OPTIMUM PLASMA LENS FOR FOCUSSING OF HIGH-CURRENT ION BEAMS

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The optimum plasma lens, intended for the focussing of high-current ion beams, has been investigated.
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INTRODUCTION

Optimum plasma lens (PL) realized in experiments [1] at weak magnetic fields, when the electron Larmor radius is comparable with PL radius, has been theoretically researched. The optimum PL is lens, in which the perturbations are not excited and particle density is uniform. Three possible reasons of perturbation damping in PL have been researched: namely, finite time of ion movement through PL, finite time of electron renovating in it, proximity of PL parameters to optimum ones. It has been shown, that the vortical perturbations are not excited in PL, if the overbalance of ions by electrons is close to limiting one, determined from a condition of balance upset of radial forces confining rotating electrons in the region of finite radius: magnetic confining, centrifugal and electrical scattering forces. At the overbalance of ions by electrons, close to limiting one, the vortical perturbations are not excited in PL. Also it has been shown that the spatial uniformity of electron density is easier supported in the optimum PL.

The collective field instability suppression in PL due to electron density increase on radius in near wall region has been considered. The influence of such electron distribution in PL on focussing of ion beam has been estimated and simulated numerically. It has been shown, that the aberrations, called by such electron density distribution, are small.

INFLUENCE OF PLASMA VORTICAL PERTURBATIONS ON ION BEAM FOCUSSING IN PLASMA LENS

In short cylindrical electrostatic PL for ion beam focusing the vortical perturbations can be excited in wide on radius, $R-r_t<r<R$, with width, r_t, near wall region. Two kinds of vortical perturbations are excited. Namely, quick vortical perturbations are excited due to development of the resonant hydrodynamic instability of interaction of electron vortical perturbations of PL with ions. Also slow vortical perturbations are excited, which phase velocities are much smaller then the electron drift velocity in PL crossed fields. The excitation of the vortical perturbations leads to anomalous electron radial transport, therefore, to decrease of electron density and to PL focusing quality deterioration. The numerical simulation shows (see Fig. 1) that if the overbalance of ions by electrons decreases monotonically due to vortical perturbation excitation in wide near wall layer, which width equals 4/7 of PL radius, then ion beam can be focused six times.

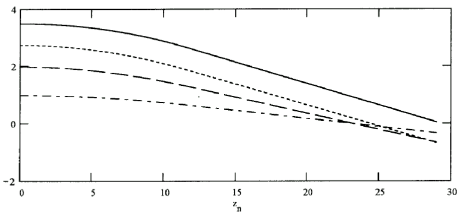

Fig. 1. Focusing of ion beam by plasma lens if n_e-n_i decreases in its circumferential region, $R-r_t<r<R$, monotonically on radius to 0 due to excitation of vortical perturbations

At increase of the PL magnetic field, H_o, electron confining properties of the magnetic field increase and, though vortical perturbations are excited, PL focusing quality is improved. The latter dependence has been observed in [1].

Let us consider damping of the vortical perturbation excitation in PL. Namely, the optimum PL is considered to be the lens, in which vortical perturbations are not excited and particle density is homogeneous.

Vortical perturbations are not excited in PL, if the overbalance of ions by electrons $\Delta n \equiv n_{eo}-n_{io}$ is closed to limit Δn_{th}. Let us determine Δn_{th} from the condition of balance upset of the radial forces, confining rotating electrons of the bunch in the region of finite radius: magnetic confining, centrifugal and electrostatic scattering forces $\omega_{He}V_\theta m_e=m_eV_\theta^2/r-eE_r$. From this expression one can obtain that two last forces exceed first one if overbalance of ions by electrons, $\Delta n \equiv n_{eo}-n_{io}$, is not smaller then Δn_{th}, determined by $\Delta n_{th}=H_o^2/8\pi m_e c^2$. If Δn could be formed such that the following inequality is executed $\Delta n>\Delta n_{th}$, then the electron cloud could propagate freely transversally to magnetic field. Then the vortical perturbations can not be excited in PL in the limiting case $\Delta n=\Delta n_{th}$. Let's explain it. The instability development of the vortical perturbation excitation leads to the electron bunch formation. But at Δn, closed to Δn_{th}, the electron bunches can not be formed due to the vortical perturbation excitation, because any electron bunches formation leads at once to their destruction by centrifugal and electric scattering forces.

The following expression $\Delta n_{th}=H_o^2/8\pi m_e c^2$ shows observed in [1] quadratic relation of optimum electron density n_{eo} on optimum H_o.

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The expression $\Delta n_{th}=H_o^2/8\pi m_e c^2$ also shows fulfillment of criterion that the radius, $r_{He}=eE_r/m_e\omega^2_{He}$, of the radial oscillations of the electrons in crossed radial electric, E_r, and longitudinal magnetic, H_o, fields should be approximately equal to the PL radius, R. This criterion is used in [1] for determination of the optimum H_o.

Let us show that PL with Δn, equal $\Delta n=\Delta n_{th}$, is the lens, in which the electron density homogeneity is supported more easy. Really, if in any part of PL the electron density is such that the overbalance of ions by electrons obeys $\Delta n>\Delta n_{th}$, then this electron density overbalance is quickly disappeared by centrifugal and electrostatic scattering forces. If in any part of PL the electron density is such that the overbalance of ions by electrons obeys $\Delta n<\Delta n_{th}$, then the vortical perturbations are excited in this region, which provide anomalous spatial diffusion of electrons, and non-uniformity of their density is flattened.

SUPRESSION OF EXCITATION OF VORTICAL PERTURBATIONS IN OPTIMUM PLASMA LENS

Let us derive the dispersion relation and show that the oscillation excitation in the cylindrical PL is damped at its optimum parameters.

We take into account that the beam ions pass through the plasma lens of length L during time, approximately equal $\tau_i=L/V_{bi}$. The electrons are renovated in PL also during finite time, τ_e. Damping of perturbations of densities and velocities of electrons and ions at recovery of their unperturbed values we describe, using $\nu_i\equiv1/\tau_i$, $\nu_e\equiv 1/\tau_e$.

We use the electron, ion hydrodynamic eq.s

$$\partial_t\mathbf{V}+\nu_e(\mathbf{V}-\mathbf{V}_{\theta o})+(\mathbf{V}\nabla)\mathbf{V}=(e/m_e)\nabla\varphi+[\boldsymbol{\omega}_{He},\mathbf{V}]-(V^2_{th}/n_e)\nabla n_e$$

$$\partial_t n_e+(n_e-n_{eo})/\tau_e+\nabla(n_e\mathbf{V})=0 \quad (1)$$

$$\partial_t\mathbf{V}_i+\nu_i(\mathbf{V}-\mathbf{V}_{bi})+(\mathbf{V}_i\nabla)\mathbf{V}_i=-(q_i/m_i)\nabla\varphi ,$$

$$\partial_t n_i+(n_i-n_{io})/\tau_i+\nabla(n_i\mathbf{V}_i)=0 \quad (2)$$

and Poisson eq. for the electrical potential, φ ,

$$\Delta\varphi=4\pi(en_e-q_in_i) \quad (3)$$

Here $\mathbf{V}$, n_e are the velocity and density of electrons; V_{th} is the electron thermal velocity; $\mathbf{V}_{\theta o}$ is the electron azimuth drift velocity in crossed fields of PL; $\mathbf{V}_i$, n_i, q_i, m_i are the velocity, density , charge and mass of ions.

As the sizes of vortical perturbations are much greater than the electron Debye radius, $r_{de}\equiv V_{th}/\omega_{pe}$, then the last term in (1) can be neglected. Here $\omega_{pe}\equiv(4\pi n_{oe}e^2/m_e)^{1/2}$, n_{oe} is the unperturbed electron density.

From eq.s (1) one can derive non-linear eq.s

$$d_t[(\alpha-\omega_{He})/n_e]=[(\alpha-\omega_{He})/n_e]\partial_zV_z-\alpha\nu_e/n_e,$$

$$d_tV_z+\nu_eV_z=(e/m_e)\partial_z\varphi \quad (4)$$

describing both transversal and longitudinal electron dynamics. Here

$$d_t=\partial_t+(\mathbf{V}_\perp\nabla_\perp) , \partial_t\equiv\partial/\partial t , \partial_z\equiv\partial/\partial z \quad (5)$$

$\mathbf{V}_\perp$, V_z are the transversal and longitudinal electron velocities, α is the vorticity, the characteristic of electron vortical motion, $\alpha\equiv\mathbf{e}_z\mathrm{rot}\mathbf{V}$.

Taking into account higher linear terms, from (1) one can obtain

$$\mathbf{V}_\perp\approx(e/m\omega_{He})[\mathbf{e}_z,\nabla_\perp\varphi]+(e/m\omega^2_{He})(\partial_t+\nu_e)\nabla_\perp\varphi \quad (6)$$

From (6) we derive

$$\alpha\approx-2eE_{ro}/rm\omega_{He}-(eE_{ro}/m)\partial_r(1/\omega_{He})+$$

$$+(e/m\omega_{He})\Delta_\perp\phi+(e/m)(\partial_r\phi)\partial_r(1/\omega_{He})+$$

$$+(e/m)(\partial_t+\nu_e)\mathbf{e}_z[\nabla_\perp,\omega^{-2}_{He}\nabla_\perp\phi]\equiv\mu\omega_{He} ,\ \nabla\varphi\equiv\nabla\phi-\mathbf{E}_{or} \quad (7)$$

Here E_{or} is the radial focusing electric field, ϕ is the electric potential of the vortical perturbation; $-2eE_{ro}/rm\omega_{He}=(\omega^2_{pe}/\omega_{He})(\Delta n/n_{oe})\equiv\eta\omega_{He}$, $\Delta n\equiv n_{oe}-q_in_{oi}/e$.

From (3), (7) $\alpha\approx(\omega^2_{pe}/\omega_{He})\delta n_e/n_{eo}$ approximately follows. Thus the vortical motion begins, as soon as the electron density perturbation, δn_e, appears.

We use that, as it will be shown below, the characteristic frequencies of perturbations approximately equal to ion plasma frequency, ω_{pi}.

As beam ions have large mass and propagate through PL with fast velocity V_{ib}, we will describe their dynamics in linear approximation. We derive ion density perturbation from eq.s (2)

$$\delta n_i=-n_{io}(q_i/m_i)\Delta\varphi/(\omega-k_zV_{ib}+i\nu_i)^2 \quad (8)$$

Here k, ω are wave number and frequency of perturbation, V_{ib} is the unperturbed longitudinal velocity of the ion beam. Substituting (8) in Poisson eq. (3), one can obtain

$$\beta\Delta\varphi/4\pi e=\delta n_e,\ \beta=1-\omega_{pi}^2/(\omega-k_zV_{ib}+i\nu_i)^2,\ n_e=n_{oe}+\delta n_e \quad (9)$$

Let us consider instability development in linear approximation. Then we search the dependence of the perturbation on z, θ in the form $\delta n_e\propto\exp(ik_zz+il_\theta\theta)$. Then from (4) we derive

$$d_t(\omega_{He}/n_e)(1-\mu)=\alpha\nu_e/n_e- \quad (10)$$

$$-(e\omega_{He}/m_en_{eo})ik_z^2\varphi(1-\mu)/(\omega-l_\theta\omega_{\theta o}+i\nu_e),\ \omega_{\theta o}\equiv V_{\theta o}/r.$$

From (5), (6), (9), (10) we obtain, using the radial gradient of the short coil magnetic field, the following linear dispersion relation, describing the instability development

$$1-(1-\eta)\omega^2_{pe}(l_\theta/r)\partial_r(1/\omega_{He})/k^2(\omega+i/\tau_e-l_\theta\omega_{\theta o})- \quad (11)$$

$$-\omega^2_{pi}/(\omega+i/\tau_i-k_zV_{bi})^2-(1-\eta)\omega^2_{pe}k_z^2/k^2(\omega+i/\tau_e-l_\theta\omega_{\theta o})^2=0.$$

It is necessary to note, that at optimum PL parameters, i.e. at $\eta=1$, last two members in (11) equal zero.

Let us mean the quick vortical perturbations those, which phase velocities, V_{ph}, are closed to azimuth velocity of the electron drift, $V_{ph}\approx V_{\theta o}$. For them from (11) we derive in approximation $k_z=0$, $\omega=\omega^{(o)}+\delta\omega$, $|\delta\omega|<<\omega^{(o)}$ and neglecting τ_e, τ_i

$$\omega^{(o)}=\omega_{pi}=l_\theta\omega_{\theta o} ,\ \omega_{\theta o}=(\omega^2_{pe}/2\omega_{He})(\Delta n/n_{oe}),$$

$$\Delta n\equiv n_{oe}-q_in_{oi}/e \quad (12)$$

$$\delta\omega=i\gamma_q,\ \gamma_q=(\omega_{pe}/k)[(1-\eta)(\omega_{pi}/2)(l_\theta/r)|\partial_r(1/\omega_{He})|]^{1/2}$$

Here n_{oi} is the unperturbed ion density. At obtaining (12) we used a validity of an inequality

$$(1-\eta)(\Delta n/n_{oe})(r/\omega_{He})\omega^2_{pe}|\partial_r(1/\omega_{He})|<<m_e/m_i \quad (13)$$

It is fulfilled at a weak overbalance of beam ion space charge by electrons of PL, $\Delta n/n_{oe}<<1$, at small radial non-uniformity of ω_{He}, small plasma density, $\omega_{pe}/\omega_{He}<<1$ and for PL, closed to optimum one. From (12) one can see that in PL, which parameters are closed to optimum ones, the excitation of the quick vortical perturbations is damped.

From (12) it follows $l_\theta=(m_e/m_i)^{1/2}(\omega_{He}/\omega_{pe})(n_{oe}/\Delta n)$, that for typical parameters of experiments, $\Delta n/n_{oe}\approx0.1$, the perturbations with $l_\theta>1$ are excited at a large magnetic field and at small electron density.

As γ_q grows with r, taking into account τ_e, τ_i can lead to those perturbations with r smaller then critical value cannot be excited.

Let us introduce slow vortical perturbations as those ones, whose phase velocity is much less than azimuth velocity of the electron drift, $V_{ph} \ll V_{\theta o}$. We derive for them from (11) in approximation $k_z=0$ and neglecting τ_e, τ_i the following expressions

$$\gamma_s=(\sqrt{3}/2^{4/3})[\omega^2_{pi} l_\theta(\omega^2_{pe}/2\omega_{He})(\Delta n/n_{oe})]^{1/3}$$

$$k^2=-(1-\eta)(1/V_{\theta o})\omega^2_{pe}\partial_r(1/\omega_{He})\ ,\ \mathrm{Re}\omega_s=\gamma_s/\sqrt{3} \quad (14)$$

Here γ_s is the growth rate of excitation of slow vortical perturbations of small amplitudes, $\mathrm{Re}\omega_s$ is the real part of the frequency. As γ_s grows with r, then taking into account τ_e, τ_i should lead to that perturbations with r smaller than some value are not excited. In other words, the vortical perturbations are excited far from the PL axis.

(14) is obtained in approximation of a validity of a following inequality $l_\theta \gg (n_{oe}/\Delta n)2\omega_{pi}\omega_{He}/\omega^2_{pe}$. From (14) one can show that really the following inequality $V_{ph} \ll V_{\theta o}$ is fulfilled, and not large l_θ are excited. From (21) it also follows that in optimum PL the slow vortical perturbations are not excited. One can also show that the radius of localization of the slow vortex r_s is less then the radius of localization of the quick vortex, $r_s<r_q$.

So, one can see that the quick vortical perturbations are excited at relatively small plasma density, and at parameters of optimum PL the quick vortical perturbations are not excited. The slow vortical perturbations are excited at large plasma density, however growth rate of their excitation decreases for parameters, closed to optimum PL parameters, and the growth rate equals zero for optimum PL parameters.

SIMULATION OF HIGH-CURRENT ION BEAM FOCUSSING IN PLASMA LENS

Let us numerically simulate the ion beam focusing in high-current PL of different structures. In particular, let us compare the results with ion beam focusing in the ideal high-current PL. The ideal PL is considered to be the cylindrical homogeneous electron cloud of finite length, L, and of radius, R, which overbalances and focuses ion beam. The results of beam focusing in the ideal PL are shown in Fig. 2. On a horizontal axis the longitudinal coordinate (cm) is postponed. On a vertical axis the current radius (cm) of the focused beam ion is postponed. At first ions are focused inside PL. After escaping a lens the ions are focused on inertia. One can see apparent result, that beam ions are focused in a point at some longitudinal coordinate.

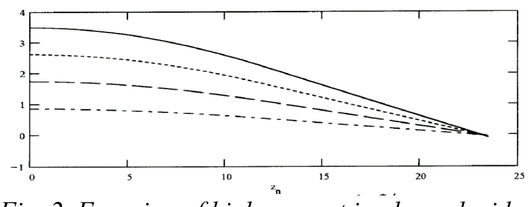

Fig. 2. Focusing of high-current ion beam by ideal plasma lens.

Let us compare now the focusing in the ideal PL with focusing in the real lens. Namely, the spatial structure of magnetic field lines of short cylindrical permanent magnet or short coil such that the electron cloud of PL can be solid cylinder of length L and radius R with hollow cones of radius R and with cones lengths Z_1 and Z_2 on the ends. In Fig. 3 the simulation results of ion beam focusing in such PL are presented.

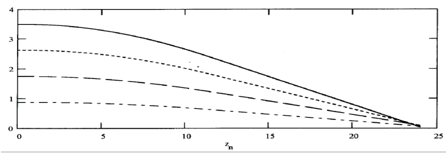

Fig. 3. Focusing of ion beam by plasma lens if electron column has hole cones on its ends.

One can see that all trajectories do not converge in one point, however if the altitudes of hollow cones are small, the aberrations are small.

In Fig. 1 the results of the ion beam focusing in PL, in which in wide near wall region, $R-r_t<r<R$, the strong vortical perturbations are excited, are shown. These perturbations lead in the region of their excitation to anomalous radial transport of electrons, and certainly to strong decrease of electron density in this region.

The excitation of the oscillated fields in the near wall region of PL can be damped by providing of the positive radial gradient of the electron density. Certainly, this gradient of density could lead to aberrations of focused beam. The numerical simulation, presented in Fig. 4, shows that aberrations are not large.

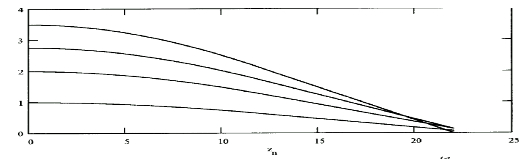

Fig. 4. Focusing of ion beam by plasma lens if electron density grows in region, $R-r_t<r<R$, on 1/3.

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