# TRANSFORMATION RATIO AT WAKEFIELD ACCELERATION IN DIELECTRIC RESONATOR

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The expressions for the wakefield have been derived and it has been shown that the transformation ratio in the dielectric resonator is significantly higher than the transformation ratio in the case of waveguide at large number of injected trains of identical bunches.

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## INTRODUCTION

The transformation ratio determines the maximal energy, to which the electrons are accelerated in the dielectric resonator for some energy of electron driver-bunches. It is widely investigated in the case of the wakefield excitation in dielectric waveguide [1, 2]. However, the dielectric resonator has an advantage for the electron acceleration [3, 4] and therefore it also widely investigated [5, 6]. Therefore, we investigate the transformation ratio at the wakefield excitation in the dielectric resonator [7]. The transformation ratio is determined as a ratio of the energy, gained by the accelerated bunch, to the energy of the sequence of bunches, exciting wakefield. In many cases transformation ratio can be concluded to the ratio of maximum accelerating wakefield experienced by witness bunch to the maximum slowing down wakefield experienced by driver bunches.

One rectangular in longitudinal direction bunch, each of length, equal to half of wave-length, is injected in each wavelength. The charges of bunches have the step shaping, so that every step consists of several bunches of identical charges in each. The charge of bunches of next step grows according to linear law 1:3:5 ... .

## TRANSFORMATION RATIO AT WAKEFIELD EXCITATION IN A DIELECTRIC RESONATOR AT CHARGE PROFILING OF SHORT STEPPED SEQUENCE OF BUNCHES BY LINEAR LAW

The transformation ratio at wakefield excitation in a dielectric resonator at charge profiling of short stepped sequence of bunches by linear law is considered. For that we consider the resonator of length L, in which a sequence of bunches are injected, each of length $\Delta\xi_b = \lambda/2$. Here $\lambda$ is the wavelength. The sequence of bunches is selected with the step shaping of charges of bunches (see Fig. 1), so that every step consists of $N_0$ bunches of identical charges in each (Fig. 1).

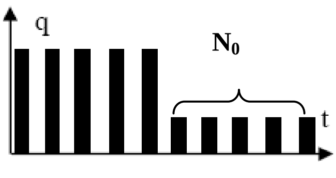


*Fig. 1. Temporal charge evolution of considered train of bunches*

The heights of steps (charge of bunches of next step) grows according to linear law 1: 3: 5: … . At arriving of the front of the first bunch to the end of the resonator the excited wakefield train is occurred in longitudinal direction inside of a spatial interval of length $L_1 = L(1 - V_g/V_0) + \Delta\xi_b \le L$. Here $V_g$ is the group velocity, $V_0$ is the velocity of bunches. The length of wakefield train after an arbitrary longitudinal point $z$, to which a bunch arrives, is equal to $z(1 - V_g/V_0) + \Delta\xi_b \le z$. The amount of bunches, which wakefields are coherently added in a point z, equals $\left[(1 - V_g/V_0)(z/\lambda) + \Delta\xi_b/\lambda\right](\omega_m/\omega)$. $\omega$ is the wave frequency, $\omega_m$ is the repetition frequency of the bunches. On the interval $0 < z < z_1$, where $z_1 = \lambda\left[(\omega/\omega_m) - \Delta\xi_b/\lambda\right]/(V_0/V_g - 1)$, $L \le z_1$, the wakefield is excited only by a single bunch. $z_1$ is determined by that in this point a next bunch comes up the back front of wakefield train, excited by a previous bunch

$$z_1/V_g + \Delta\xi_b/V_0 = z_1/V_0 + 2\pi/\omega_m .$$

There are two cases:

1) the case when the next inequality is satisfied

$$z(1 - V_g/V_0) < \Delta\xi_b , \ \Delta\xi_b < z .$$

Then the length of the leading and trailing edges of train, excited by bunch, equals $z(1 - V_g/V_0)$. The length of the wakefield train equals $\Delta\xi_b$. In other words near the boundary of injection the length of the wakefield train is determined by the length of the bunch.

2) Far from the boundary of injection, when the next inequality is satisfied

$$z(1 - V_g/V_0) > \Delta\xi_b ,$$

the length of the leading and trailing edges of wakefield train equals $\Delta\xi_b$. The length of the train equals $z(1 - V_g/V_0)$. In other words far from the boundary of injection the length of the wakefield train is determined by $z(1 - V_g/V_0)$.

We choose such parameters, that the front of (i+1)-th bunch comes up the trailing edge of i-th wakefield train at the end of resonator (at $z = L$). This condition can be written down in the following view

$$L\left(1 - V_g/V_0\right) = \lambda\left(V_g/V_0\right)\left(\omega/\omega_m\right). \quad (1)$$

Let us show that there are the integer of wavelengths on length of train at the end of the resonator

$$\left(V_0 - V_g\right)L/V_g = \lambda q_n, \ q_n = 1, 2, 3, \dots . \quad (2)$$

Indeed from (1) we have

$$\left(L/V_0 V_g\right)\left(V_0 - V_g\right) = 2\pi/\omega_m .$$

From here one can derive

$$\left(L/\lambda V_g\right)\left(V_0 - V_g\right) = \left(V_0/\lambda\right)\left(2\pi/\omega_m\right) = \omega/\omega_m = q_n .$$

If ratio $\omega/\omega_m$ equals integer (condition (3)) the condition (2) is satisfied automatically.

At the repetition frequency of the bunches

$$\omega/\omega_m = p_r = 1, 2, 3, \dots . \quad (3)$$

(the sequence of resonant bunches) the excited wakefields are added at the end of the resonator $z = L$ so, that the wakefield becomes a monochromatic wave.

Waves of frequencies $\omega_2$

$$\omega_2 = q_3\omega, \ q_3 = 2, 4, \dots \quad (4)$$

cannot be excited, because on length of bunch the integer of wavelengths is occurred.

For the use of the resonator of large length $L$, it is necessary according to expression

$$L/\lambda = \left(V_g/V_0\right)\left(\omega/\omega_m\right)/\left(1 - V_g/V_0\right) \quad (5)$$

to use $V_g$, close to $V_0$, and/or to use large $\omega/\omega_m$ with taking into account that the resonator length $L$ should include an integer of wavelengths

$$L = N\lambda, \ N = 1, 2, 3, \dots \quad (6)$$

or odd number of half-wavelengths

$$L = M\lambda/2, \ M = 1, 3, \dots . \quad (7)$$

But the bunches interchange by energy with a backward wave in the case (7). One can show that if the following inequality is satisfied

$$R(\lambda/4L) \ll 1 \quad (8)$$

the energy exchange of bunches with a backward wave is small in comparison with an energy loss of bunches in a decelerating wakefield of a forward wave.

The fields are added coherently after their complete passage through the resonator in forward and backward directions with $V_g$ during time $2L/V_g$. From here one can derive the number of identical bunches in every step (see Fig. 1)

$$N_0 = \left(2L/\lambda\right)\left(V_0/V_g\right)\left(\omega_m/\omega\right). \quad (9)$$

This scheme is possible to use train of witness bunches. Using (9) one can derive that the following number of bunches

$$N_{br} = \left(N_0/2\right)\left(V_g/V_0\right) \quad (10)$$

are simultaneously in the resonator. This number is truncate to a large value. The forward wave is modulated. At the end of resonator the forward wave becomes a homogeneous wave. As a result the backward wave is also homogeneous wave.

To become an excited wakefield a monochromatic wave at the end of resonator, the repetition frequency of the bunches should follow the condition (2) $\omega = p_r\omega_m$, $p_r = 1, 2, 3, \dots$. Because the bunches are resonant along the resonator, then for transformation ratio increase it is necessary, that during the time of return of backward wave from z=L to $z = 0$ the wakefield phase should shift by the odd number of half-wavelengths

$$2L/\lambda = \left(V_g/V_0\right)q_0, \ q_0 = 1, 3, 5, \dots . \quad (11)$$

The conditions of transformation ratio increase due to this method are not satisfied in the case $L/\lambda = P$, $P = 1, 2, 3, \dots$. Then one can use $2L/\lambda = M$, $M = 1, 3, \dots$.

We will derive longitudinal distribution of the single-mode wakefield in a dielectric resonator for different times for the case $\omega/\omega_m = 3$, $L/\lambda = 9/2$, $V_g/V_0 = 0.6$, $N_0 = 5$, $q_0 = 15$, $N_{br} = 2$. We take into account that on times $t > 2\pi/\omega_m$ two bunches are simultaneously in the resonator. Then temporal charge evolution of considered train of bunches, of longitudinal distribution of the wakefield in the resonator at the moment $t = L/V_0 - \pi/\omega$ and the longitudinal distribution of the wakefield of forward wave at the moment $t = 2L/V_g + L/V_0 - \pi/\omega$ looks like, shown in Fig. 1, Fig. 2 and Fig. 3.

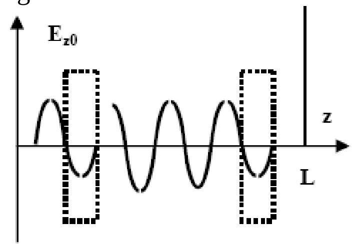


*Fig. 2. The longitudinal distribution of wakefield in a resonator at the moment* $t = L/V_0 - \pi/\omega$

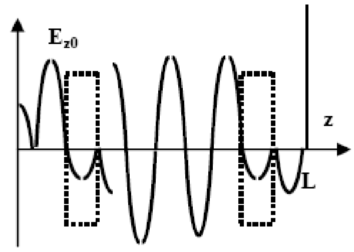


*Fig. 3. The longitudinal distribution of forward wakefield in a resonator at the moment* $t = 2L/V_g + L/V_0 - \pi/\omega$

We find that excited wakefield equals in the hole resonator at the moment $t = L/V_0 - \pi/\omega$

$$E_z(z, t) = E_0(r) \times$$

$$\left\{\left[\theta\left(V_0 t - z\right) - \theta\left(V_0 t - \lambda/2 - z\right)\right]0.5\sin\left(k\left(V_0 t - z\right)\right) + \right.$$

$$\left[\theta\left(V_0 t - \lambda/2 - z\right) - \theta\left(V_g\left(t - \pi/\omega\right) - z\right)\right]\sin\left(k\left(V_0 t - z\right)\right) +$$

$$+\left[\theta\left(V_0 t - \lambda\,\omega/\omega_m - z\right) - \theta\left(V_0 t - \lambda\,\omega/\omega_m - \lambda/2 - z\right)\right]\times$$

$$\times 0.5\sin\left(k\left(V_0 t-\lambda\,\omega/\omega_m-z\right)\right)+ \quad (12)$$
$$[-\theta\left(V_g t-\lambda\left(V_g/V_0\right)\left(1/2+\omega/\omega_m\right)-z\right)+$$
$$+\theta\left(V_0 t-\lambda\,\omega/\omega_m-\lambda/2-z\right)]\sin\left(k\left(V_0 t-\lambda\,\omega/\omega_m-z\right)\right)\}.$$

At the moment $t=2\left(N_p-1\right)L/V_g+L/V_0-\pi/\omega$ the excited wakefield of the forward wave in hole resonator equals

$$E_z(z,t)=E_0(r)\times$$
$$\{\left[\theta\left(V_0 t-z\right)-\theta\left(V_0 t-\lambda/2-z\right)\right]0.5\sin\left(k\left(V_0 t-z\right)\right)+$$
$$+N_p[\theta\left(V_0 t-\lambda/2-z\right)-$$
$$-\theta\left(V_g\left(t-\pi/\omega\right)-z\right)]\sin\left(k\left(V_0 t-z\right)\right)+$$
$$+\left[\theta\left(V_0 t-\lambda\,\omega/\omega_m-z\right)-\theta\left(V_0 t-\lambda\,\omega/\omega_m-\lambda/2-z\right)\right]\times$$
$$\times 0.5\sin\left(k\left(V_0 t-\lambda\,\omega/\omega_m-z\right)\right)+$$
$$+N_p[-\theta\left(V_g t-\lambda\left(V_g/V_0\right)\left(1/2+\omega/\omega_m\right)-z\right)+$$
$$\theta\left(V_0 t-\lambda\,\omega/\omega_m-\lambda/2-z\right)]\sin\left(k\left(V_0 t-\lambda\,\omega/\omega_m-z\right)\right)$$
$$-\left(N_p-1\right)\left[\theta\left(L-z\right)-\theta\left(V_0 t-z\right)\right]\sin\left(k\left(V_0 t-z\right)\right)-$$
$$-\left[\theta\left(V_g t-\lambda\left(V_g/V_0\right)\left(1/2+\omega/\omega_m\right)-z\right)-\theta\left(z\right)\right]\times$$
$$\times\left(N_p-1\right)\sin\left(k\left(V_0 t-z\right)\right)- \quad (13)$$
$$-\left[\theta\left(V_g\left(t-\pi/\omega\right)-z\right)-\theta\left(V_0 t-\lambda\,\omega/\omega_m-z\right)\right]\times$$
$$\times\left(N_p-1\right)\sin\left(k\left(V_0 t-z\right)\right)\}.$$

$N_p$ is the number of injected trains of identical bunches. One can see that the wakefield amplitude increases in $N_p$ times in comparison with (12). Then the transformation ratio equals

$$R=2N_p=2N/N_0\,, \quad (14)$$

$N$ is the number of injected bunches.

## CONCLUSIONS

The conditions have been formulated, when the wakefield pulses, excited by all consistently injected bunches, are coherently added and the witness-bunch interacts with full pulse. The conditions have been formulated, when decelerating longitudinal wakefield for all bunches is small that provides a large transformation ratio. The expressions for the wakefield have been derived and it has been shown that the transformation ratio is significantly higher than the transformation ratio in the case of waveguide at large number of injected trains of identical bunches.


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## КОЭФФИЦИЕНТ ТРАНСФОРМАЦИИ ПРИ КИЛЬВАТЕРНОМ УСКОРЕНИИ В ДИЭЛЕКТРИЧЕСКОМ РЕЗОНАТОРЕ


***В.И. Маслов, И.Н. Онищенко***



Получены выражения для кильватерного поля и показано, что коэффициент трансформации в диэлектрическом резонаторе значительно превышает его в случае волновода при значительном количестве инжектированных профилированных цугов одинаковых сгустков.


## КОЕФІЦІЄНТ ТРАНСФОРМАЦІЇ ПРИ КІЛЬВАТЕРНОМУ ПРИСКОРЕННІ В ДІЕЛЕКТРИЧНОМУ РЕЗОНАТОРІ


***В.І. Маслов, І.М. Онищенко***

Отримано вирази для кільватерного поля і показано, що коефіцієнт трансформації в діелектричному резонаторі значно перевищує його у випадку хвилеводу при значній кількості інжектованих профільованих цугів однакових згустків.