_\theta$ denotes averaging over the poloidal oscillations of the equilibrium magnetic field. According to [21], one has approximately $\mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_\parallel^e = (3/2)\pi_\parallel^e\boldsymbol{\nabla}\cdot\mathbf{b}$, where $\pi_\parallel^e$ is the electron parallel viscosity scalar of the form

$$\pi_\parallel^e = -\frac{4}{3}n_0 M_e\frac{R^2q^2}{\epsilon^{3/2}}\nu_e\mathbf{V}_e\cdot\boldsymbol{\nabla}\ln B_0, \quad (13)$$

where $\mathbf{V}_e$ is the electron velocity. Then representing $\sigma_\parallel = e^2 n_0/(M_e\nu_e)$, equation (12) reduces to

$$\langle J_{bs}\rangle = -2en_0\frac{R^2q^2}{\epsilon^{3/2}}\langle\langle(\boldsymbol{\nabla}\cdot\mathbf{b})(\mathbf{V}_e\cdot\boldsymbol{\nabla}\ln B_0)\rangle_\theta\rangle. \quad (14)$$

Leaving the dominant terms in the radial projection of the plasma motion equation, one can express the current j_y in terms of the radial derivative of the plasma pressure, $j_y = (c/B_0)\,\partial p/\partial r$. At the same time, the y-projection of the plasma perpendicular velocity yields

$$V_y = -\frac{cE_r}{B_0} + \frac{c}{en_0B_0}\frac{\partial p_i}{\partial r}. \quad (15)$$

Then (14) is transformed to

$$\langle J_{bs}\rangle = -\frac{\epsilon^{1/2}c}{B_\theta}\left[\left\langle\frac{\partial p_e}{\partial r}\right\rangle + en_0\left(\langle E_r\rangle - \frac{B_\theta}{c}\left\langle V_\parallel\right\rangle\right)\right]. \quad (16)$$

To find $\left\langle V_\parallel\right\rangle$ one should turn to the equation of parallel plasma motion. Averaging this equation over the poloidal oscillations of the equilibrium magnetic field and the magnetic island surfaces, one has

$$\left\langle\left\langle\mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_\parallel^i\right\rangle_\theta\right\rangle + \langle\langle\mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_\perp^i\rangle_\theta\rangle = 0. \quad (17)$$

Here $\boldsymbol{\pi}_\parallel^i$ and $\boldsymbol{\pi}_\perp^i$ are the ion parallel and perpendicular viscosity tensors, respectively. The contribution of the parallel viscosity into (17) could be represented as it was done for electrons (see, e.g., (13)). The term with perpendicular viscosity is approximated by [22] $\left\langle\mathbf{b}\cdot\boldsymbol{\nabla}\cdot\boldsymbol{\pi}_\perp^i\right\rangle_\theta = -4M_i n_0\mu_{\perp i}\partial^2 V_{\parallel i}/\partial x^2$, where $\mu_{\perp i}$ is the ion perpendicular viscosity coefficient. For magnetic island problem one can use an estimation $\partial^2/\partial x^2 \simeq -1/W^2$, where W is the magnetic island width. Then, qualitatively, (17) could be represented as

$$\epsilon^{1/2}\nu_i\left[\left\langle V_\parallel\right\rangle - \frac{c}{B_\theta}\left(\langle E_r\rangle - \frac{1}{en_0}\left\langle\frac{\partial p_i}{\partial r}\right\rangle\right)\right] + \frac{\mu_{\perp i}\left\langle V_\parallel\right\rangle}{W^2} = 0. \quad (18)$$

Substituting $V_\parallel$ from (18) into (16) we arrive at the following expression for the averaged bootstrap current

$$\langle J_{bs}\rangle = -\frac{\epsilon^{1/2}c}{B_\theta}\left[\left\langle\frac{\partial p_e}{\partial r}\right\rangle + \frac{W^2}{W^2+W_\mu^2}\left\langle\frac{\partial p_i}{\partial r}\right\rangle + \frac{W_\mu^2}{W^2+W_\mu^2}en_0\langle E_r\rangle\right], \quad (19)$$

where $W_\mu \simeq \left(\mu_{\perp i}/\epsilon^{1/2}\nu_i\right)^{1/2}$ is the characteristic island width governed by the ion perpendicular viscosity.

Using (19) leads to the following generalization of (5) [19]

$$\frac{\tau_s}{r_s}\frac{dW}{dt} = r_s\Delta' + \beta_p\frac{r_sC_{bs}}{W}\left(\frac{W^2}{W^2+W_*^2} - \frac{C_\mu W_\mu^2}{W^2+W_\mu^2}\frac{\omega}{\omega_{*pi}}\right), \quad (20)$$

where ω_{*pi} is the ion diamagnetic drift frequency and C_μ is a numerical coefficient of the order of unity.

The value W_μ increases with increasing the perpendicular viscosity, while, according to (6), the critical island width W_* increases with increasing the perpendicular heat conductivity. Therefore, when all the transport coefficients are sufficiently large, so that $(W_\mu, W_*) > W$, and the island rotation frequency is of the order of characteristic diamagnetic drift frequencies, $\omega \simeq \omega_{*pi}$, equation (20) reduces to

$$\frac{\tau_s}{r_s}\frac{dW}{dt} = r_s\Delta' - \beta_p\frac{r_sC_{bs}C_\mu}{W}\frac{\omega}{\omega_{*pi}}. \quad (21)$$

Thus, in the presence of the perpendicular viscosity the conventional viewpoint that the bootstrap current always drives the NTMs proves to be non-universal. According to (21), bootstrap effect is stabilizing for islands rotating to the ion diamagnetic drift direction, $\omega/\omega_{*pi} > 0$, and destabilizing in the contrary case, $\omega/\omega_{*pi} < 0$.

The kinetic analysis of [20] confirms the above-given results of [19].

5. In addition to the transport threshold model, the ITER's work [11] introduced the notion of the polarization current threshold model of NTMs. A contribution to development of this model is the work [23] aiming at incorporating the anomalous perpendicular viscosity effect into it.

It is not obligatory that the beta threshold of NTMs is governed by nonlinear effects, i.e. by some version of the generalized Rutherford equation. It is possible that this threshold is determined by a linear mode instability. Such a possibility is discussed in [24].

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References

[1] Chang Z *et al* 1995 *Phys. Rev. Lett.* **74** 4663
[2] Zohm H *et al* 1997 *Plasma Phys. Control. Fusion* **39** B237
[3] La Haye R J *et al* 1997 *Fusion Energy* 1996 vol 1 (Vienna: IAEA) p 747
[4] Gates D A *et al* 1997 *Fusion Energy* 1996 vol 1 (Vienna: IAEA) p 715
[5] Huysmans GTA 1999 *Nucl. Fusion* **39** 1227
[6] Isayama A *et al* 1999 *Plasma Phys. Control. Fusion* **41** 35
[7] Kislov D A *et al* 2001 *Nucl. Fusion* **41** 1473
[8] Koslowski H R *et al* 2000 *Nucl. Fusion* **40** 821
[9] Reimerdes H *et al* 2002 *Phys. Rev. Lett.* **88** 105005
[10] Lazzaro E *et al* 2000 *Phys. Rev. Lett.* **84** 6038
[11] ITER Physics Expert Group on Disruptions, Plasma Control, and MHD, ITER Physics Basis Editors 1999 *Nucl. Fusion* **39** 2251
[12] Qu W X and Callen J D 1985 *UWPR855 Report of University of Wisconsin*
[13] Carrera R *et al* 1986 *Phys. Fluids* **29** 899
[14] Furth H P, Killeen J and Rosenbluth M N 1963 *Phys. Fluids* **6** 459
[15] Rutherford P H 1973 *Phys. Fluids* **16** 1903
[16] Fitzpatrick R 1995 *Phys. Plasmas* **2** 825
[17] Gorelenkov N N *et al* 1996 *Phys. Plasmas* **3** 3379
[18] Konovalov S V, Mikhailovskii A B, Shirokov M S and Tsypin V S 2002 *Plasma Phys. Control. Fusion* **44** 579
[19] Konovalov S V, Mikhailovskii A B and Tsypin V S 2002 Effect of anomalous perpendicular viscosity on bootstrap drive of neoclassical tearing modes *Plasma Phys. Control. Fusion* (to be published)
[20] Konovalov S V, Mikhailovskii A B, Shirokov M S and Tsypin V S 2002 Transport threshold model of neoclassical tearing modes allowing for anomalous perpendicular viscosity *Phys. Plasmas* (to be published)
[21] Mikhailovskii A B 1998 *Plasma Phys. Control. Fusion* **40** 1907
[22] Mikhailovskii A B and Lakhin V P 1995 *Plasma Phys. Rep* **21** 271
[23] Shirokov M S, Mikhailovskii A B, Konovalov S V, Tsypin V S, 2002 *Polarization Current Threshold Model of Neoclassical Tearing Modes in the Presence of Anomalous Perpendicular Viscosity*, this conference
[24] Shirokov M S, Mikhailovskii A B, Konovalov S V, Tsypin V S, 2002 *Spontaneous generation of beta-limiting MHD modes in tokamaks*, this conference