NUMERICAL STUDY OF THE MERCIER PLASMA STABILITY AND EQUILIBRIUM PLASMA CURRENTS IN AN L=2 YAMATOR

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In paper [1] a new stellarator-type magnetic system having a high magnetic well value was proposed (later it got the name YAMATOR). In the present work the MHD plasma stability depending from value of the total vertical magnetic field for the magnetic configuration of *l*=2 variant of a similar system is investigated numerically. To investigate the plasma stability using the Mercier stability criterion, we apply the numerical method based on the calculation of the terms contained in Mercier criterion by tracing the magnetic field lines in the given stellarator magnetic field [3,4]. The magnetic field of helical windings is calculated by the Biot-Savart law. From our analysis of the MHD plasma stability in the trap under consideration it follows that the Mercier stability criterion is satisfied for a great part of the plasma-confinement region, and the violation of this criterion can take place only for the magnetic surfaces from the external part of the plasma column.

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1. INTRODUCTION

In Ref. [1] a new stellarator system has been proposed (later it got the name YAMATOR, see [2]) which is characterized by an increased magnetic well value. The magnetic field of such a system is produced by pairs of helical conductors with oppositely directed currents of the same value. The conductors wind with equal pitch values on the nested tori with the same major radii R_0 and different small radii r_1 and $r_2 = r_1 + \Delta r$. High magnetic well values in the proposed stellarator systems suggest a possibility of an MHD stability of plasma in such systems.

As it is known, the Mercie stability criterion is widely used to study the MHD plasma stability in stellarator magnetic configurations both analytically and numerically [6,7]. To investigate the plasma stability using the Mercier stability criterion in YAMATOR, we apply the numerical method based on the calculation of the terms contained in Mercier criterion by tracing the magnetic field lines in the given stellarator magnetic field [3,4]. The magnetic field of helical windings is calculated by the Biot-Savart law.

In order to study stability properties, one must calculate the equilibrium plasma currents. We here present the calculated results of the $< j_{II}^2 > / < j_\perp^2 >$ ratio (where j_{II} and $j_\perp$ are the equilibrium plasma-current components parallel and perpendicular to the magnetic field, respectively). The reduction of this ratio is one of the main points of the stellarator trap optimization (see, e.g., [8]).

2. METHODS AND RESULTS OF NUMERICAL INVESTIGATION

For the Mercier criterion evaluation the integration along the magnetic field lines was used with the method derived in [3,4]. For the case of a low-pressure plasma with a finite local pressure gradient this criterion can be presented in the form of the inequality

$$D < 1/4 \tag{1}$$

where

$$D = p' \frac{X_{GB} - X_{hS}}{(X_S - p' X_{hB})^2} \tag{2}$$

with $X_{GB} = \left\langle \nabla\psi \cdot \nabla B^2 / (B^2 |\nabla\psi|^2) \right\rangle / X_B$, $X_{hS} = \langle hS \rangle / X_B$, $X_{hB} = \left\langle hB^2 / (|\nabla\psi|^2) \right\rangle / X_B$, $X_S = \langle S \rangle / X_B$ and $X_B = \left\langle B^2 / (|\nabla\psi|^{2)} \right\rangle$. Here h is related to the equilibrium parallel current density $j_{II} = p' hB$ (the prime denotes the derivation with respect to ψ, ψ is a magnetic surface label), $S = (B \times \nabla\psi) \cdot \nabla \times (B \times \nabla\psi) / |\nabla\psi|^4$ and $\langle A \rangle$ is determined by integration along the magnetic field line $\langle A \rangle = \int A dl / B \Big/ \int dl / B$.

For evaluation of the $\langle j_{II}^2 \rangle$ to $\langle j_\perp^2 \rangle$ ratio we considered the parameter

$$\gamma_{PS} = \int j_{II}^2 \delta V \Big/ \int j_\perp^2 \delta V \tag{3}$$

where the integrals imply integration over the volume the layer between closely spaced magnetic surfaces. For calculating these integrals the integration along the magnetic field lines [5] was used.

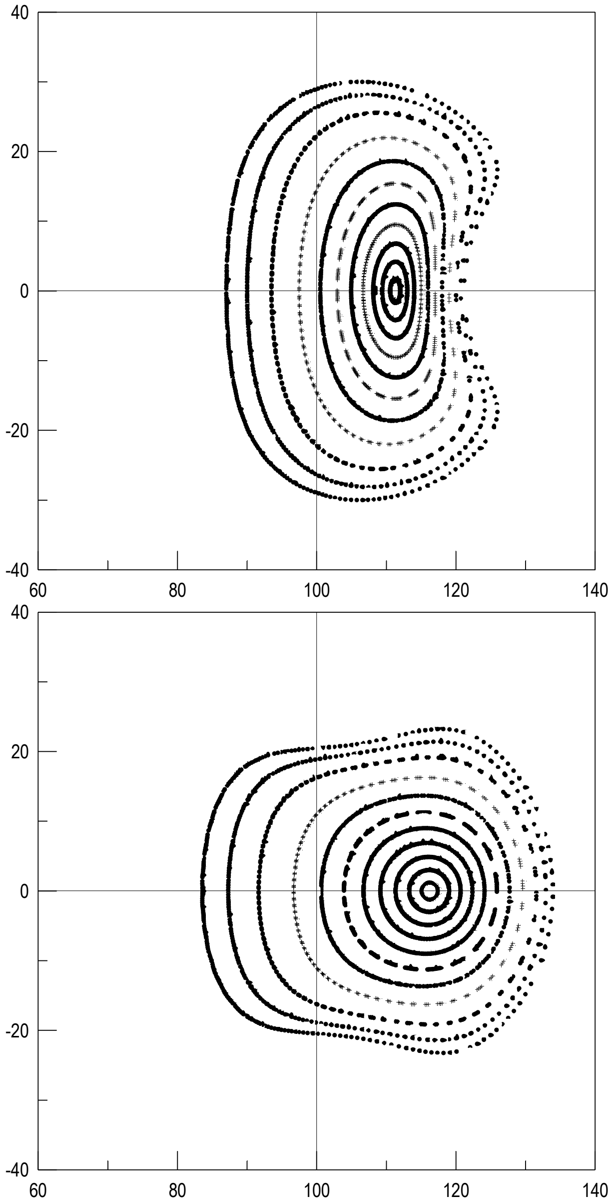

FIG.1. Magnetic surface cross-section for $B_{\perp} = 0$.

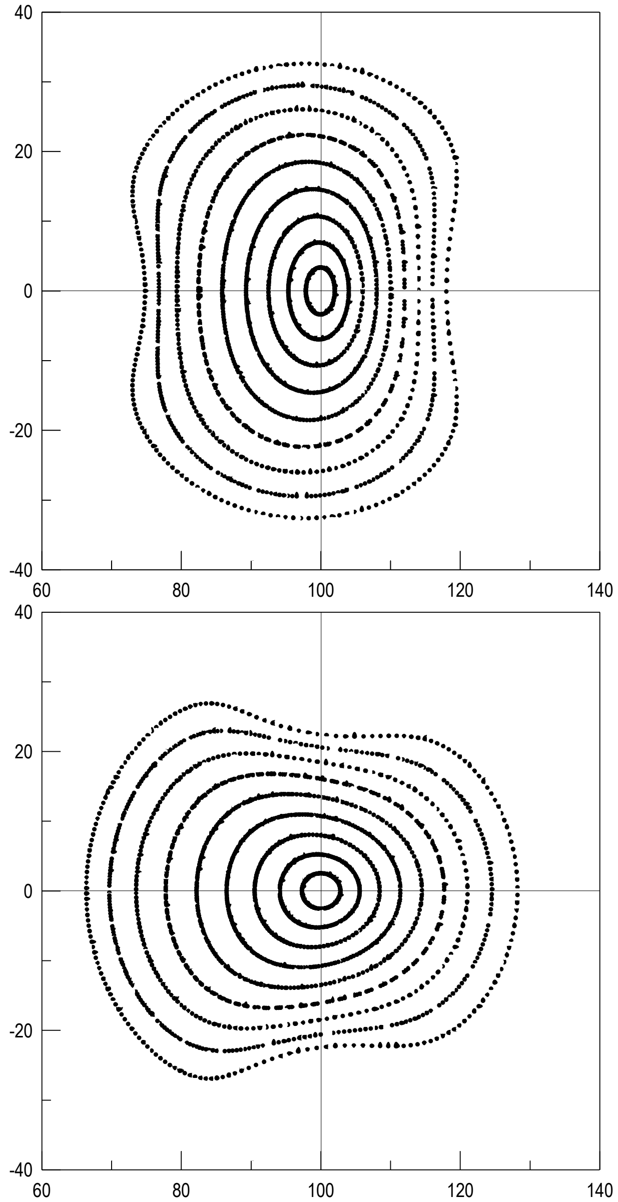

FIG.2. Magnetic surface cross-section for $B_{\perp} \neq 0$.

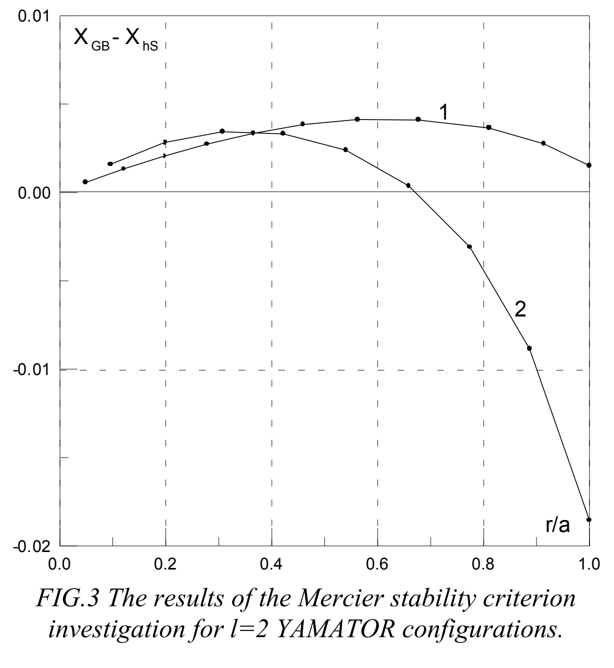

FIG.3 The results of the Mercier stability criterion investigation for l=2 YAMATOR configurations.

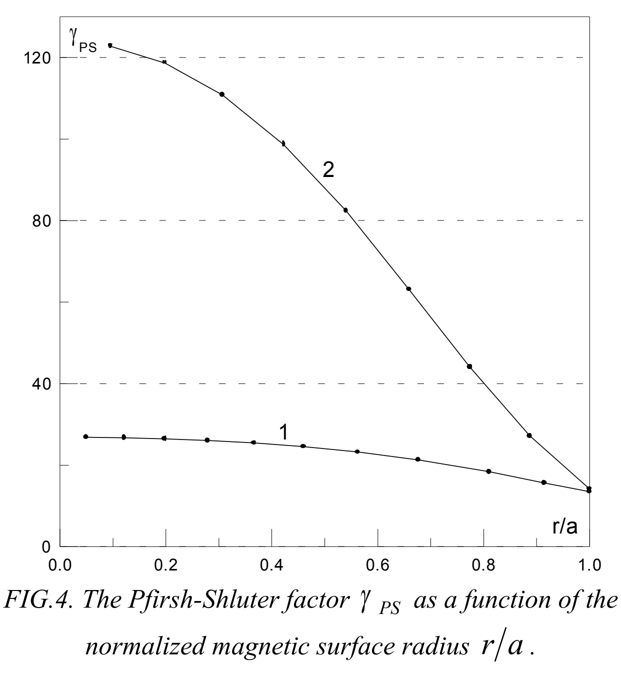

FIG.4. The Pfirsh-Shluter factor γ_{PS} *as a function of the normalized magnetic surface radius* r/a.

The $l = 2$ variant of the proposed system chosen for the numerical evaluation has the following parameters, $r_1/R_0 = 0.3$, $r_2/R_0 = 0.45$, $\Delta r/R_0 = 0.15$. The evaluation was performed assuming thin conductors placed at the toroidal surfaces along the helical line $\theta = n\varphi$ where θ and φ are poloidal and toroidal angles and n is a number of poloidal turns of the helical line after one toroidal turn ($n = 3$). The initial magnetic field of the system consisted from the field of helical conductors and toridal magnetic field $B_\varphi = B_0 R_0/\rho$ (ρ is the distance from the main axis of the torus). The magnetic field of the helical conductors has been calculated with the help of Biot-Savart law. In order to study the possibility to change the Pfirsh-Shluter factor and the Mercier stability criterion the calculation was performed both for the initial configuration and the analogous configuration with the additional uniform vertical magnetic field. The value of vertical field corresponded to the return of the magnetic axis to the circular axis of torus. The magnetic surface cross-sections for these two cases are shown in Figs. 1 and 2.

The results of the Mercier stability investigation for these two configurations are presented in Fig.3 where the difference $X_{GB} - X_{hS}$ as a function of the normalized magnetic surface radius r/a is shown. The curve 1 in the Fig.3 corresponds to the first configuration ($B_\perp = 0$), the curve 2 shows the result corresponding to the second configuration ($B_\perp \neq 0$). From (2) follows that for $p' < 0$ (the natural profile of p) and for $X_{GB} - X_{hS} > 0$ the D value is smaller than zero. In this case the Mercier criterion (1) is satisfied. It is seen from Fig.3 that this takes place for all r/a in the first configuration and for $r/a < 0.67$ in the second configuration. It turns out that in this case the denominator in (2) is of such a value that the condition (1) is true for any p'. At the same time for r/a in the limits from 0.67 to 1.0 in the configuration with the additional vertical magnetic field the condition (1) is violated for a rather small magnitude of p' (for $p' < 0$).

So, from the results follows that the Mercier stability criterion will be satisfied for all magnetic surfaces of the initial l=2 YAMATOR configuration and for a great part of the magnetic surfaces of the configuration with the additional vertical magnetic field. On the other hand, in the second case the violation of the Mercier stability criterion can take place for the magnetic surfaces from the external part of this configuration.

The corresponding radial dependencies of the Pfirsh-Shluter factor γ_{PS} are shown in Fig.4. The curve 1 shows the result corresponding to the initial l=2 YAMATOR configuration, and the curve 2 corresponds to the configuration with the additional vertical magnetic field that shifts the magnetic configuration to the inner side of torus. It follows from the Fig.4 that for the $B_\perp = 0$ configuration the value of the γ_{PS} factor varies from 14 to 27. For the configuration with the additional vertical magnetic field this parameter turns out to be even greater ($\gamma_{PS} = 14 - 123$). For the both configurations γ_{PS} factor has the same order of magnitude that theoretical estimate $2/\iota^2$ (equal to $17 - 165$ for the configuration with the additional vertical field and $23 - 28$ for the configuration without such a field) for the case of a stellarator with circular transverse cross sections of the magnetic surfaces and a planar magnetic axis.

3.CONCLUSION

In conclusion, we briefly summarize all of the results obtained. As it follows from the numerical investigation, the initial l=2 YAMATOR configuration is characterized by increased values of the Pfirsh-Shluter factor γ_{PS}. The stability Mercier criterion in this case is satisfied for all magnetic surfaces of this configuration (when $p' < 0$).

For the configuration with the additional vertical magnetic field the Pfirsh-Shluter factor is somewhat greater than that for the $B_\perp = 0$ configuration. The stability Mercier criterion for this configuration is satisfied in its central region. For rather large r/a values in the limits from 0.67 to 1.0 the violation of the Mercier stability criterion can take place.

Note, that we have considered questions related to the Pfirsh-Shluter factor and the Mercier stability criterion for $l = 3$ variant of the proposed system [2] too. The obtained in this case results appeared to be similar to the results for $l = 2$ system.

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