SCATTERING OF ELECTROMAGNETIC WAVE BY LINEAR CHAIN OF CHARGED PARTICLES

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The Thomson scattering of a plane monochromatic electromagnetic wave by a linear chain of periodically spaced charged particles is investigated theoretically. It is obtained a functional dependence for the total power and angular distribution of this radiation as a function of the number of the charged particles and the distance between them. The coherence effects for a linear chain of pointlike bunches are discussed.

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1. INTRODUCTION

Investigations of the electromagnetic radiation coherence from the charged particles bunches is of considerable interest for a number of plasma and beam physics applications (see, e.g. [1,2]). These investigations are extremely impotent for the researches directed on the short wavelength coherent radiation in free – electron lasers (FEL), which are expected generate coherent electromagnetic radiation at wavelengths on the order of tens of angstroms [3,4]. For a number of charged particles bunches, the dependence of the total intensity of scattered electromagnetic wave (EMW) on character distance between scatterers in bunch has been investigated in [5-7]. In this paper the results of investigations of the angular distribution and the total intensity of radiation for a linear charged particles chain under the Thomson scattering of EMW are presented.

2. FORMULATION OF THE PROBLEM

As a model of the bunch configuration let us consider a linear chain of a finite number of identical charged particles scattering the plane monochromatic linearly polarized EMW. The main physical reasons for the choice of this model are follows. The Thomson scattering of EMW by charged particles is the fundamental physical process of electromagnetic field emission. Secondly, for the ultra–relativistic beam energy range a plane undulator field in the beam rest frame is similar to the EMW. On another hand, by varying the distance between point charge-radiators and their total number can be control the level of radiation coherence by the individual bunch, and by the chain of pointlike bunches as well.

Let us consider a linear chain of N identical charged particles, with mass m and charge q. In this chain the distance between two neighboring particles is d. The plane monochromatic EMW with the frequency ω, the wave number $k=\omega/c$ and the amplitude E propagates along the OZ axis ($x=y=0$), where the scatterers are situated in: $E_{ext}(r,t)=e_x E\cos(\omega t-kz)$. It is necessary to determine the total radiation intensity of the scattered radiation and angular distribution of the energy flux of this radiation as a function of the number of scatterers N and the period of their sequence d.

The angular distribution of the energy flux of the scattered radiation and total intensity of this radiation will be calculated from the formula for dipole radiation of charges in its wave zone [8]. Indeed, at a distance $r=\left(x^2+y^2+z^2\right)^{1/2}$, considerably larger than the bunch linear dimension ($r>>L_N=(N-1)d$), the energy flux density of the radiation into the solid angle element $d\Omega=\sin\vartheta d\vartheta d\varphi$ can be expressed in terms of the total dipole momentum of all charges of the bunch $\mathbf{D}(t)$:

$$dI_{tot}(\vartheta,\varphi)=\frac{1}{4\pi c^3}\left\langle\left[\ddot{\mathbf{D}}(t')\,\mathbf{n}\right]^2\right\rangle d\Omega\ , \qquad (1)$$

$$\mathbf{D}(t')=\mathbf{e}_x qa\sum_{s=1}^{N}\cos\left[\omega t'(r)-(1-\cos\vartheta)kz_s\right].$$

Here $\mathbf{e}_x$ is the unit vector along the OX axis, $a=qE/(m\omega^2)$ is the amplitude of particle oscillations in the EMW field, ϑ is the angle between the unit vector $\mathbf{n}$ toward the observation point and positive direction of the OZ axis, $\mathbf{n}=\mathbf{r}/r$, z_s is the longitudinal coordinate of the charge of the number s, the angular bracket mean a time average over the field period $T=2\pi/\omega$, $t'(r)=t-r/c$ is the retarded time.

Angular distribution of the radiation. In the considered model of the bunch the right-hand side of Eq. (1) is a rather simple function of the external bunch parameters:

$$dI_{tot}(\vartheta,\varphi;\theta)=K_N(\vartheta;\theta)\,dI_{inc}^{(N)}(\vartheta,\varphi)\ , \qquad (2)$$

$$K_N(\vartheta;\theta)=1+\frac{2}{N}\sum_{s=1}^{N}(N-s)\cos\left[\theta s(1-\cos\vartheta)\right], \qquad (3)$$

$$dI_{inc}^{(N)}(\vartheta,\varphi)=\frac{3}{8\pi}N I_1\left(1-\sin^2\vartheta\cos^2\varphi\right)d\Omega\ . \qquad (4)$$

Here $dI_{inc}^{(N)}(\vartheta,\varphi)$ is the angular distribution of intensity of incoherent radiation of the bunch in solid angle element $d\Omega$, $I_1=\frac{q^2a^2\omega^4}{3c^3}=\frac{8\pi}{3}\left(\frac{q^2}{mc^2}\right)^2\frac{cE^2}{8\pi}$ is the intensity of individual charge radiation, $\theta=kd$ is the dimensionless period of scatterer sequence in the chain, $K_N(\vartheta;\theta)$ is the bunch coherence factor of the radiation into the given solid angle element $d\Omega$.

Performing the summation over the charge number *s* in the right–hand side of the Eq. (3) we can obtain the follow expression for the coherence factor:

$$K_N(\vartheta;\theta)=2\operatorname{cosec}\mu\,\sin(N\mu)\cos[(N-1)\mu]-1-\frac{\operatorname{cosec}^2\mu}{2}\left\{\cos[2(N-1)\mu]-\left(1-\frac{1}{N}\right)\cos(2N\mu)-\frac{1}{N}\right\},\quad(5)$$

where $\mu(\vartheta;\theta)=\theta(1-\cos\vartheta)/2$, $\operatorname{cosec}z=1/\sin z$.

Total power. Integrating the right–hand side (2) over the total solid angle ($0\le\vartheta\le\pi$; $0\le\varphi\le2\pi$) gives the following explicit formula for the total radiation intensity of the bunch considered [7]:

$$I_{tot}(\theta)=K_N^{(tot)}(\theta)I_{inc}^{(N)},\quad I_{inc}^{(N)}=N\,I_1,\quad(6)$$

$$K_N^{(tot)}(\theta)=1+3\sum_{s=1}^{N}\left(1-\frac{s}{N}\right)\left[\cos\rho_s+\left(\rho_s-\frac{1}{\rho_s}\right)\sin\rho_s\right]\frac{\cos\rho_s}{\rho_s^2}.\quad(7)$$

Here we expressed the total radiation intensity $I_{tot}(\theta)$ in terms of the total incoherent radiation intensity of the charges $I_{inc}^{(N)}$, $\rho_s=\theta s$. In this way the coherence factor for the total radiation intensity of the bunch defined by Eq. (7), as in [7].

3. RESULTS AND DISCUSSION

Eqs. (2)–(7) present the complete solution of the problem in the explicit analytical form. The last of them generalizes the asymptotic results of the classical Thomson scattering theory in case of the idealized model of the bunch configuration considered (linear periodic chain of scatterers). In the particular case of two scatterers (N=2), these formulae are in good agreement with those obtained earlier in [5]. In a theory and applications of the microwave electronics based on bremsstrahlung radiation the functional dependencies of coherence factor on the external bunch parameters N and θ are of fundamental interest. Below we will describe these dependencies in more detail using their analytical asymptotics and graphs (calculated for the particular values of scatterers number N).

Analytical asymptotics. For small and large distances between charges from the Eq. (7) follow the classical results [8]. Indeed, for small bunch dimensions ($(N-1)\theta<<1$) its right–hand side takes the form:

$$K_N^{(tot)}(\theta)=N\left[1-7\left(N^2-1\right)\theta^2/60\right].$$

In the alternative limiting case (when the distance between neighboring scatterers *d* is greater than wavelength $\lambda=2\pi/k$, i.e. $\theta>>1$) the overall contribution of coherent interaction between charges-scatterers in the bunch

$$\delta_N(\theta)\equiv K_N^{(tot)}(\theta)-1\approx\frac{3}{2\theta}\sum_{s=1}^{N}\left[\left(\frac{1}{s}-\frac{1}{N}\right)\sin2\theta s\right],\quad(8)$$

is of the order $\theta^{-1}<<1$.

For the values of the period *d,* multiplied to half wave length ($\theta=\pi s$; s=1,2,3,..), this contribution decreases inversely to the square of this period:

$$\delta_N(\pi s)=\frac{1}{2s^2}\left\{1-\frac{6}{\pi^2}\left[\psi'(N)-\frac{1}{N}\left(C+\psi(N)\right)\right]\right\}.\quad(9)$$

Here C=0,577... is the Euler's constant; ψ(z) is Psi (Digamma) function; $\psi(z)\equiv\Gamma'(z)/\Gamma(z)$; Γ(z) is the gamma function and $\psi'(z)\equiv d\psi/dz$.

For the great numbers N (N>>1) the last formula takes the form $\delta_N(\pi s)=1/(2s^2)$

Differentiating the right-hand side of the Eq. (7) over θ we can see, that at points $\theta=\theta_s=\pi s$ the derivative $\delta_N'(\theta)$ is positive and increases linearly with the number of particles N and decreases in inversely proportional to the distance between them θ_s: $\delta_N'(\theta_s)=3(N-1)/(2\theta_s)$.

The functional dependence of coherence factor in the angular distribution of radiation $K_N(\vartheta;\theta)$ on the ϑ is more complicated, including two external parameters – θ and N (see (5)). Nevertheless, some general characteristics of this dependence can be obtained in analytical form directly from the Eq. (5). In fact, for small bunch dimensions ($L_N<<1$) the coherence factor is at absolute maximum $K_N(\vartheta;\theta)=N$ for all values of the angle ϑ. For a period of charge locations $\theta\approx\pi$ the bunch radiation anisotropy appears: in the directions ϑ and $\vartheta_*\equiv\pi-\vartheta$ the values of coherence factor begin to differ from one another: $K_N(\vartheta_*;\theta)\neq K_N(\vartheta;\theta)$. Then the distances between the charges are greater than wavelength of scattered radiation ($\theta>2\pi$), the coherence factor $K_N(\vartheta;\theta)$ becomes substantially a non-monotonous function of the angle ϑ. In particular, for the directions ϑ_s, imposed by the equation

$$\theta(1-\cos\vartheta_n)=2\pi n\ ;\ n=0,1,2,3,...,[\theta/\pi],\quad(10)$$

this factor is at its maximum

$$K_N(\vartheta_n;\theta)=N.\quad(11)$$

Here the square bracket means the integer part of the bracket number.

The half-widths of these extreme (in the angle ϑ) are inversely proportional to the bunch length L_N :

$$|\Delta\vartheta|\approx\left[\theta\sqrt{N^2-1}\right]^{-1}.\quad(12)$$

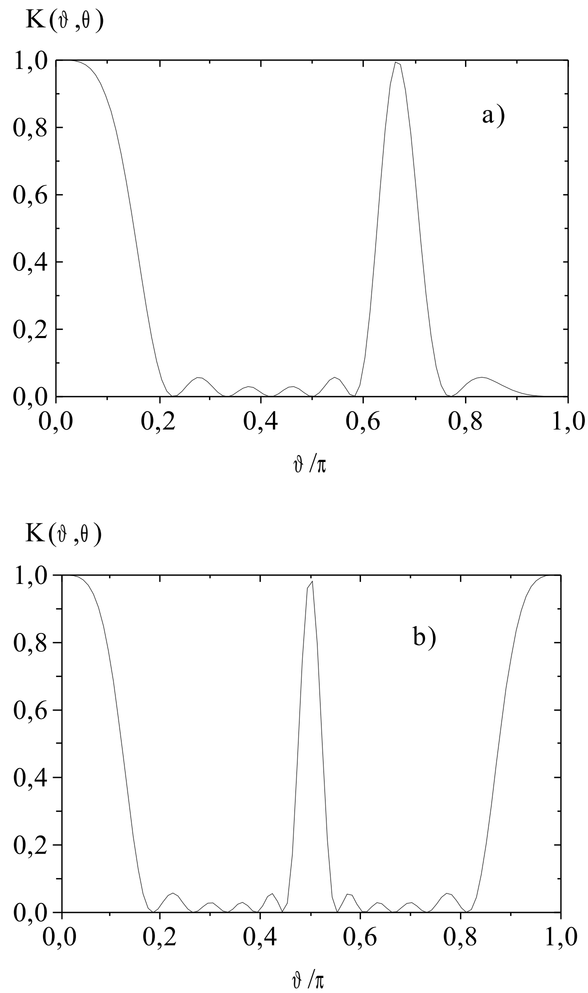
The exceptions are the angles ϑ=0 and ϑ=π for distances between charges, multiplied to half-length of the scattered wave: for them the half-widths of the extrema decrease with increasing of the bunch length in inverse proportion to the square root of this length:

$$|\Delta \vartheta| \approx \left[\theta \sqrt{N^2 - 1}\right]^{-1/2} .$$

It should be noted that the coherence factor $K_N^{(tot)}(\theta)$ drops quickly with increase of the bunch dimension (the graphs for the coherence factor $K_N^{(tot)}(\theta)$ as function of the number of scatterers N and the distance between them *d* are presented in [7]). Corresponding extreme in bremsstrahlung radiation angular spectrum power of the bunch (see Eqs. (10), (11) and figure) describe physically only the result of the coherent summation of bremsstrahlung radiation fields of individual scatterers phased by the regular field of scattered wave. In the simplest case N=2 this effect was investigated in details in [5].

The numerical calculations of the function $K_N(\vartheta;\theta)$ have been carried out.

Figure shows the coherence factor $K_N(\vartheta;\theta)$ as a function of the angle ϑ for two particular values of the parameter θ for the number of scatterers N=6.



Coherence factor $K_N(\vartheta;\theta)$ as a function of the angle ϑ for the parameters N=6, $\theta=4\pi/3$ (a), 2π (b)

Using the formulae presented above one can describe the coherence effects in periodic chain of pointlike bunches separated by arbitrary distances. Let N bunches, each of them contains M charged particles, situated along the OZ axis so that the distance between each two neighbors bunches in the chain is *d*. Then, make the substitution q for Mq and m for Mm in equation (4), and using Eqs. (2), (6) we obtain following expressions for angular distribution of energy flux of the scattered radiation:

$$dI_{tot}(\vartheta,\varphi;\theta) = M\ K_N(\vartheta;\theta) dI_{inc}^{(MN)}(\vartheta,\varphi) , \qquad (13)$$

and total intensity of this radiation

$$I_{tot}(\theta) = M\ K_N^{(tot)}(\theta) I_{inc}^{(MN)} , \qquad (14)$$

$$dI_{inc}^{(MN)}(\vartheta,\varphi) = M\ dI_{inc}^{(N)}(\vartheta,\varphi)\ ,\ I_{inc}^{(MN)} = M\ I_{inc}^{(N)} ,$$

were $dI_{inc}^{(MN)}(\vartheta,\varphi)$, $I_{inc}^{(MN)}$ are angular distribution and total intensity of incoherent radiation by all charges in the chain, respectively. The functions $K_N(\vartheta;\theta)$ and $K_N^{(tot)}(\theta)$ in this equations have the form (3), (5) and (7).

It is show from Eqs. (13), (14) that coherence factors in total intensity $K_{MN}^{(tot)}(\theta)$ and angular distribution $K_{MN}(\vartheta,\theta)$ for the considered periodic linear chain of pointlike bunches of charged particles may be write as

$$K_{MN}^{(tot)}(\theta) = M\ K_N^{(tot)}(\theta)\ ,\ K_{MN}(\vartheta,\theta) = M\ K_N(\vartheta,\theta) .$$

Thus, Eqs. (13) and (14) with expressions for coherence factors (5)-(7) define the total intensity of the scattered radiation and angular distribution of energy flux of this radiation under the Thomson scattering of EMW by periodic chain of pointlike bunches.

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