EXCITATION OF OSCILLATIONS IN THE VIRCATOR WITH PREMODULATION

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The electronics flows dynamic in vircator premodulation diode has been studied. The problem on stationary states and disturbances in the first order approximation of interacting contrary flows-direct and reflected from virtual cathode. The expression describing for electron flows moving and theirs fields has been found. The excitation of oscillation in premodulation diode has been studied. The oscillation spectrum in electronics flows system travelling of premodulation diode vircator was found.

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INTRODUCTION

The first vircator that made use of the electron flow premodulation was the virtode. The latter, described as the generator of electromagnetic microwave radiation, has become widely known since publication of paper [1]. Its action was based, first, on the property of the virtual-cathode (VC) of electron flow to excite electric oscillations at a frequency close to the electron plasma frequency $\boldsymbol{\omega_{pe}}$ [2] of the flow in the VC region and, secondly, on the electron flow premodulation [3] in the cathode region at the oscillation frequency using the waveguide that transports a part of oscillation energy to the cathode gap, thereby realizing the positive feedback.

1. THE VIRCATOR WITH PREMODULATION

After the virtode experiment [1] premodulations began to perform in a specially created gap [4, 5] with the use of this positive feedback.

The key diagram of the experiment is presented in Fig. 1,a.

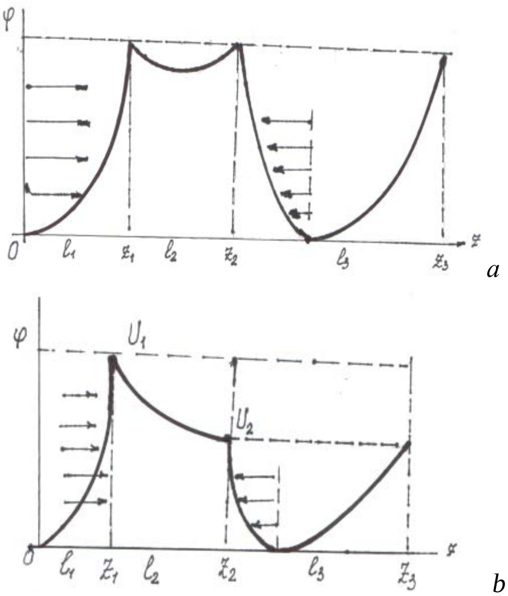


Fig. 1. Basic diagram of the premodulation vircator: without external action in the premodulation gap (a); with direct electron flow deceleration (b)

The device made in accordance with the diagram includes the cathode gap, $0<Z<Z_1$, the premodulation gap, $Z_1<Z<Z_2$, and the VC gap, $Z_2<Z<Z_3$.

Whereas in the virtode [1] the feedback is realized due to the signal imposed to the cathode gap from the VC, in the premodulation vircator the signal from the VC region imposes to the premodulation gap.

It has appeared that the device based on this premodulation scheme of the electron flow has lower generation efficiency. The premodulation scheme located in a special gap is capable to operate in the single-mode regime only at small supercriticality of the electron flow. The appearance of frequencies in the generation spectrum at great supercriticality have not been previously interpreted with certainly.

The present study is just intended to elucidate of this questions.

The generation scheme with three functional gaps also permits to realize the "two-generator" mode of excitation of electric oscillations in the given device. We have in mind the possibility of oscillation excitation in the premodulation region because of the instability of the electron flow decelerated in the diode [9 - 11] (Fig. 1,b) and also the oscillation excitation in the VC gap caused to its instability. The two gaps exchange of their electron flows, namely, the direct flow that goes from the cathode gap to the premodulation gap, and the flow reflected in the opposite direction. In principle, electron flows 1 and 2 may perform the feedback between the gaps as the oscillation sources. It is hoped that at certain parameters the resonance can be attained between the oscillation sources and the required amplitude of oscillations can be provided.

The similar scheme was realized in the experiment [6], where oscillations were excited both as VC and as an electron flow oscillating around the anode grid [7].

Our consideration will be carried out with the replacement of the scheme of the device by equivalent diodes [13]. In our case we shall have three diodes: the forming (accelerating) diode, the premodulation diode and the diode for VC creation. Each of the diodes presents a relatively autonomy electrodynamic region, through the boundaries of which electron flows can freely pass (or pass with the absorption coefficient *f*) [12].
Take into account of this considerations let us consider the following process scheme (see Fig. 1,b).

Fig. 1,b shows the behavior of the potential φ in the electron flow. In the cathode gap $0<Z<Z_1$ the potential increases by the Child-Langmuir-Boguslavsky law (three-halves power law) up to the U_1 value. On exit from the gap the electron flow has the density n_0 and

the velocity v_0, sufficient to form the VC in the $Z_2 \le Z \le Z_3$ region. In the premodulation gap the potential drops down to U_2 without forming the potential minimum in the gap. A part R of the flow reflects from the VC and comes back to the $Z_1 \le Z \le Z_2$ region. For the description of the processes in the diode gaps we shall use the quantities, dimensionless by the initial values of the first flow: velocity $\bar{V}_j = \frac{V_j}{v_0}$, density $\bar{n}_j = \frac{n_j}{n_0}$, potential $\bar{\varphi} = \frac{e\varphi}{m\, v_0^2}$, coordinates $\varsigma = \frac{Z}{l_2}$, $\bar{t} = \frac{v_0}{L_2} t$. Henceforth the bars above quantities are omitted.

The propagation of the flow in the premodulation gap is described by the equation of motion

$$\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial \zeta} = \frac{\partial \varphi}{\partial \zeta}, \quad (1)$$

the continuity equation

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial \zeta}\left(n_j v_j\right) = 0, \quad \text{j=1, 2} \quad (2)$$

and the Poisson equation

$$\frac{\partial^2 \varphi}{\partial \zeta^2} = q_1 n_1 + q_2 n_2. \quad (3)$$

Further on, these equations are used to describe nonlinear stationary inhomogeneous states of electron flows and to solve linearized differential equations.

2. STATIONARY STATES OF ELECTRON FLOWS IN THE PREMODULATION GAP

We find the stationary states of flows in the diode (further all the quantities are marked with the superscript 0).

At $\partial / \partial t = 0$, from the set (1) - (3) we obtain the following conservation laws:

$$\frac{\left(V_j^0\right)^2}{2} - \varphi^0 = 0, \quad (4)$$

$$n_j^0 V_j^0 = D_j, \quad (5)$$

where $D_1 = 1; D_2 = R$ and the Poisson equation of the form

$$\frac{d^2 \varphi^0}{d\varsigma^2} = \frac{q^*}{\sqrt{2\varphi^0}}, \quad (6)$$

where $q^* = q(1+R)$, $q = \frac{4\pi e^2 n_0 l_2^2}{m\, v_0^2} = \frac{\omega_{pe}^2 l_2^2}{v_0^2}$, ω_{pe} is the Langmuir frequency.

First, we shall study the direct flow motion, for which $V_1^0(\varsigma) = \sqrt{2\varphi^0(\varsigma)}$. In the description of the flow we use the variable τ, which is introduced by

$$\frac{d\varsigma}{d\tau} = V_1^0(\tau). \quad (7)$$

Equation (6) is modified into

$$\frac{d^2 V_1^0}{d\tau^2} = q^*, \quad (8a)$$

and is easily integrated as

$$V_1^0(\tau) = \frac{q^*}{2}\left(\tau - \tau_1\right)^2 + \mathrm{B}, \quad (9)$$

$$\varsigma_1(\tau) = \frac{q^*}{6}\left(\tau - \tau_1\right)^3 + \mathrm{B}\,\tau + \hat{\varsigma}_1. \quad (10)$$

The parameters τ_1, B (and ζ_j) can be found from the boundary conditions

$$V_1^0\Big|_{\zeta=0,\tau=0} = 1,\; V_1^0\Big|_{\zeta=1,\tau=T_1} = V_L = \sqrt{\frac{U_2}{U_1}},\; \varsigma\big|_{\tau=0} = 0,$$

$$\varsigma_1\big|_{\tau=T_1} = 1, \quad (11)$$

in which T_1 is the travel time, by using T_1:

$$\tau_1 = \frac{\mathrm{T}_1}{2} + \frac{1 - V_L}{q^* \cdot \mathrm{T}_1}, \quad (12)$$

$$B = \frac{1}{2}(!+V_L) - \frac{q^*}{8} T_1^2 - \frac{(1 - V_L)^2}{2q^* \cdot \mathrm{T}_1^2}. \quad (13)$$

The value T_1 can be found from the equation

$$-\frac{q^*}{12}\mathrm{T}_1^3 + \frac{\mathrm{T}_1}{2}\left(1 + V_L\right) - 1 = 0. \quad (14)$$

Note that the variable τ, introduced by eq. (7), is the current Lagrangian travel time of direct-flow electrons. The corresponding Lagrangian start time τ_0 is the time of electron departure from the grid with the coordinate $\varsigma = \varsigma_1$ (put to be equal to zero in the present consideration).

To obtain the transit time T_1 from set (11) we derive equation (13).

The parameters B and $\hat{\varsigma}_1$ are found directly from the equations of set (11), formula (14) and the solution of eq. (13).

The motion of the reflected flow is described in a similar way, proceeding from the equation

$$\frac{d^2 V_2^0}{d\tau^2} = -q^*. \quad (8b)$$

The solution of eq. (8,b) has the form:

$$V_2^0(\tau) = -\frac{q^*}{2}\left(\tau - \tau_2\right)^2 - \mathrm{B}, \quad (15)$$

$$\varsigma_2(\tau) = -\frac{q^*}{6}\left(\tau - \tau_2\right)^2 - \mathrm{B}\,\tau + \hat{\varsigma}_2, \quad (16)$$

where $\tau_2 = \frac{\mathrm{T}_2}{2} - \frac{1 - V_L}{q^* \mathrm{T}_2}$. (17)

For the stationary states can be proven that

$$\mathrm{T}_2 = \mathrm{T}_1 \equiv \mathrm{T}. \quad (18)$$

It has appeared that the travel time T_1 of the direct flow, which has a high initial velocity but is decelerated in the premodulation gap, and the travel time T_2 of the reflected flow that has a lower velocity, but is accelerated in the mentioned gap, are equal. And it provides stationary motion of the flows.

3. THE FIRST APPROXIMATION

We shall find the spectrum of oscillations, which may arise in the premodulation gap against the background of the steady-state condition.

The time- and coordinate-dependent variables can be written as

$$F(\varsigma,t)=F^{0}(\varsigma)+\tilde{F}(\varsigma,t), \qquad (19)$$

where $F^{0}(\varsigma)$ is the stationary quantity.

The deviations from stationary values are assumed to be small and from (1) we obtain a linearized set of equations. The time dependence of the deviations will be sought for in the form

$$\tilde{F}(\varsigma,t)=\eta(\varsigma)e^{-i\omega t}. \qquad (20)$$

For the oscillation amplitudes F(ζ, ω), where the tildes are omitted, we obtain the following set of equations:

$$(-i\omega)V_j+\frac{d}{d\zeta}\left(V_j^0V_j\right)=\frac{d\Psi}{d\zeta},$$
$$(-i\omega)n_j+\frac{d}{d\zeta}\left(n_j^0V_j+V_j^0n_j\right)=0,\ j=1,2, \qquad (21)$$
$$\frac{d^2\Psi}{d\zeta^2}=q_1n_1+q_2n_2.$$

The set (21) reduces to the system of four ordinary differential equations with variable coefficients. Generally speaking, the search for its solution presents a serious problem. In our case we shall find an approximate solution of the system, making use of the parameter q smallness dictated by physics considerations. The point is that the realization of the above - given approach is based on the use of the instability described in [9, 10], which may arise in the decelerated flow only if q<0.3.

The related issue whether at this q the virtual cathode arises in the third gap ($Z_1\leq Z\leq Z_2$) of the premodulation vircator [4, 5] is easily decided owing to a strong dependence of the Bursian- Pavlov parameter on the gap length and the initial velocity of electrons. In particular, for the third gap we have

$$q_{B3}=q_{B2}\cdot\left(\frac{l_3}{l_2}\right)^2V_L^{-3}, \qquad (22)$$

where V_L is the direct flow velocity at the exit of the premodulation gap. In this case for VC generation in the third gap the following inequality should be fulfilled [2, 12]:

$$q_{B3}\rangle\frac{16}{9}. \qquad (23)$$

If it take into account that for the beam decelerated in the second gap we have $V_L<1$, then it can be concluded that even due to the appropriate choice of the gap lengths ratio, inequality (23) can be fulfilled. As a result the VC arises in the third gap. The VC divides the incident electron flow into two flows – direct and reflected. The reflected flow comes back to the second gap and cross through it. The counter-propagating flows, i.e., the incident flow and the flow reflected in the second gap, form a two-flow system, the stationary flowing of which is just considered in this section. As a result we get the answer to the question: oscillations of what frequency with what increments and in what parameter region can be excited in the premodulation gap.

The incident (1st) and reflected (2nd) flows penetrate into each other and by their fields exert mutual effect on each other and on itself.

We describe the flow propagation and the scheme of the mutual influence.

First, we find the perturbations of flow densities and velocities under the action of the space self-charge perturbations only. To this we use the solution of the following set of equations

$$(-i\omega)V_{j1}+\frac{d}{d\zeta}\left(V_j^oV_{j1}\right)=\frac{d\Psi_j}{d\zeta},$$
$$(-i\omega)n_{j1}+\frac{d}{d\zeta}\left(n_j^0V_{j1}+V_j^0n_{j1}\right)=0, \qquad (24)$$
$$\frac{d^2\Psi_j}{d\zeta^2}=q_jn_j,\ j=1,2.$$

The solution of the set (24) has been found in [10] and has the form:

$$\tilde{V}_{j1}=\frac{e^{\sigma_j\theta_j}}{V^0}\{D_{1j}(\theta_j-1)+D_{2j}\left[\theta_j(\theta_j-1)+1-2\gamma_j\right]+$$
$$+\overline{C}_j\left[1-2\gamma_j-\frac{2}{\sigma}(\theta_j-1)-(\theta_j-1)^2\right]e^{-\sigma_j\theta_j}\}, \qquad (25)$$

$$\breve{n}_{j1}=\frac{\gamma_j}{(V_j^0)^2}\{D_{1j}e^{\sigma_j\theta_j}\left(\sigma_j-\frac{\theta_j-1}{\gamma_j}\right)-D_{2j}e^{\sigma_j\theta_j}[\sigma_j(1-2\theta_j)-2-$$
$$-\frac{\theta_j-1}{\gamma_j}(1-2\theta_j)]+\overline{C}_j\sigma_j\left[2(\theta_j-1)-\frac{\theta_j-1}{\gamma_j}\left(-\frac{2}{\sigma_j^2}-1+2\gamma_j(\theta_j-1)^2\right)\right]\}, \qquad (26)$$

$$\Psi_j=\frac{D_{1j}}{\sigma_j}(e^{\sigma_j\theta_j}-1)-\frac{D_{2j}}{\sigma_j}\{e^{\sigma_j\theta_j}\left(1-2\theta_j+\frac{2}{\sigma_j}\right)-1-\frac{1}{\sigma_j}\}+$$
$$\overline{C}_j\sigma_j\left[\left(-\frac{2}{\sigma_j^2}+2\gamma_j-1\right)\theta_j+\frac{(\theta_j-1)^3+1}{3}\right]+D_j, \qquad (27)$$

where

$$\sigma_j=\frac{i\omega}{\sqrt{q_j\gamma_j}},\gamma_1=\frac{1}{2(B+1)},\gamma_2=\frac{1}{2(B+V_L)},\theta_j=\tau\sqrt{q_j\gamma_j},$$

$V_L=\sqrt{\frac{U_2}{U_1}}$, D_{1j}, D_{2j}, $\overline{C}_j$, D_j are the integration constants, $\overline{C}_1=\frac{C_1}{1+R}$, a $\overline{C}_2=\frac{R}{1+R}C=RC_1$: $\overline{C}$ is the integration constant of the Poisson equation of the set (24) and B is determined by (13).

The expressions obtained for the potential disturbances, $\tilde{\Psi}_j$, make it possible to find the velocity $\tilde{V}_{j2}$ and density $\tilde{n}_{j2}$ disturbances, which describe the mutual influence of the flows. The corresponding sets of equations have the forms:

$$(-i\omega)\tilde{V}_{j2}+\frac{d}{d\zeta}\left(V_j^o\tilde{V}_{j2}\right)=\frac{d\Psi_l}{d\zeta}, \qquad (28)$$
$$(-i\omega)\tilde{n}_{j2}+\frac{d}{d\zeta}\left(n_j^0\tilde{V}_{j2}+V_j^0\tilde{n}_{j2}\right),$$

where j=1,2, a l =2,1. So, the solution of the system (21) takes the form:

$$\widetilde{V}_j = \widetilde{V}_{j1} + \widetilde{V}_{j2},$$

where $\widetilde{V}_{j1}$ is determined by formula (25), and

$$\widetilde{V}_{j2} = \frac{1}{V^0}\{D_{1l}\theta_\ell e^{\sigma_l\theta_l} + D_{2l}\left(\theta_l^2 - \theta_l\right)e^{\sigma_l\theta_l} + \overline{C}_j\left[-\theta_l^2 + 2\theta_l\left(1 - \frac{1}{\theta_l}\right) - 2\gamma_l + \frac{2}{\sigma_j}\right]\}, \quad (29)$$

j=1, 2, and l=2, 1, correspondingly;

$$\widetilde{n}_j = \widetilde{n}_{j1}(\tau) + \widetilde{n}_{j2}(\tau),$$

$$\widetilde{n}_{j1} = \gamma_j\left\{D_{1j}\left(\sigma_j + \frac{1}{\gamma_j}\right) - D_{2j}\left(\sigma_j - 2 + \frac{1}{\gamma_j}\right) + \widetilde{C}_j\left(-\frac{2}{\sigma_j\gamma_j}\right)\right\}, \quad (30)$$

$$\widetilde{n}_{j2} = D_{1l}\{I\} + D_{2l}\{II\} + \overline{C}_l\{III\}.$$

where j=1,2, l=2,1.

The solution of the set (28) has the form:

$$\widetilde{V}_{j2} = \frac{1}{V^0}\{D_{1l}\theta_l e^{\sigma_l\theta_l} + D_{2l}\left(\theta_l^2 - \theta_l\right)e^{\sigma_l\theta_l} + \overline{C}_l\left[-\theta_l^2 + 2\theta_l\left(1 - \frac{1}{\sigma_l}\right) - 2\gamma_l + \frac{2}{\sigma_l}\right]\}, \quad (31)$$

$$\widetilde{n}_{j2} = D_{1l}\{I\} + D_{2l}\{II\} + \overline{C}_l\{III\}, \quad (32)$$

where

$$\{I\} = \left(1 + \frac{\sigma_2}{2c^2}\frac{\overline{x} - a}{b_2}\right)\overline{x}e^{\sigma_2 x} - \frac{\sigma_2}{4c^3 b_2^2}e^{\sigma_2\overline{x}}\cdot a\ln\frac{\overline{x} - a + c}{\overline{x} - a - c},$$

$$\{II\} = \left(1 + \frac{\sigma_2(\overline{x} - a)}{2c^2 b_2}\right)e^{\sigma_2\overline{x}}\left(\overline{x}^2 - \overline{x}\right) - \frac{\sigma_2}{4c^3 b_2^2}e^{\sigma_2\overline{x}}\left[2c\overline{x} - \left(a + c^2 - a^2\right)\ln\frac{\overline{x} - a + c}{\overline{x} - a - c}\right]$$

$$\{III\} = \left(1 + \frac{\sigma_2}{2c^2}\cdot\frac{\overline{x} - a}{B}\right)\left[\frac{2}{\sigma_2} - 2\gamma_2 + \left(2 - \frac{2}{\sigma_2}\right)\overline{x} - \overline{x}^2\right] -$$

$$- \frac{\sigma_2}{4c^3 B^2}e^{\sigma_2\overline{x}}\cdot\{2ce^{-\sigma_2\overline{x}}\cdot\{\left(-\frac{2}{\sigma_2} - \overline{x} - a + 2\right) +$$

$$+\left[2a - a^2 - c^2 - 2\gamma_2 + 4c\,(a-1) + \frac{2}{\sigma_2}(1-a) + 2c\sigma_2\left(1 - 3a + a^2 + c^2 + 2\gamma_2\right)\right]J_+ +$$

$$+\left[a^2 + c^2 - 2a + 2\gamma_2 + 4c\,(a-1) + \frac{2}{\sigma_2}(1-a) + 2c\sigma_2\left(1 - 3a + a^2 + c^2 + 2\gamma_2\right)\right]J_-,$$

where $J_+ = \int\limits_o^{\overline{x}}\frac{e^{-\sigma_2 x}dx}{(x-a)^2 - c^2}$; $J_- = \int\limits_o^{\overline{x}}\frac{e^{-\sigma_2 x}(x-a)}{(x-a)^2 - c^2}$ (33)

and $\overline{x} = \overline{\theta}_2 = 1 + \sqrt{\frac{1 + b_1}{V_l + b_1}}$; $a = \frac{1}{\kappa}$, $c = \frac{\sqrt{1 - 2\gamma_1}}{\kappa}$;

$$\kappa = \sqrt{1 - 2\gamma_1(1 - V_L)},\ b_1 = \frac{q^*}{2}\tau_2^2,\quad b_2 = \frac{q^*}{2}\tau_1^2.$$

The procedure of finding the approximate solution can be continued. For this purpose it is necessary to find the potential Ψ_3 disturbance related to the disturbances of the densities $\widetilde{n}_{12}$ and $\widetilde{n}_{22}$ by the equation

$$\frac{d^2\widetilde{\Psi}_3}{d\zeta^2} = q_j\left(\widetilde{n}_{12} + \widetilde{n}_{22}\right). \quad (34)$$

Knowing $\widetilde{\Psi}_3$ and using the system similar to the system (28), we find the $\widetilde{V}_{j3}$ and $\widetilde{n}_{j3}$ corrections to the velocity and density disturbances, which corresponds to $\widetilde{\Psi}_3$. This procedure may go to infinity. However, based on the smallness of the parameter q_j, we shall restrict ourselves to the solutions of (23) - (27), and (31), (32). The corrections to these solutions are proportional to q^3_j.

At each point of space the potential disturbances represent a sum of flow disturbances

$$\overline{\Psi} = \widetilde{\Psi}_1 + \widetilde{\Psi}_2, \quad (35)$$

which are given by formula (27).

The disturbances of velocities and densities are obtained using formulas (25) and (26).

4. THE DISPERSION EQUATION

The spectrum of oscillations will be found, which may arise in the gap under consideration against the background of the steady-state condition. The dispersion equation (DE) for the premodulation diode will be found with the use of the boundary conditions, which corresponds to the requirement both through the electrode-potential maintaining circuit and electron flows don't superinduce the oscillations in the gap under study. This means that the following conditions should be fulfilled:

$$\begin{cases} V_1(0) = 0, n_1(0) = 0, \\ V_2(1) = 0, n_2(1) = 0, \\ \Psi(0) = 0, \Psi(1) = 0. \end{cases} \quad (36)$$

The application of these conditions to expressions (25) - (27), (29), (30) leads to 5 linear algebraic equations for the coefficients D_{11}, D_{12}, D_{21}, D_{22} and C. Generally the requirement that the equation system determinant should equal zero leads to the dispersion equation:

$$\begin{aligned} &\alpha\{1 + \alpha - e^{2\alpha}(1-\alpha) - 4G\alpha^3\} + \\ &\frac{R}{\theta_1^2}\{\left(H_0 + H_1\alpha + H_2\alpha^2 + H_3\alpha^3 + H_4\alpha^4\right) + \\ &e^{2\alpha}\left(M_0 + M_1\alpha + M_2\alpha^2\right) + \\ &+ Ae^{2\alpha}[\left(F_0 + F_1\alpha + F_2\alpha^2 + F_3\alpha^3\right) + \\ &e^{2\alpha}\left(\Phi_0 + \Phi_1\alpha + \Phi_2\alpha^2 + \Phi_3\alpha^3\right)]J_+ + \\ &+[\left(S_0 + S_1\alpha + S_2\alpha^2 + S_3\alpha^3\right) + \\ &e^{2\alpha}\left(N_0 + N_1\alpha + N_2\alpha^2 + N_3\alpha^3\right)]J_-\} = 0, \end{aligned} \quad (37)$$

where

$$H_0 = \frac{(1+k)^3}{8\gamma_1}P =$$

$$= \frac{(1+k)^3}{8\gamma_1}[2 - \frac{1}{k} - \frac{1}{k^2} - \frac{8\gamma_1}{k^2} + 2\gamma_1\frac{1 + 2\gamma_1}{k^2(1 - 2\gamma_1)}],$$

$$H_1 = \frac{(1+\kappa)^2}{2\kappa^2} - \frac{(1+\kappa)^2}{4}\{[\frac{1}{\kappa(1 - 2\gamma_1)}(\kappa^2 - 1 - 2\gamma_1 - \frac{(\kappa - 2)\gamma_1}{\sqrt{1 - 2\gamma_1}}) - \frac{1 - \kappa^2 + 2\gamma_1}{\kappa^2}] + 2 - \frac{1}{\kappa} - \frac{1}{\kappa^2} - \frac{8\gamma_1}{\kappa^2}2\gamma_1\frac{1 + 2\gamma_1}{(1 - 2\gamma_1)\kappa^2}\},$$

$$H_2=\frac{1}{4}\left(3-4\gamma_1-\frac{1}{1-2\gamma_1}\right)-\frac{1}{8}\left(5+6\gamma_1-\frac{1}{1-2\gamma_1}\right)-$$
$$\frac{1}{4}\left(7+2\gamma_1-\frac{1}{1-2\gamma_1}\right)\frac{1}{\kappa}-\frac{1}{2}(3+\gamma_1)\frac{1}{\kappa^2},$$

$H_3=0$, $H_4=\frac{1}{3\kappa^2}\left(6\gamma_1+1-\kappa-2\kappa^2\right)$,

$$M_0=\frac{(1+\kappa)^3}{8\gamma_1}\left[2-\frac{1}{\kappa}-\frac{1}{\kappa^2}-\frac{8\gamma_1}{\kappa^2}+2\gamma_1\frac{1+2\gamma_1}{(1-2\gamma_1)\kappa^2}\right],$$

$$M_1=\frac{(1+\kappa)^2}{2\kappa^2}+\frac{(1+\kappa)^2}{8\gamma_1}\Big\{2(\gamma_1-1-\kappa)\left[2-\frac{1}{\kappa}-\frac{1}{\kappa^2}-\frac{8\gamma_1}{\kappa^2}+2\gamma_1\frac{1+2\gamma_1}{(1-2\gamma_1)\kappa^2}\right]+$$
$$(1-\kappa)\cdot\left[\frac{2\gamma_1}{1-2\gamma_1}\cdot\frac{1}{\kappa(\kappa+1)}\left(\kappa^2-1-2\gamma_1-\frac{\gamma_1(\kappa-2)}{\sqrt{1-2\gamma_1}}\right)-2\gamma_1\frac{1-\kappa^2+2\gamma_1}{\kappa^2(1+\kappa)}\right]\Big\},$$

$$M_2=(\kappa+1)\frac{\kappa^2-1-2\gamma_1}{2\kappa^2}+\frac{(1+\kappa)^2}{4\gamma_1}(\gamma_1-1-\kappa),$$

$$A=\frac{\gamma_1}{2(1-2\gamma_1)^{3/2}}\left(1-\frac{1}{\kappa^2}\right),$$

$$\Phi_0=\frac{(\kappa+1)^2}{2},\Phi_1=(\kappa+1)\left(\gamma_1-\kappa-2\sqrt{1-2\gamma_1}\right),$$

$$\Phi_2=2\left[\gamma_1-\kappa-1+(3\kappa-2\gamma_1)\sqrt{1-2\gamma_1}\right],\Phi_3=4\sqrt{1-2\gamma_1}\frac{\kappa-2}{\kappa+1}(\gamma_1-1-\kappa),$$

$$F_0=-\frac{(1+\kappa)^2}{2};F_1=(\kappa+1)\left(-1+2\sqrt{1-2\gamma_1}-\gamma_1\right),$$

$$F_2=2\left[\sqrt{1-2\gamma_1}(-\kappa+2+2\gamma_1)-\gamma_1\right];F_3=-4\gamma_1\sqrt{1-2\gamma_1}\frac{\kappa-2}{\kappa+1},$$

$$S_0=-\frac{(1+\kappa)^2}{2};S_1=-(\kappa+1)\left(1+2\sqrt{1-2\gamma_1}+\gamma_1\right);$$

$$S_2=-2\left[\sqrt{1-2\gamma_1}(\kappa+2+2\gamma_1)+\gamma_1\right];S_3=-2\gamma_1\sqrt{1-2\gamma_1}\frac{\kappa+2}{\kappa+1},$$

$$N_0=\frac{(1+\kappa)^2}{2};N_1=(1+\kappa)\left(\gamma_1+2\sqrt{1-2\gamma_1}-\kappa\right);$$

$$N_2=2\left[(2\gamma_1-\kappa)\sqrt{1-2\gamma_1}+\gamma_1-1-\kappa\right];N_3=4\sqrt{1-2\gamma_1}\frac{\kappa+2}{\kappa+1}(\gamma_1-1-\kappa).$$

In equation (2) we have $G=\frac{2\gamma_1+\frac{1}{3}\bar{\theta}_1^2-\bar{\theta}_1}{2\bar{\theta}_1^2}$, (38)

where $\bar{\theta}_1=1+\kappa,\ \kappa=\sqrt{1-2\gamma_1(1-V_L)}.$

The constant γ_1 is calculated by the formula:

$$q^*=\frac{1}{\gamma_1}\left[-\left(1-\frac{1}{3\gamma_1}\right)+\frac{1}{3}\left(V_L+2-\frac{1}{\gamma_1}\right)\sqrt{1-2\gamma_1(1-V_L)}\right].\quad(39)$$

In the single-flow case R=0, conditions (36) lead to the known equation (see [9 - 11]):

$$e^{2\alpha}(1-\alpha)+4G\alpha^3-\alpha-1=0,$$

where $2\alpha=\sigma\theta_1$, and $G=\frac{1-2\gamma(V_L+2)+\sqrt{1-2\gamma(1-V_L)}}{\sigma\left[1+\sqrt{1-2\gamma(1-V_L)}\right]^2}$.

Conditions (36) are in accordance with the expression taken from [9 - 11] and derived when describing the instability, which arises in the electron flow slowing down in the planar diode.

Equation (37) makes it possible to find the complex quantity $\alpha=-Q+iP$, where P and Q is the dimensionless frequency and decrement (increment), appropriately.

The integrals J_+ and J_- play an essential role in eq. (37):

$$J_+=\int_0^{\bar{\theta}_2}\frac{e^{\frac{2Q}{\bar{\theta}_2}x}\cos\left(\frac{2p}{\bar{\theta}_2}x\right)}{x-a+c}dx-i\int_0^{\bar{\theta}_2}\frac{e^{\frac{2Q}{\bar{\theta}_2}x}\sin\left(\frac{2p}{\bar{\theta}_2}x\right)}{x-a+c}dx,$$
$$J_-(a,c)=J_+(a,-c).$$

In the expressions for J_+ and J_-, we have $a=\frac{1}{\kappa},c=\frac{\sqrt{1-2\gamma_1}}{\kappa},\bar{\theta}_2=1+a$.

A detailed analysis of the integrals shows that although the denominators of the integration elements go to zero, the integrals J_+ and J_- converge.

The derived DE gives the frequency and increment of possible oscillations as a function of *q* and R and V_0 (flow velocity at the entry to the gap with the VC). This DE coincides with the DE for the electron flow slowing down in the diode [9 - 11] at R=0.

We give below the numerical solution of the DE, which gives the frequency P, the increment (decrement) Q as a function of *q* for the set of parameters R and V_L varying in the range from 0 to 1. As example, Fig. 2 shows Q(q) and P(q) at V_L=0.5 and at a substantial variation of the parameter R.

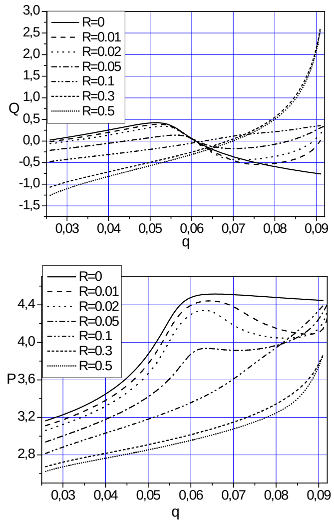


Fig. 2. Dispersion characteristics of one of the harmonic oscillation modes under changes of the reflection coefficient R: frequencies (a); oscillation increments (decrements) (b)

At R=0 the solutions agree with the results from [9 - 11].

At small R (R≤0.05), just as in the single-flow case, the instability (Q>0) takes place, which changes to a stable solution at q>0.06. As R increases, i.e. with an increasing role of the reflected flow, the instability in the region of low *q* disappears, however at high q≥0.08 the two-flow instability manifests itself, having the increments as higher as the reflection coefficient R gets higher.

The reflection coefficient R is related to the parameters of the diode gaps by the formula [12]:

$$R = \left\{1 - \left[\frac{1}{9\ q_3}\left(1+\sqrt{1+9q_3}\right)\right]^2\right\}^{1/2}, \quad (40)$$

where $q_3 = q_2\left(\frac{l_3}{l_2}\right)^2 \frac{1}{V_L^3}, q_2 = q,$ (41)

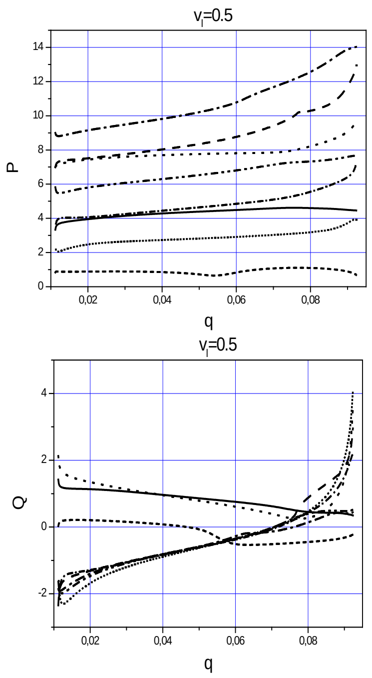
q_3 and l_3 are the parameters of the VC gap, q_2 and l_2 are the parameters of the premodulation gap.

Solutions of the dispersion equation were obtained for the case of $\left(\frac{l_3}{l_2}\right)^2 = 10$ and different V_L (V_L=0.2; V_L=0.5; V_L=0.7).

A qualitative correspondence between the results obtained at different V_L is observed. We give here the results for the V_L=0 case.

Fig. 3,a,b shows dispersion characteristics for the harmonic oscillation modes, which may be excited in the premodulation gap due to both the instability of the slowing down beam (low *q*) and the two-beam instability (high *q*). Since the frequency is dimensionless by v_0/l_2, and in the premodulation gap we have $v_0 \approx 10^{10}$cm/s, and l_2 equals a few centimeters, we draw conclusion that in Fig. 3,a the frequency P is dimensionless by the value belonging to the microwave range.

In this case for some modes the increments appear to be of the same order of magnitude), and this may permit a good excitation of the modes.



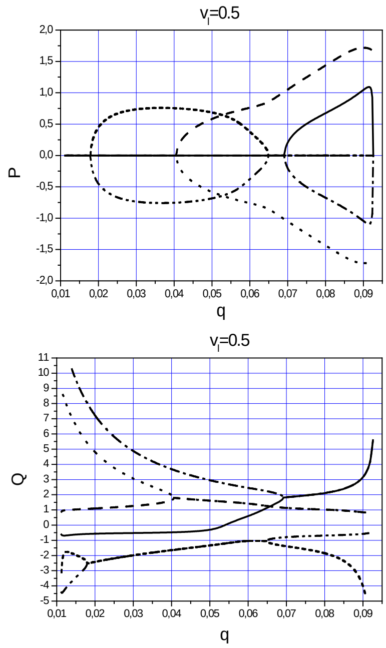
a

b

Fig. 3. Dispersion characteristics of oscillation modes excited at VC-consistent premodulation gap operation: frequency (a) and increment (decrement) of oscillations (b)

Fig. 4,a,b show the solutions for aperiodic modes, which in a certain *q* range become harmonic.

For example, the solution shown in the figures by dots is aperiodic at $q < 0.04$, whereas at $q > 0.04$ the solution becomes unstable (Q>4) harmonic solution.



a

b

Fig. 4. Dispersion characteristics of nonperiodic oscillation modes excited at VC-consistent premodulation gap operation: frequency (a) and increment (decrement) of oscillations (b)

CONCLUSIONS

In this paper the possibility of oscillation excitation in the premodulation gap of the vircator for feedback realization has been investigated.

The problem has been solved on the interaction of two colliding electron flows – direct flow and the flow reflected from the VC. In particular, a nonlinear steady state of electron flows as a function of the coordinate has been found.

Based on these coordinate functions a linearized set of five differential equations has been solved for the two-flow system under study.

The results of work have been used for determining spectral characteristics of the excited oscillations.

We have derived the dispersion equation that relates the oscillation frequencies and increments to the parameters of geometry and flows. The solutions have been obtained numerically, and the calculated data have been illustrated by a number of graphic pictures.

It has been shown that in the system of two electron flows that penetrate to the premodulation gap of the vircator, a variety of oscillation modes get excited. At that, the oscillation frequencies belong to the microwave range and the increments are of the same (or higher) order of magnitude as the frequency. At a low reflected-flow density, the excitation is realized owing to the instability of the slowing down flow, and with an increase in the mentioned density the excitation is realized owing to the two-flow instability. Large increments of the excited oscillations encourage us to hope for possible realization of the feedback in both the “wave” [1, 4, 5] and the “beam”, i.e. due to the action of the reflected flow on the incident flow in the premodulation gap.

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ВОЗБУЖДЕНИЕ КОЛЕБАНИЙ В ВИРКАТОРЕ С ПРЕДМОДУЛЯЦИЕЙ

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Исследована динамика электронных потоков и возбуждение колебаний в предмодуляционном диоде виркатора. Найден спектр колебаний в системе электронных потоков, проходящих через предмодуляционный диод виркатора.

ЗБУДЖЕННЯ КОЛИВАНЬ У ВІРКАТОРІ З ПЕРЕДМОДУЛЯЦІЄЮ

О.Г. Мележик, А.В. Пащенко, С.С. Романов, І.М. Шаповал

Досліджена динаміка електронних потоків та збудження коливань у передмодуляційному діоді віркатора. Знайдено спектр коливань у системі електронних потоків, що проходять через передмодуляційний діод віркатора.