MICROWAVE FLUCTUATION REFLECTOMETRY
(a theoretical view)

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1. Introduction

Fluctuation reflectometry is widely used technique providing information on the plasma low frequency turbulence. It is often used for monitoring the density fluctuations behavior in the tokamak discharge, in particular, for indication of the transition to improved confinement regimes. The plasma probing by ordinary or extraordinary microwave is used in this diagnostics and the reflected wave spectral broadening is usually measured. Technical simplicity and operation at a single access to plasma are among its merits, which however cause interpretation problems related to localization of measurements and wave number resolution. In order to improve the fluctuation reflectometry wave number selectivity a more sophisticated correlative technique, using simultaneously different frequencies for probing was proposed [1, 2]. The data interpretation in this technique is based on the hypothesis that the wave backscattered off long wavelength fluctuations dominating in the turbulence spectra comes from the cut-off layer. Therefore, the turbulence correlation length is often determined from the shift between two cut-offs at which the coherence of two reflectometry signals at different frequencies vanishes [1]. In contradiction to the above approach, the numerical analysis performed in [3] in one dimensional model, in the framework of Born approximation, had shown slow decay of coherence with increasing frequency difference of reflectometer channels. According to [3], poor localised small angle scattering responsible for this effect plays a significant role in producing the fluctuation reflectometry signal.

2. Linear 2D model

The rigorous theoretical analysis of fluctuation reflectometry carried out in this Section in the frame of linear 2D model, applicable in the case of low turbulence level, when the probing line is observable in the spectrum of reflected wave, had shown that the scattering efficiency is maximal for poorly localised small angle scattering along the incident wave ray trajectory as well. However unlike the 1D model, in 2D one it is produced by fluctuations running in poloidal direction and possessing finite wave number. Thus it is not possible, even in a stationary model, to rule this effect out by re-normalization of the density profile. As in 1D model, small angle scattering results in slow decay of the correlation function of two signals and, consequently, makes estimation of the turbulence correlation scale in linear regime of scattering questionable. Our analyses is based upon the reciprocity
theorem in the form introduced by Piliya [4] which gives the scattering signal received by the antenna

$$A_s(\omega-\Omega)=\frac{i\omega\sqrt{P_i}}{16\pi}\int\frac{\tilde{n}_\Omega(\vec{r})}{n_c}E_{za}^2(\vec{r})d^3\vec{r}\ , \tag{1}$$

where $A_s(\omega - \Omega)$ is the signal amplitude in the waveguide, normalised so that $|A_s(\omega - \Omega)|^2$ gives the spectral power density of the scattering signal received by antenna and a single antenna operation is assumed; $\tilde{n}_\Omega(\vec{r})$ is a spectral harmonic of density perturbation at frequency Ω; P_i is the probing power and E_{za} gives the distribution of the probing wave electric field in plasma in the case of unit incident power. In the case of slab linear background density profile $n = n_c x / L$ the probing electric field can be represented as

$$E_{za}(x,y)=\int\frac{dk_y}{2\pi}W(x,k_y)f\left(k_y\right)e^{ik_y y}\ , \tag{2}$$

where $\left|f\left(k_y\right)\right|^2$ is the antenna diagram in the poloidal direction and the field spatial distribution for each poloidal harmonic is expressed in terms of integral representation for the Airy function

$$\begin{aligned}&W(x,k_y)=\\&=\sqrt{\frac{8\omega l}{c^2}}\exp\left\{i\int_0^{x_c(k_y)}k_x(x',k_y)dx'-i\frac{\pi}{4}\right\}\times\\&\times\int_{-\infty}^{\infty}\exp\left\{i\left(\frac{p^3}{3}+\alpha p+\xi p\right)\right\}dp\end{aligned} \tag{3}$$

where $l = (Lc^2/\omega^2)^{1/3}$ is the Airy length; $\xi = (x-L)/l$; $\alpha = -k_y^2 l^2$; $k_x\left(x_c,k_y\right)=0$ and $k_x\left(x,k_y\right)=\sqrt{\frac{\omega^2}{c^2}\left(1-\frac{x}{L}\right)-k_y^2}$. Following the approach of [5] we represent density fluctuations as a Fourier integral

$$\tilde{n}_\Omega\left(x,y\right)=\int\frac{d\kappa dq}{\left(2\pi\right)^2}e^{-i\kappa(x-L)-iqy}\delta n_\Omega\left(q,\kappa\right), \tag{4}$$

where κ and q are correspondingly "radial" and "poloidal" components of the fluctuation wave vector and finally obtain the fluctuation reflectometry signal as a superposition of signals, produced by density spectral harmonics

$$\begin{aligned}&A_S\left(\omega-\Omega\right)=\frac{e^2\ell^2}{\sqrt{\pi}mc^2}\sqrt{P_i}\int\frac{dqdk_y d\kappa}{\left(2\pi\right)^3}\times\\&\times f\left(k_y\right)f\left(q-k_y\right)C\left(\kappa,q,k_y\right)e^{i\Psi}\delta n_\Omega\left(q,\kappa\right)\end{aligned}\ , \tag{5}$$

where Ψ is the phase of scattered wave, which in paraxial case, $\left|k_y\right| << \omega/c$ and $\left|k_y-q\right| << \omega/c$, takes the form

$$\Psi \simeq \frac{4}{3}\frac{L\omega}{c} - \frac{Lc}{\omega}\left[k_y^2 + \left(q-k_y\right)^2\right] + \\ +\frac{1}{2}\left(\frac{c}{\omega}\right)^3 L\left[k_y^4 + \left(q-k_y\right)^4\right].$$

The factor $C\left(\kappa,q,k_y\right)$ in (5) is the scattering efficiency introduced in [5] as the integral

$$C(\kappa,q,k_y) = \sqrt{\frac{i\pi}{\beta}} \times \\ \times \int_{-\infty}^{\infty} \frac{\exp\left\{i\left(\frac{\beta^3}{12}\xi^3 - \frac{\beta\sigma}{2}\xi - \frac{\delta}{4\beta}\xi^{-1} - (\xi-1)s\right)\right\}}{(\xi-1+i\varepsilon)\sqrt{\xi+i\varepsilon}} d\xi \qquad (6)$$

Where $\beta = \kappa\ell$; $s = \kappa L$; $\delta = \ell^4 q^2\left(q-2k_y\right)^2$; $\sigma = \ell^2\left[-k_y^2 - \left(q-k_y\right)^2\right]$.

The realistic assumption that the long scale component satisfying the condition $\beta \ll 1$ dominates in the turbulence spectrum as well as modeling of the antenna diagram by the Gaussian dependence

$$f\left(k_y\right) = \sqrt{2\sqrt{\pi}\rho}\exp\left[-k_y^2\rho^2/2\right] \qquad (7)$$

allows one after averaging to obtain the following expression for the CCS of two scattering signals at frequencies ω_0 and ω in terms of the turbulence spectrum

$$\left\langle A_s(\omega_0)A_0^*(\omega_s)\right\rangle_\Omega = \\ = 4\pi\left(\frac{e^2}{mc^2}\right)^2 P_i l^3\rho^2 \exp\left\{-i\frac{4L\Delta\omega}{c}\right\} J(\Delta\omega) \qquad (8)$$

where $\Delta\omega = \omega - \omega_0$,

$$J(\Delta\omega) = \\ \iint dq d\kappa \frac{\exp\left\{-q^2\rho^2/2 + i\,\Delta\omega/\omega(q^2\,Lc/\omega + 2kL)\right\}\left|\delta n_{\Omega,\kappa,q}\right|^2}{\sqrt{\kappa^2\rho^4 + \left[2\,Lc/\omega\,\kappa + L(cq/\omega)^2\right]^2}} \qquad (9)$$

and the turbulence is supposed to be statistically homogeneous and stationary, so that the following relation holds for the average cross-correlation spectrum

$$\left\langle \delta n_\Omega\left(q,\kappa\right)\delta n_\Omega^*\left(q',\kappa'\right)\right\rangle = (2\pi)^2\left|\delta n\right|^2_{\Omega,q,\kappa} \times \\ \times\delta\left(q-q'\right)\delta\left(\kappa-\kappa'\right)\delta\left(\Omega-\Omega'\right) \qquad (10)$$

In the case $\rho^2 << 2Lc/\omega$, when the cut off is situated in the wave zone of the antenna, the main contribution in (9) is provided by fluctuations satisfying condition $\kappa = -cq^2/2\omega$. These fluctuations are responsible for small angle scattering of the probing beam all over the plasma, which can be shown in terms of trajectories of incident and scattered rays applicable for $\rho^2 << 2Lc/\omega$. Namely four conditions should be fulfilled for the incident and scattered rays to guarantee the generation and reception of the scattering signal. The first two of them are the usual Bragg resonance conditions for radial and poloidal wave numbers written at the scattering point x_s: $k_x\left(x_s,k_{sy}\right) = k_x\left(x_s,k_{iy}\right) - \kappa$ and $k_{sy} = k_{iy} - q$, whereas the third and fourth are needed for compensation of the poloidal shift of the incident and scattered rays. They read as $y_s = y_a + \Delta y_i$ and $y_a = y_s + \Delta y_s$, where $\Delta y_\alpha = \int_{x_a}^{x_s} v_{g\alpha y}/v_{g\alpha x}dx$, $v_{g\alpha\beta}$ are the group velocities components for incident and scattered waves and integrals are taken along the ray trajectories. In the case of linear density profile and long scale fluctuations $q/2 \ll \omega/c$ the above conditions result in the following values of incident and scattered poloidal wave numbers

$$k_{sy} = -q\frac{k_x\left(0,0\right) - k_x\left(x_s,0\right)}{2k_x\left(0,0\right)},$$

$$k_{iy} = q\frac{k_x\left(0,0\right) + k_x\left(x_s,0\right)}{2k_x\left(0,0\right)}$$

In addition they fix the poloidal coordinate of the scattering point y_s and radial wavenumber of fluctuation producing the scattering signal to

$$y_s = y_a - \frac{qc}{\omega}x_s \text{ and } \kappa = -\frac{q^2c}{2\omega} \qquad (11)$$

For a model Gaussian turbulence spectrum

$$\left|\delta n\right|^2_{\Omega,q,\kappa} = \frac{4\pi\left|\delta n\right|^2_\Omega}{\mathrm{K}Q}\exp\left[-\frac{\kappa^2}{\mathrm{K}^2} - \frac{q^2}{Q^2}\right] \qquad (12)$$

we obtain

$$\left\langle A_s(\omega_0)A_0^*(\omega_s)\right\rangle_\Omega = P_i\frac{\omega^3\rho}{c^3\mathrm{K}Q}\frac{\left|\delta n_{\Omega,\kappa,q}\right|^2}{n_c^2}\exp\left\{-i\frac{4L\Delta\omega}{c}\right\}\times \\ \times\frac{\sqrt{\pi}\ln\left\{4\pi/\Delta\omega(1/2 + 1/Q^2\rho^2 - 2i\Delta\omega/\omega\cdot Lc/\omega\rho^2) - \gamma\right\}}{\sqrt{1/2 + 1/Q^2\rho^2 - 2i\Delta\omega/\omega\cdot Lc/\omega\rho^2}} \qquad (13)$$

where $\varphi \simeq 3.48$, $\gamma \simeq 0.577$. The slow, logarithmic, decay of correlation described by the above formula is caused by small value of radial wave number of fluctuations responsible for small angle scattering $\kappa \sim -\frac{Qc}{2\omega}Q \ll Q$.

The contribution of the poorly localized small angle scattering to the fluctuation reflectometry signal is less important when small radial scales are somehow suppressed in the turbulence spectrum. For example, for the model spectrum

$$\frac{\left|\delta n_{\Omega,\kappa,q}\right|^2}{n_c^2} = 4\pi^{3/2}\frac{\left|\delta n_\Omega\right|^2}{n_c^2}\exp\left\{-\frac{\kappa^2 + q^2}{Q^2}\right\}\frac{\left|\kappa + cq^2/2\omega\right|}{Q^3} \qquad (14)$$

in which the fluctuations satisfying small angle scattering condition (11) are totally suppressed the CCS is determined by the backscattering in the cut off and takes in the case $\rho^2 << 2Lc/\omega$ the form

$$\left\langle A_s(\omega_0)A_0^*(\omega_s)\right\rangle_\Omega = P_i\frac{\pi^{3/2}}{2}\frac{\omega^3\rho^2}{c^3}\frac{\left|\delta n_\Omega\right|^2}{n_c^2}\times \\ \times\exp\left\{-i\frac{4L\Delta\omega}{c} - \left(\frac{\Delta\omega}{\omega}QL\right)^2\right\} \qquad (15)$$

The slow logarithmic kind of coherence dependence of CCF on normalized frequency shift has been computed also for more realistic isotropic spectrum $\left|\delta n\right|^2_{\Omega,\kappa,q}/n_c^2 \sim Q^3/\left(q^2 + \kappa^2 + Q^2\right)^{3/2}$. It is shown in figure 1 a,b. The decay of coherence here is still much slower than that predicted by (15), which provides no possibility for the turbulence correlation lengths determination.

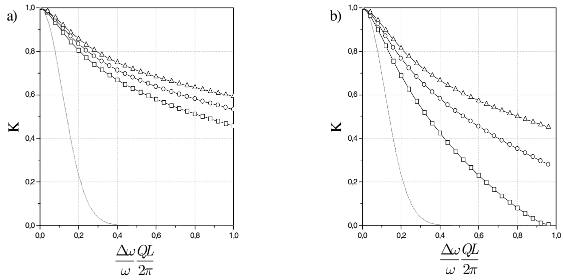

Fig. 1 Dependence of coherence on normalized cut offs separation.

a) $L = 50\frac{c}{\pi\omega}$ b) $L = 10\frac{c}{\pi\omega}$, where

—△— - $Q = 0.4\,\omega/c$; —○— - $Q = 0.2\,\omega/c$

—□— - $Q = 0.1\,\omega/c$, ------- - formula (15)

The coherence dependence on normalized frequency shift computed for isotropic spectrum $\frac{|\delta n|^2_{\Omega,q,\kappa}}{n_c^2} \sim \frac{\kappa^2 + q^2}{Q^2}\exp\left[-\frac{\kappa^2 + q^2}{Q^2}\right]$, in which large scale component of the turbulence is suppressed, is shown in figure 2 a,b. It consists of two parts: at smaller cut off separation the coherence decays quickly at the rate close to that prescribed by (15) and shown in figure 2 by the broken line. This decay is deeper for smaller density scale length. However at larger cut off separation the quick decay saturates and is followed by a logarithmic type behavior. Only in the case of the longest correlation length corresponding to $Q = 0.1\,\omega/c$ and strong density gradient $L = 10\,c/\pi\omega$ this logarithmic behavior was not seen.

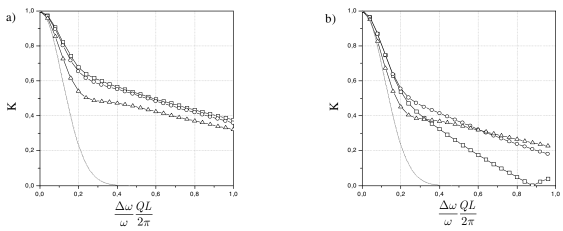

Fig. 2 Dependence of coherence on normalized cut offs separation.

a) $L = 50\frac{c}{\pi\omega}$ b) $L = 10\frac{c}{\pi\omega}$, where

—△— - $Q = 0.4\,\omega/c$; —○— - $Q = 0.2\,\omega/c$;

—□— - $Q = 0.1\,\omega/c$, ------- - formula (15)

In the above treatment no exact dispersion relation was supposed for the density fluctuations, which is realistic in the case of high turbulence level, however in the case of weak turbulence one can expect a drift wave type of dispersion $\Omega = Q_d v_d$, where v_d is the electron diamagnetic drift velocity. Modeling this kind of dispersion by a spectrum $|\delta n|^2_{\Omega,q,\kappa} \sim \exp\left[-\left(\frac{\kappa^2}{K^2} + \frac{(q - Q_d)^2}{Q^2}\right)\right]$, we get the coherence dependence shown in figure 3 a,b for $L = [100;20]\frac{c}{2\pi\omega}$; $\mathrm{K} = 0.3\,\omega/c$, $Q = 0.3Q_d$, $Q_d = [0.4;0.2;0.1]\,\omega/c$. The coherence decreases much slower here than prescribed by (15) for backscattering in the cut off. However the decay is quicker for large poloidal wave numbers Q_d.

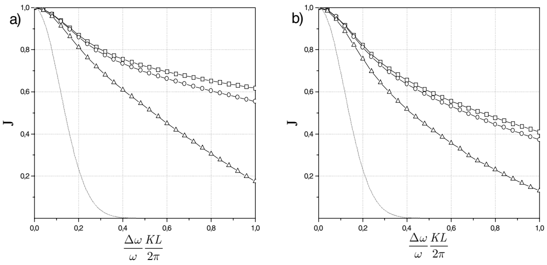

Fig. 3 Dependence of coherence on normalized cut offs separation.

a) $L = 50\frac{c}{\pi\omega}$ b) $L = 10\frac{c}{\pi\omega}$, where

—△— - $Q_d = 0.4\,\omega/c$; —○— - $Q_d = 0.2\,\omega/c$;

—□— - $Q_d = 0.1\,\omega/c$, ------- - formula (15)

In contradiction to above predictions a quick decay of coherence is often observed in experiment [1, 2]. In some cases the coherence is suppressed at cut off separation comparable to the vacuum wavelength [1]. A linear theory is unable to explain these results, moreover one should not expect it to account for these observations made under conditions when no probing line was observable in the reflected spectrum [2]. Nevertheless the linear theory elucidates the way in which a non-linear approach should be developed. Taking into account that according to [5] the scattering efficiency is maximal for fluctuations possessing small radial component of wave vector, comparable to the density scale length, and making use of the fact that long scales are dominant in the tokamak turbulence one can conclude that the transition to nonlinear regime occurs via small angle multi-scattering. From the mathematical point of view it is quite natural under above assumptions to base a non-linear model describing fluctuation reflectometry upon the WKB approximation, which is not limited to low-level fluctuations.

3. Non-linear 1D model

In this Section we focus on investigation within 1D model, which, as it will be shown below, has a potential to describe the main features of fluctuation reflectometry both in linear and non-linear regime. Neglecting the possibility of numerous reflections of the probing wave from the plasma boundary, metallic walls and the cut off, which is a reasonable supposition for the case of large fusion machines we obtain the amplitude of the wave reflected from the cut off in the framework of WKB approximation as

$$E_{zs}(\omega,t) = E_{zi}\exp(i\phi_r), \qquad (16)$$

where E_{zi} is the incident wave amplitude; its phase, ϕ_r, consists of two parts $\phi_r = \phi_0 + \delta\varphi$;

the unperturbed part of the phase is given by $\phi_0 = 2\int_0^{x_c} k_0(x,\omega)dx - \pi/2$, where $k_0(x,\omega)$ is the O-mode wave number given by

$$k_0(x,\omega) = \sqrt{\frac{\omega^2}{c^2} - \frac{4\pi e^2 n(x)}{m_e c^2}} \qquad (17)$$

and x_c is a cut-off point so that $k_0^2(x_c,\omega) = 0$ and $n(x_c) = n_c$.

The fluctuating part of the reflected signal phase is given by the first order expression

$$\delta\varphi(t,\omega) = -\frac{\omega^2}{c^2} \cdot \int_0^{x_c} \frac{\delta n(x,t)}{n_c} \frac{dx}{k_0(x,\omega)}. \qquad (18)$$

Supposing the density fluctuation correlation length, l_c, to be much smaller than the system size $l_c \ll x_c$ and taking into account that in this case a random value $\delta\varphi(t,\omega)$ is a sum of many independent random values we conclude that it's evolution should be a normal random process.

In this case it is easy to calculate the average reflected field

$$\langle E_{zs}(\omega_k,t_k)\rangle = E_{zi} \exp\left(i\phi_0(\omega_k) - \frac{\sigma_{kk}}{2}\right), \qquad (19)$$

and the cross-correlation function of signals in two frequency channels $j \neq k$

$$K_{jk} = E_{zi}(\omega_j) E_{zi}^*(\omega_k)\left\{\exp\left[\sigma_{jk}\right] - 1\right\}\times$$
$$\times \exp\left\{i\left[\phi_0(\omega_j) - \phi_0(\omega_k)\right] - \frac{\sigma_{kk}+\sigma_{jj}}{2}\right\} \qquad (20)$$

where subscript *"j"* stands for (t_j,ω_j) and the matrix σ_{jk} is a phase perturbation correlation function

$$\sigma_{jk} = \langle\delta\varphi_j, \delta\varphi_k\rangle = \frac{\omega_j^2\omega_k^2}{c^4}\int_0^{x_c(\omega_j)} dx' \times \qquad (21)$$
$$\times \int_0^{x_c(\omega_k)} \frac{\langle\delta n(x',t_j)\delta n(x'',t_k)\rangle}{n_c^2} \frac{dx''}{k_0(x',\omega_j)k_0(x'',\omega_k)},$$

$\langle ...\rangle$ is the statistical averaging.

Formulas (19), (20) describe extinction of the probing line in the non-linear regime, when $\sigma_{kk}/2 \geq 1$ and decay of the coherence both in linear and non-linear regimes.

In the most simple case of linear density profile and statistically homogeneous turbulence, which however possesses arbitrary wave number spectrum $\tilde{n}_\kappa^2(t_j - t_k)$ related to the correlation function of density fluctuations by expression

$$\langle\delta n(x',t_j)\delta n(x'',t_k)\rangle = \qquad (22)$$
$$= \delta n^2 \int_{-\infty}^{\infty} \tilde{n}_\kappa^2(t_j - t_k)\exp\left[i\kappa(x'-x'')\right]\frac{d\kappa}{2\pi},$$

where δn^2 determines the density perturbation level and $K_n(0,0) = 1$, the cross-correlation function (21) can be represented as

$$\sigma_{jk} = 4\frac{\omega^2 x_c(\omega_j)}{c^2}\frac{\delta n^2}{n_c^2}\int\frac{d\kappa}{2\pi}\frac{\tilde{n}_\kappa^2(t_j-t_k)}{|\kappa|}\times \qquad (23)$$
$$\times\left\{e^{i\kappa\Delta}F\left[\sqrt{\kappa x_c(\omega_j)}\right]F^*\left[\sqrt{\kappa x_c(\omega_k)}\right]\right\}$$

where $\Delta = x_c(\omega_j) - x_c(\omega_k)$ and $F(s) = \int_0^s \exp(i\varsigma^2)d\varsigma$ is a Fresnel integral. It should be emphasized, that poor localized small angle scattering, which in 1D model is produced by small wave numbers, makes substantial contribution to this integral because of $|\kappa|^{-1}$ singularity, so that for large plasma, where $x_c(\omega_1) \gg l_c$, its value can be estimated with logarithmic accuracy as

$$\sigma_{11} \simeq \frac{\omega^2 l_c x_c(\omega_1)}{c^2}\cdot\frac{\delta n^2}{n_c^2}\ln\frac{x_c(\omega_1)}{l_c} \qquad (24)$$

$$\sigma_{12} \simeq \frac{\omega_1^2 x_c(\omega_1) l_c}{c^2}\cdot\frac{\delta n^2}{n_c^2}K_n(0,t_1-t_2)\ln\frac{\Delta}{l_c} \qquad (25)$$

According to (24) the transition to the non-linear regime where the probing line is no longer observable in the reflected spectrum occurs when the following criteria is fulfilled:

$$\frac{\omega^2 l_c x_c(\omega_1)}{c^2}\cdot\frac{\delta n^2}{n_c^2}\ln\frac{x_c(\omega_1)}{l_c} \geq 1 \qquad (26)$$

For smaller density perturbation level

$$\frac{\omega^2 l_c x_c(\omega_1)}{c^2}\cdot\frac{\delta n^2}{n_c^2}\ln\frac{x_c(\omega_1)}{l_c} \ll 1 \qquad (27)$$

the probing line is dominant in the reflected spectrum and the Born approximation approach is correct. It results in the following simplified formula for the cross-correlation function:

$$K_{12} = E_{zi}(\omega_1)E_{zi}^*(\omega_2)\times \qquad (28)$$
$$\times\sigma_{12}\exp\left\{i\left[\phi_0(\omega_1) - \phi_0(\omega_2)\right]\right\}.$$

According to (25), (28) the coherence decays very slow with growing separation of cut offs. Such a logarithmic decay was observed in 1D numerical linear full-wave consideration [3] and obtained above in 2D linear analytical theory, based upon full-wave Born approximation.

It is clear that under condition (27) the fluctuation reflectometry is providing data on the frequency spectrum of large-scale component of turbulence and is not capable of its coherence length estimation. The behavior of the cross-correlation function at the transition from linear to non- linear regime is shown in Fig. 4, where it is plotted against normalized cut off separation Δ/l_c for different level of turbulence possessing Gaussian spectrum. As it is seen for small density perturbations $\delta n/n_c < 10^{-3}$ the decay of

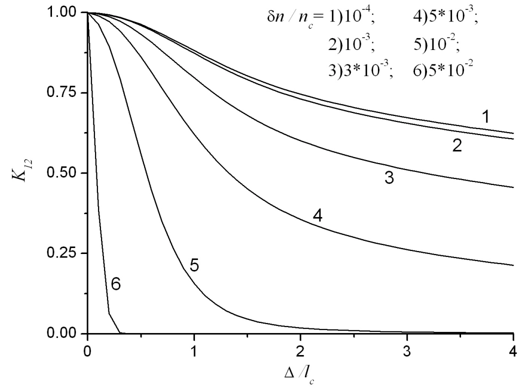

Fig. 4 Dependence of the normalized cross-correlation function on cut off separation for different turbulence levels.

$2\pi c/\omega = 0.52\text{cm}; l_c = 2\text{cm}; x_c = 60\text{cm}$.

coherence is slow independently of the turbulence level, however at the larger perturbations $\delta n/n_c > 2\times10^{-3}$ when condition (16) holds the situation is different. The high coherence region is narrower and its width decreases with growing turbulence level. At $\delta n/n_c > 2\times10^{-2}$ it is already much smaller then the turbulence correlation length. In this case the small angle scattering is already deep in the non-linear regime so that a strong inequality

$$\frac{\omega^2 l_c x_c(\omega_1)}{c^2}\cdot\frac{\delta n^2}{n_c^2} \gg 1 \qquad (29)$$

is valid. Under this condition the correlation in two channels is not negligible only for close enough cut offs $\Delta \ll l_c$ and for time difference $|t_1 - t_2|$ shorter than the turbulence correlation time $|t_1 - t_2| \ll t_c$. Using these inequalities we can represent σ_{jk} in the form of Taylor series expansion in Δ/l_c and $(t_1 - t_2)/t_c$. Finally we obtain

$$K_{12}(\Delta, t_1 - t_2) \simeq E_{zi}(\omega_j)E_{zi}^*(\omega_k)\times$$
$$\times\exp\left\{i\left[\phi_0(\omega_j) - \phi_0(\omega_k)\right] + \sigma_{12} - \frac{\sigma_{11}+\sigma_{22}}{2}\right\},$$
$$\sigma_{12} - \frac{\sigma_{11}+\sigma_{22}}{2} \simeq -\left\{\frac{(\kappa_r\Delta)^2}{2} + \frac{\left[\Omega_r(t_1 - t_2)\right]^2}{2}\right\} \qquad (30)$$

Coming to analysis of a spatially inhomogeneous turbulence case, let us assume, for the sake of simplicity, that only the level of density perturbations is variable

$$\left\langle \delta n_j' \delta n_k \right\rangle = \delta n^2\left(\frac{x' + x''}{2}\right)\int_{-\infty}^{\infty}\frac{\tilde{n}_\kappa^2(t_j - t_k)\mathrm{e}^{i\kappa(x'-x'')}\,d\kappa}{2\pi} \qquad (31)$$

We suppose below that the turbulence inhomogeneity scale, *a*, is smaller than the distance between the plasma edge and cut off, but larger than the turbulence correlation length so that an equality $x_c(\omega_1) \gg a \gg l_c$ holds. After substituting the above expression into (21) we obtain the following expression for the phase correlation function:

$$\sigma_{12} = 4\frac{\omega^2 x_c(\omega_1)}{c^2}\times \qquad (32)$$
$$\times\int_0^\infty d\xi\,\frac{\delta n^2\left(\frac{x_c(\omega_1)+x_c(\omega_2)}{2} - \xi\right)}{n_c^2}S(\xi,\Delta,\tau),$$

where the kernel S is determined by the turbulence spectrum

$$S(\xi,\Delta,\tau) = \int_0^{\pi/2} K_n(\Delta - 2\xi\cos 2\varphi, \tau)\,d\varphi \qquad (33)$$

The localization of fluctuation reflectometry is prescribed by asymptotic behavior of the form-factor S at large $\xi \gg l_c$, which in the case of large cut offs separation $\Delta \gg l_c$, $\xi - 2\Delta \gg l_c$ takes the following form:

$$S(\xi,\Delta,\tau) \simeq \frac{\bar{K}_n(\tau)l_c}{2\sqrt{4\xi^2 - \Delta^2}} \qquad (34)$$

where $\bar{K}_n(\tau) = \frac{1}{l_c}\int_{-\infty}^{\infty} K_n(\varsigma,\tau)\,d\varsigma$. For growing distance from the cut off the form–factor *S* decreases, however not fast enough to guarantee the dominant contribution of the cut off in the signal. The value of the phase correlation function is very sensitive to behavior of the turbulence level far from cut off and is determined by it in the case of density perturbations growing towards the edge.

The situation is different in the case of small cut off separation important for high turbulence level (29). Under condition $\Delta \ll l_c$ and $\xi \gg l_c$ the kernel S takes the following asymptotic form:

$$S(\xi,\Delta,\tau) \simeq \frac{\bar{K}_n(\tau)l_c}{4\xi}\left(1 + \frac{\Delta^2}{8\xi^2}\right) \qquad (35)$$

As it is seen, the first term in the right hand side of (35), being responsible for frequency broadening at high turbulence level, is only slowly decreasing with coordinate, in agreement with (32). On the contrary, the second term proportional to the quantity Δ^2, which describes the coherence suppression at growing cut off separation, is a stronger function of ξ. The dependence of

$$S(\xi,\Delta,\tau) - \frac{\bar{K}_n(\tau)l_c}{4\xi} \sim \bar{K}_n(\tau)l_c\frac{\Delta^2}{32\xi^3}$$

guarantees the dominant input of the nearest vicinity of cut off into the correlation decay at high turbulence amplitude. It should be mentioned here that the positive sign of this term indicates the tendency of increasing coherence due to fluctuations situated far from the cut off. Expression (30) was used to evaluate the frequency broadening of the reflectometry spectra Ω_r and parameter κ_r in the case of inhomogeneous turbulence distribution, shown in Fig 5. This distribution consists of homogeneous background turbulence $\delta n_0/n_c = 2\times10^{-2}$ and additional turbulent layer $\delta n_a(x)/n_c = \delta_a f(x)$ situated between the plasma

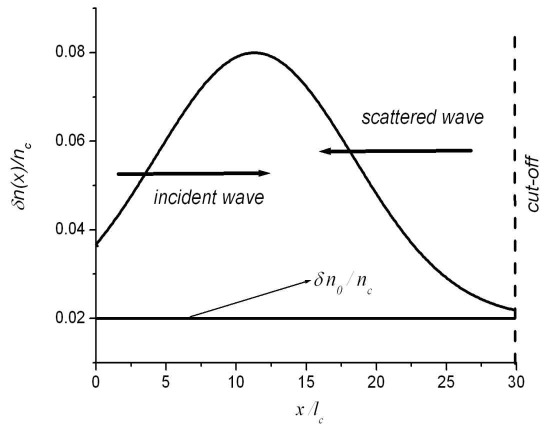

Fig. he modeled turbulence level distribution

$$\delta n(x) = \delta n_a(x) + \delta n_0 .$$

edge and cut off. The spatial distribution of turbulence in this layer $f(x)$ was not altered, however its level δ_a was varied. Dependence of normalized parameters $\Omega_r t_c$ and $\kappa_r l_c$ on the ratio of additional density perturbation in the cut off to the background one is shown in Fig. 6.

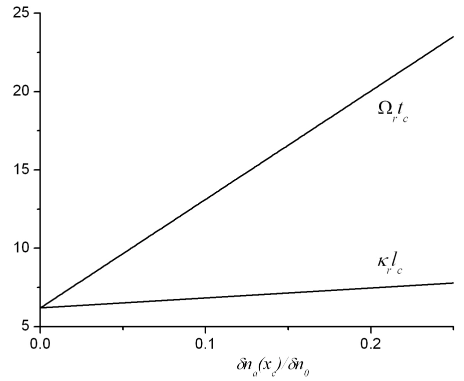

Fig. Dependence of the normalized signal fre uenc broadening and inverse correlation length on additional densit perturbation in the cut off
$\delta n_a(x_c)/\delta n_0$. $2\pi c/\omega = 0.52$cm; $l_c = 2$cm; $x_c = 60$cm.
he modeled turbulence level distribution

$$\delta n(x) = \delta n_a(x) + \delta n_0 .$$

As it is seen the behavior of these two parameters is very different. The frequency broadening changes drastically at a minor variation of the density perturbation in the cut off, whereas the parameter $\kappa_r l_c$ varies only slightly. This difference is explained by different spatial origin of these parameters. As it was mentioned above, the frequency broadening is determined by small angle scattering caused by turbulence in a wide plasma region, where the growth of density perturbation was large, whereas suppression of coherence in non-linear regime is determined by a narrow region in the cut off vicinity, where the turbulence was enhanced only by 20%.

4. Conclusions.

According to the above analysis the volume small angle scattering from long radial scale component of density turbulence plays an important and in some cases dominant role in formation of fluctuation reflectometry signal. For low level density perturbations, when the linear approach is valid and, from experimental viewpoint, the probing line is observable in the reflected spectrum, it cause the slow decrease of the coherence, shown above for different fluctuation spectra and complicates interpretation of fluctuation reflectometry data, leading to degradation of wave number resolution. Backscattering in the cut off plays dominant role in radial correlation reflectometry signal only if the long radial scales satisfying condition $\kappa = -q^2 c / 2\omega$ are suppressed in the spectrum or in the case of small density scale length $L << 2\omega / Q^2 c$ leading to saturation of the scattering efficiency at large radial scales. Only under these conditions the wave number resolution with radial correlation reflectometry appears to be possible at low turbulence level. The transition of the fluctuation reflectometry from linear to non-linear scattering regime has been followed in the present study. In non-linear regime the reflected signal spectrum broadening is also not localized to the cut-off and determined by the wide region of plasma in which the turbulence is situated. In spite of this, the coherence decay in radial correlation reflectometry deep in non-linear regime is only sensitive to the turbulence level in the cut-off. The role of poor localized small angle scattering in coherence decay is not so strong there. As it was shown it is rather enhancing coherence than suppressing it. In non-linear regime another effect, negligible at small density perturbation level comes into play. Namely, it is phase mismatch produced by the part of plasma between cut offs evanescent for a smaller frequency of the probing wave. The coherence decays quickly if this phase mismatch's average absolute value, produced by smaller scales, exceeds π, thus making two signals statistically independent. Based on this mechanism we can foresee that such a localization improvement persists in the 2D model under development now. However it should be stressed that in this non-linear regime the cut off separation at which the coherence of two signals is suppressed provide us with information not on correlation length but on the ratio of its square root and the density perturbation amplitude.

Acknowledgments.

The work was supported by RFBR grants 01-02-17926; 00-15-96762, 02-02-06633, 02-02-17589, RFBR-NWO 047.009.009 and INTAS grants INTAS-01-2056 and YSF 2001/1-131.

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