PEPPER-POT DIAGNOSTIC METHOD TO DEFINE EMITTANCE AND TWISS PARAMETERS ON LOW ENERGIES ACCELERATORS

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The new complex mathematical algorithm to determine beam transverse emittance data and the Twiss parameters from intensity measured with pepper-pot diagnostic device on rf low energies accelerators is described.
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1. INTRODUCTION

In order to describe the performance of a beam on rf accelerators one have to consider the total beam intensity. Full characterization of an external beam requires knowledge of its hyperemittance.

Early the eminence data measurement was made by slit methods [1]. Such methods are simply and reliable but the measurements are carried in both directions separately. Therefore slit procedures are so long time continues.

Using co-called pepper-pot method it gives possibility to measure and extract the emittance data in both transverse phase spaces at the same time. Such device already was tested on lineal accelerator UNILAC GSI (Darmstadt, Germany) [2].

In this paper the complex mathematical algorithm for analysis of experimental data by computer codes using such measured method is presented. The new more effective graphic algorithm for the determination emittance and the Twiss parameters is described.

2. PEPPER-POT MEASURED METHOD

The main element of the pepper-pot device is pepper-pot plate with a regular array of identical holes (hole diameter is about 0.2 mm and distances between two holes are 2.5 mm [1]) arranged over its whole surface. The sample beamlets hit a viewing screen situated in the angular analysis plane in a well-defined distance behind the pepper-pot plate. Therefore the two plates form a "multipinhole camera" (see Fig. 1.).

The experimental data must be represented as so many density diagrams as there are holes in the pepper-pot plate. The information of emittance is extracted from size, shape and location of the light spots observed on the viewing screen.

On the first measured step to determine the correspondence between the spot picture and the real physical dimensions the calibration process a parallel light beam from a laser is used. The coordinates on the image, correlated to the location of the holes in the pepper-pot plate can be obtained from the center of intensity of each light spot. It would be matrix (Xh_r, Yh_p); $r = 1...rmax$; $p = 1...pmax$, where $rmax$ and $pmax$ - are hole numbers in horizontal and vertical directions. The lightest point on the laser image (XC, YC) defines the beam center. As example, Fig. 2 shows the light spots from laser and real beam O^{3+} from diagnostic pepper-pot device for UNILAC, GSI [1].

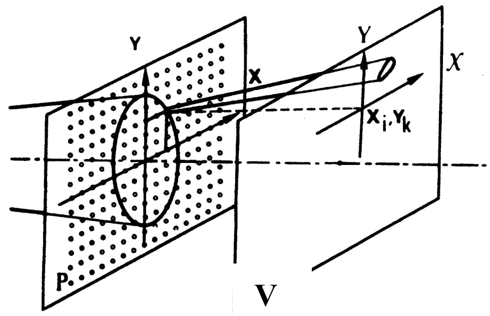

Fig. 1. *Pepper-pot diagnostic device: P - pepper-pot plate; V - viewing screen with one spot from the hole (X_j, Y_k)*

The experimental results for the future calculation are represented as 2-dimensional numerical intensity distribution matrix $I_{i,j}=I(x_i,y_j)$, where $i = 1...I_{np}$, $j = 1...J_{np}$. Here I_{np} and J_{np} are pixel numbers in horizontal and vertical directions. For the GSI diagnostic device [3]: $I_{np} = 1280$, $J_{np} = 1024$, $rmax = pmax = 15$.

3. THE SMOOTHING PROCEDURE

As already mentioned one has to notice, that in case the experimental data for intensity in dependence of the coordinate are not smooth enough an incorrect determination of emittances may result. Therefore, in a first step a smoothing process to the experimental data has to be carried out. It is suggested to use the least root-square method with Legendre polynomials as a basis [4].

In the interval $[x_0, x_n]$ the discrete function $I(x,y_j)$ for the fixed y_j can be approximate by polynomials:

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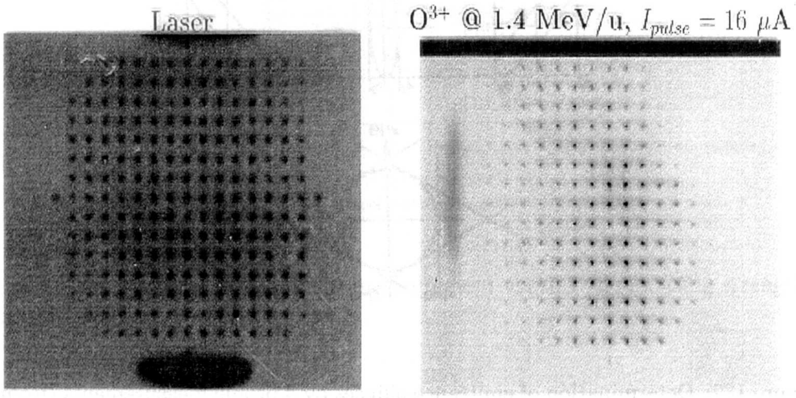

Fig. 2. *The experimental data from laser (left) and ion beam O^{3+} 1.4 Mev/u (right) from pepper-pot device (UNILAC, GSI, Darmstadt)*

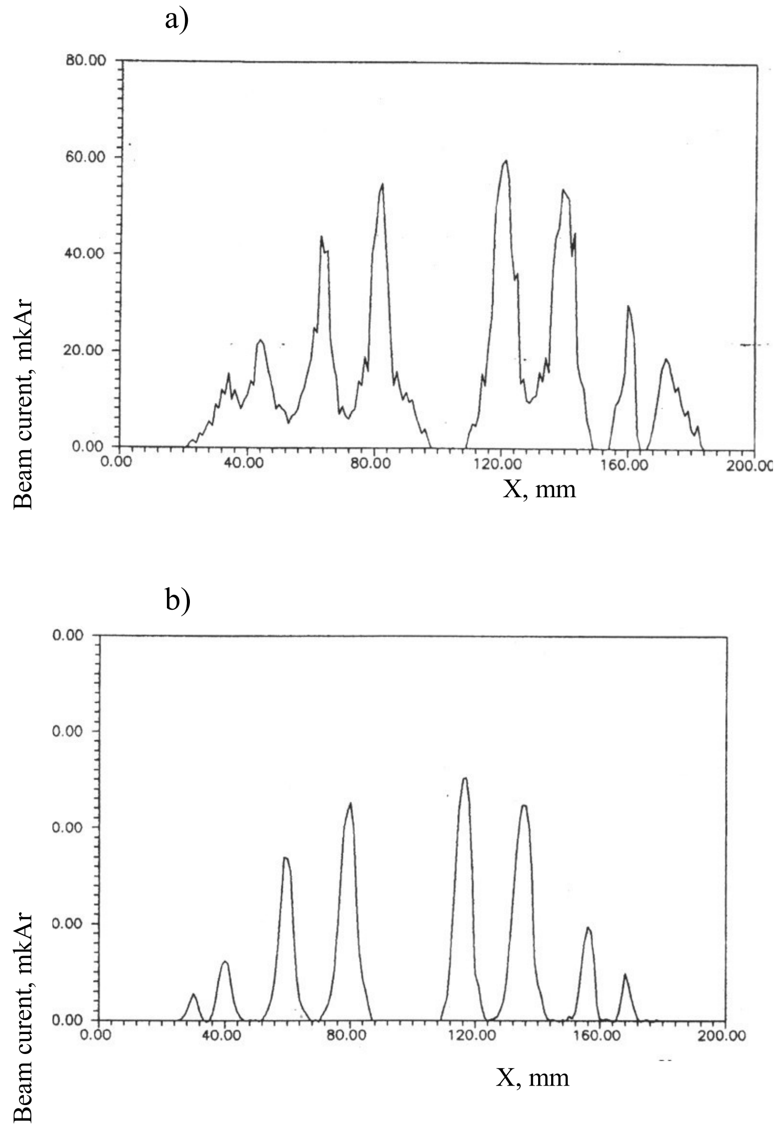

Fig. 3. *Example of the smoothing procedure:*
a) initial beam spectrum;
b) smoothed beam spectrum

$$IF(x, y_j) = \sum_{k=0}^{m} c_k \cdot L_k(x), \qquad (1)$$

with $m \le n$, and $L_k(x)$ - basis Legendre polynomials.

The unknown coefficients c_κ are found by minimization of the expression:

$$\sum_{k=0}^{n} \left[IF(x_k, y_j) - I(x_k, y_j) \right]^2 ,$$

where x_k are experimental data points in the interval $[x_0, x_n]$. Using the well-known Gauss method without choice of the main element [5] this system of the normal equations can be solved and as result the smoothing data $IF(x,y_j)$ are obtained. As an example the experimental data before and after smoothing are shown in Fig. 3.

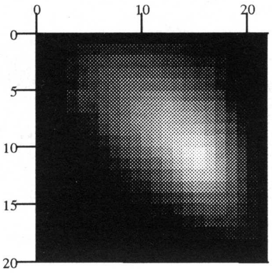

Fig. 4. Intensity distribution within one distinguished spot

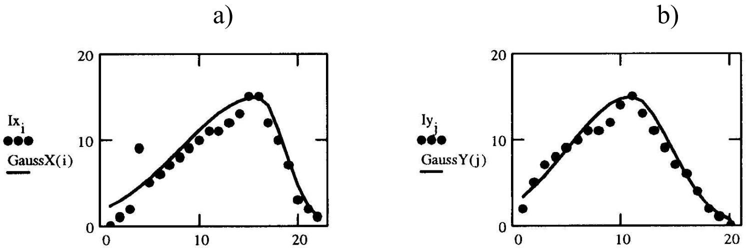

Fig. 5. *Horizontal (a) and vertical (b) projected one spot (see Fig. 4) intensity distribution: vectors Ix, Iy (points curves) and approximated Gaussian functions (solid lines)*

4. APPROXIMATION OF THE INTENSITY DISTRIBUTION FOR EACH SPOT BY GAUSSIAN FUNCTION

To get correct information about emittance and the Twiss parameters each spot has to be analyzed separately. For this meter the special cycling procedure to detect and distinguish each spot is carried out. Inside this procedure the sizes and projections for each spot are determined. Fig. 4 shows one detected spot.

Since the particle density distribution of the beam near the each peak can be represented by a Gaussian function [6] the projected spot curves can be also represented by similar shape:

$$IS(x) = IS_0 \cdot \exp\left[-\frac{(x - x_0)^2}{2 \cdot \sigma^2}\right], \tag{2}$$

where IS_0 - denotes the peak height of the spot intensity and x_0 is the peak position, σ is normal width.

It is necessary to note, that the projected curves usually will have non-symmetrical shape. Therefore the left and right sides of curves can be described separately. Preliminary the left side has to be extended to the right direction and the right curve to the left direction. So, each spot is defined by four Gaussian functions (two for each direction). Our further description is devoted to the symmetrical curves.

In order to find σ one can apply the relation:

$$Q(x) = \frac{IS(x_i - h)}{IS(x_i + h)} = \exp\left[\frac{2h \cdot (x_i - x_0)}{\sigma^2}\right], \tag{3}$$

where $h = x_i - x_{i-1}$. The logarithm of this expression is:

$$\ln Q(x) = \frac{2h \cdot (x_i - x_0)}{\sigma^2}. \tag{3.1}$$

For the function (3.1) it is used the least squares method with weight function W [7]:

$$W_i = \frac{IS(x_i + h) \cdot IS(x_i - h)}{IS(x_i + h) + IS(x_i - h)}. \tag{4}$$

The next function is minimized:

$$F(\sigma) = \sum_{i=1}^{N} W_i \cdot \left[\frac{2h \cdot (x_i - x_0)}{\sigma^2} - \ln\frac{IS(x_i - h)}{IS(x_i + h)}\right]^2, \tag{5}$$

where N - number of pixels that are involved in the approximation.
After some treatment the solution for σ is:

$$\sigma = \left[\frac{2h \cdot \sum_{i=1}^{N} W_i \cdot (x_i - x_0)^2}{\sum_{i=1}^{N} W_i \cdot \ln\frac{IS(x_i - h)}{IS(x_i + h)} \cdot (x_i - x_0)}\right]^{\frac{1}{2}}. \tag{6}$$

By this way each spot is described by Gaussian function in both directions. In Fig. 5 the projected experimental data for the light spot from Fig. 4 and the curves which they approximate are shown.

5. DEFINITION OF THE DIVERGENCE FOR THE SPOT

Using approximation by Gaussian function it is possible to find the divergence for different fractions of the maximum intensity. From (2) the formula of lineal deviation is given by:

$$\Delta x = \sigma \cdot \sqrt{2 \cdot \ln\frac{IS_0}{IS(x)}}, \tag{7}$$

where $\Delta x = x - x_0$. The spot divergence in horizontal direction is:

$$XR' = \frac{(dx + \Delta xr) \cdot h}{L},$$

$$XL' = \frac{(-dx + \Delta xl) \cdot h}{L}, \tag{8}$$

where $dx = x_0 - x_{hole}$. The same calculations for vertical divergence YD', YU' are made with $dy = y_0 - y_{hole}$. Here x_0, y_0 are the positions of the maximum intensity within a spot and x_{hole}, y_{hole} are the coordinates of the corresponding hole. Values of Δxr, Δxl, Δyd, Δyu are calculated using the formula (7):

$$\Delta xr = \sigma r \cdot \sqrt{2 \cdot \ln \frac{IN_m}{PR}},$$

$$\Delta xl = \sigma l \cdot \sqrt{2 \cdot \ln \frac{IN_m}{PR}},$$

$$\Delta yd = \sigma d \cdot \sqrt{2 \cdot \ln \frac{IN_m}{PR}},$$

$$\Delta yu = \sigma u \cdot \sqrt{2 \cdot \ln \frac{IN_m}{PR}}.$$

Number of h is determined by $h = \frac{\Delta h}{N_p}$, where Δh is the spacing between two holes and N_p is the number of pixels two holes. Value L is the distance between pepper-pot plate and the viewing screen.

Using the procedure described above, 5 matrixes (4 matrixes with divergences *XR', XL'* for horizontal direction, *YU', YD'* for vertical one and matrix *IST* representing the total spot intensities) are formed.

6. FORMATION OF THE HYPEREMITTANCE

The two-dimensional intensity distributions in horizontal and vertical phase planes have to be calculated. In the vertical plane the averages of *XRP* and *XLP* for each vertical column in matrixes *XR', XL'* are found. And in vertical plane the averages of *YDP* и *YUP* for each horizontal line in matrixes *YD', YU'* are determined.

As result for all two points from vectors *XRP* and *XLP* the horizontal coordinate of some hole is putted in conformity, while for all two points from vectors *YDP, YUP* the vertical coordinate of some hole is putted in conformity. By this way the emittance pattern is formed in two phase spaces. In Fig. 6 the example of the emittance pattern is shown.

7. DEFINITION OF THE EMITTANCE AND CALCULATION OF THE TWISS PARAMETERS BY A GRAPHIC METHOD

The phase space figure on the next step should be represented by some ideal ellipse with coordinate center and inclination angle θ. Then the square of such ellipse defines the emittance value ε. The relation between the Twiss parameters and approximated ellipse is described by Courant-Snyder invariant [6]:

$$\gamma \cdot x^2 + 2 \cdot \alpha \cdot x \cdot x' + \beta \cdot x'^2 - \varepsilon = 0. \tag{9}$$

Here α, β, $\gamma = (1+\alpha^2)/\beta$ are the Twiss parameters that should be defined.

The area enclosed within all of the phase space figure points can be estimated by such formula for the:

$$SX = \sum_{r=2}^{r\max} \frac{|XRP_{r-1} - XLP_{r-1}| + |XRP_r - XLP_r|}{2} \cdot |Xh_{r-1} - Xh_r|. \tag{10}$$

Than the emittance is [6]:

$$\varepsilon_x = \frac{SX}{\pi}. \tag{11}$$

In order to define the Twiss parameters one can apply the co-called graphic method.

Using the next coordinate transformation the ellipse is turned on the angle θ [8]:

$$x = x \cdot \cos\theta - x' \cdot \sin\theta,$$

$$x' = x \cdot \sin\theta + x' \cdot \cos\theta. \tag{12}$$

After the substitution (12) in the canonical ellipse equation [7], one gets the next expression:

$$x^2 \cdot \left[\frac{A}{K1^2} + \frac{A}{K2^2} \cdot B^2\right] + 2x \cdot x' \left[\frac{A \cdot B}{K2^2} - \frac{A \cdot B^2}{K1^2}\right] +$$

$$+ x'^2 \cdot \left[\frac{A}{K1^2} \cdot B^2 + \frac{A}{K2^2}\right] - 1 = 0, \tag{13}$$

where *K1* and *K2* are the ellipse half axis length and the coefficients *A* and *B* are found using angle θ:

$$A = \frac{1}{1 + tg^2\theta}, \quad B = tg\theta. \tag{14}$$

After multiplying of the equation (13) by ε and comparing with (9), the Twiss parameters are described by:

$$\alpha = \varepsilon \cdot \left[\frac{A \cdot B}{K2^2} - \frac{A \cdot B}{K1^2}\right],$$

$$\beta = \varepsilon \cdot \left[\frac{A \cdot B^2}{K1^2} + \frac{A}{K2^2}\right], \tag{15}$$

$$\gamma = \varepsilon \cdot \left[\frac{A}{K1^2} + \frac{A \cdot B^2}{K2^2}\right].$$

To find the inclination angle of the ellipse $tg\theta$, the average value for the angles $tg\theta_r$ of all pair XRP_r, XLP_r is calculated:

$$tg\theta_r = \frac{|XRP_r + XLP_r|}{2 \cdot Xh_r}.$$

Then value $tg\theta$ is presented as :

$$tg\theta = \frac{\sum_{r=1}^{r\max} tg\theta_r}{r\max}. \tag{16}$$

One half axis *K1* is defined as maximum value between:

$$\sqrt{\left[\frac{XRP_i + XLP_i}{2}\right]^2 + Xh_i^2} + \frac{|XRP_i - XLP_i|}{2},$$

where $i = 1, rmax$.

The next half axis is calculated from formula for ellipse area [8]: $K2 = Sx/(\pi \bullet K1)$.

When the emittance and the Twiss parameters are defined it is easy to find the ideal ellipses in horizontal and vertical phase spaces. In Fig. 7 such ellipse for the emittance pattern from Fig. 6 is shown.

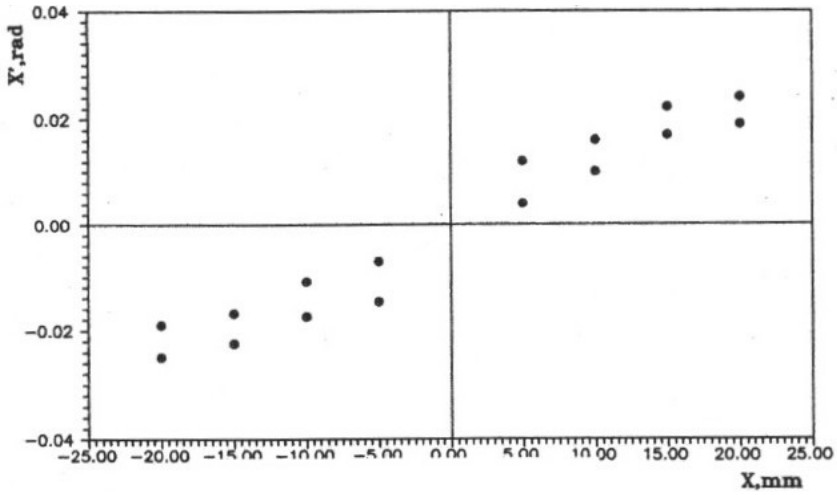

Fig. 6. *Example of the emittance phase space pattern*

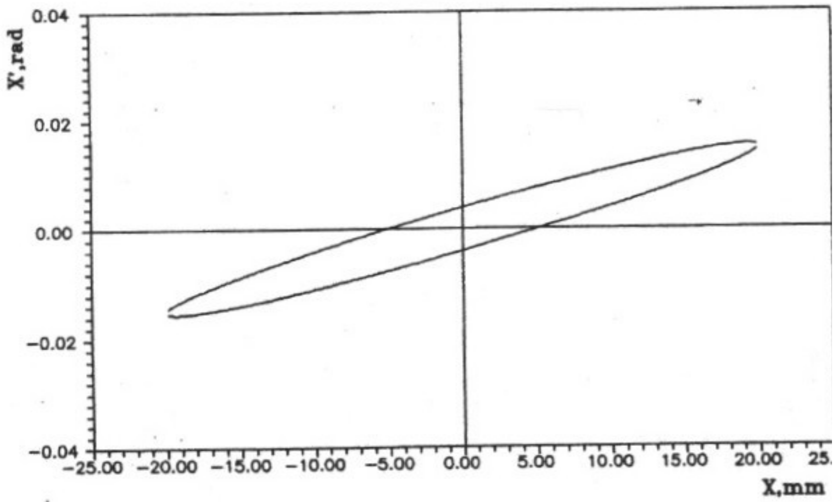

Fig. 7. *Phase space approximated ellipse for the pattern from Fig. 6.*

8. CONCLUSION

In comparison with another slit diagnostic methods the pepper-pot algorithm has some advantages. This method gives possibility to measure the emittance in both transverse phase spaces at the same time. Therefore the measurements and proceedings of the data are going faster. The mathematical algorithm, which is described in this paper, can be used not only at presented method, but at another ones also.

Pepper-pot device can be also installed in the low energies accelerators of INR, Kiev, Ukraine (for example, in the cyclotron U-240). The mathematical algorithm, which is presented here, can be applied in the complex computer code for the definition of the beam emittance and the Twiss parameters.

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