CLASSIFICATION OF SPATIALLY-NONUNIFORM EQUILIBRIUM STATES OF SUPERFLUIDS

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The classification of equilibrium states of superfluid with scalar, vector and tensor order parameters is carried out on the basis of the quasiaveragues concept. The generalization of a requirement of the residual symmetry for nonuniform equilibrium states is given. The admissible requirements of a spatial symmetry in the terms of integrals of motion are found. The connection of these requirements with helicoidal structure of vectors of a spin and spatial anisotropy is established. At some restrictions is shown, that the equilibrium structure of an order parameter can be represented as product of a nonuniform part, depending on spatial coordinates, and homogeneous part of an order parameter.

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1. INTRODUCTION

Investigations into the phenomenon of superfluidity in ^{3}He have resulted in the prediction and discovery of a number of superfluid phases. Among the discovered phases we mention the isotropic B-phase, anisotropic A- and A$_1$-phases in the presence of a magnetic field. The other phase states have not been found in experiment.

The classification of homogeneous equilibrium superfluid states in ^{3}He was carried out in [1-5] on the basis of the Ginzburg-Landau theory or with the use of group-theoretical methods.

The papers [6-8] dealt with nonuniform equilibrium states in superfluid ^{3}He. Within the framework of model expressions for a free energy, those authors have elucidated the stability conditions of helicoidal structures. In papers [9,10] the stability boundaries of the mentioned states were extended to a wider range of temperatures. The interest in this problem has increased due to its close connection with the problem of critical velocities in superfluid ^{3}He. However, no classification of equilibrium inhomogeneous states was performed.

The purpose of the present study has been to classify superfluid phases for singlet or triplet pairing on the basis of the quasiaverages concept [11,12], taking into account possible nonhomogeneous equilibrium structures. We have formulated the condition of residual symmetry of the equilibrium state and the simultaneous condition of spatial symmetry. Nonhomogeneous equilibrium structures of order parameter are found.

2. NOPMAL EQUILIBRIUM STATE OF FERMI LIQUID

The Gibbs statistical operator is given by the standard expression

$$\hat w = \exp\left(\Omega - Y_0\hat H - Y_4\hat N\right), \tag{2.1}$$

where $\hat H$ - Hamiltonian, $\hat N$ - particle number operator, $Y_0^{-1} \equiv T$ - temperature, $-Y_4/Y_0 \equiv \mu_k$ - chemical potential. For simplicity we assume that the condensed medium as a whole is at rest, and the effective magnetic field is equal to zero. The statistical operator has the following symmetry properties:

$$\left[\hat w, \hat P_k\right] = 0, \quad \left[\hat w, \hat H\right] = 0, \quad \left[\hat w, \hat N\right] = 0,$$
$$\left[\hat w, \hat S_\alpha\right] = 0, \quad \left[\hat w, \hat L_k\right] = 0, \tag{2.2}$$

$\hat P_k$ - operator of impulse, $\hat S_\alpha$ and $\hat L_i$ - operators of spin and orbital momenta. The first three relations represent the space-time translational invariance and the phase invariance. The symmetry conditions relative to the rotations in spin and configurational spaces imply the neglect of weak dipole and spin-orbit interactions at characterization of equilibrium state.

The unitary transformations $U \equiv \exp i\hat G g$, where $\hat G \in \hat{\vec S}, \hat{\vec L}, \hat N, \hat{\vec P}, \hat H$ are the integrals of motion and g denotes the parameters of transformation, leave the Gibbs statistical operator invariant

$$U\hat w U^+ = \hat w. \tag{2.3}$$

Note that the averages of the form $Sp\hat w\left[\hat G, \hat b(x)\right] \equiv 0$ at an arbitrary operator $\hat b(x)$. If $\hat b(x) \equiv \hat\Delta_a(x)$, then, taking into account that the commutators $\left[\hat G, \hat\Delta_a(x)\right]$ are linear and homogeneous in the order parameter operators $\hat\Delta_a(x)$, we come that the average normal-state equilibrium order parameters vanish

$$Sp\hat w\hat\Delta_a(x) = 0.$$

3. EQUILIBRIUM. SINGLET PAIRING OF FERMI SUPERFLUID

The quasiaverage $a(x) = \langle \hat a(x)\rangle$ in the equilibrium state with broken symmetry is given by the formula [11]

$$\langle \hat a(x)\rangle \equiv \lim_{\nu \to 0}\lim_{V\to\infty} Sp\hat w_\nu\, \hat a(x), \tag{3.1}$$

$$\hat{w}_V \equiv \exp\left(\Omega_V - Y_0\hat{H} - Y_4\hat{N} - vY_0\hat{F}\right).$$

Here the operator $\hat{F}$ is a linear functional of the order parameter operator

$$\hat{F} = \int d^3x\left(f(x)\hat{\Delta}(x) + h.c.\right),$$

$f(x)$ is the function of coordinates, conjugate of the order parameter, which sets the equilibrium values of the latter. The Fermi superfluid with singlet pairing is characterized by the scalar order parameter

$$\hat{\Delta}(x) \equiv (i/2)\hat{\psi}(x)\sigma_2\hat{\psi}(x). \tag{3.2}$$

Here $\hat{\psi}(x)$ is the Fermi operator of particle annihilation at the point x, and σ_2 – is the Pauli matrix. The order parameter operator satisfies the commutation relations

$$\left[\hat{N},\hat{\Delta}(x)\right] = -2\hat{\Delta}(x), \qquad i\left[\hat{S}_\alpha,\hat{\Delta}(x)\right] = 0,$$

$$i\left[\hat{P}_k,\hat{\Delta}(x)\right] = -\nabla_k\hat{\Delta}(x),$$

$$i\left[\hat{L}_i,\hat{\Delta}(x)\right] = -\varepsilon_{ikl}x_k\nabla_l\hat{\Delta}(x). \tag{3.3}$$

We shall consider homogeneous equilibrium states to establish possible structures of the scalar order parameter. The homogeneous equilibrium statistical operator satisfies the relation

$$\left[\hat{w},\hat{P}_k\right] = 0. \tag{3.4}$$

The analysis of homogeneous subgroups of the residual symmetry of equilibrium states is realizable on the basis of the relation

$$\left[\hat{w},\hat{T}\right] = 0, \tag{3.5}$$

where the residual symmetry generator $\hat{T}$ presents a linear combination of integrals of motion

$$\hat{T} \equiv a_i\hat{L}_i + b_\alpha\hat{S}_\alpha + c\hat{N} \equiv \hat{T}(\xi), \tag{3.6}$$

with real numerical parameters ($a_i, b_\alpha, c \equiv \xi$). According to relations (3.4), (3.5), we have

$$iSp\left[\hat{w},\hat{T}\right]\hat{\Delta}(x) = 0,$$

$$iSp\left[\hat{w},\hat{P}_i\right]\hat{\Delta}(x) = 0.$$

Whence, in view of (3.3), we have $c = 0$, as $\Delta(x) \neq 0$. The residual symmetry generator takes on the form

$$\hat{T} \equiv a_i\hat{L}_i + b_\alpha\hat{S}_\alpha. \tag{3.7}$$

The order parameter in the equilibrium state for this case looks like

$$\left\langle\hat{\Delta}(x)\right\rangle = \eta(Y)\exp i\varphi.$$

Here η is the modulus of the order parameter, and φ is the superfluid phase.

Let us now consider the states of equilibrium, which have no translational invariance (3.4). The physical possibilities of violation of this equilibrium state invariance are as follows: (i) violation of phase invariance (the superfluid impulse is nonzero), (ii) violation of symmetry relative to spin rotations (the vector of a magnetic spiral is distinct from zero), (iii) violation of rotation symmetry in the configurational space (the vector of a cholesteric spiral is not equal to zero). Such symmetry of the equilibrium states can be given by the following relation:

$$\left[\hat{w},\hat{P}_k\right] = 0,$$

$$\hat{P}_k \equiv \hat{P}_k - p_k\hat{N} - q_{k\alpha}\hat{S}_\alpha - t_{kj}\hat{L}_j, \tag{3.8}$$

here $p_k, q_{k\alpha}, t_{kj}$ are certain real parameters. The residual symmetry generator for these states now includes the operator of impulse

$$\hat{T} \equiv a_i\hat{L}_i + b_\alpha\hat{S}_\alpha + c\hat{N} + d_i\hat{P}_i. \tag{3.9}$$

According to the relations

$$iSp\left[\hat{w},\hat{T}\right]\hat{\Delta}(x) = 0,$$

$$iSp\left[\hat{w},\hat{P}_i\right]\hat{\Delta}(x) = 0$$

we come to the equalities that relate the parameters of spatial symmetry generators to the parameters of residual symmetry generator as

$$t_{kj}\varepsilon_{juv}p_v = 0, \quad \nabla_k\Delta(x) = 2ip_k\Delta(x), \tag{3.10}$$

$$a_i\varepsilon_{ikl}p_l = 0, \qquad c + d_ip_i = 0.$$

We shall find additional relationships between these parameters using the Jacobi identity for the operators $\hat{w},\hat{T},\hat{P}$ and $\hat{w},\hat{P}_i,\hat{P}_k$. Taking into account the properties of residual and spatial symmetries we have

$$Sp\left[\hat{w},\left[\hat{T},\hat{P}_k\right]\right]\hat{\Delta}(x) = 0,$$

$$Sp\left[\hat{w},\left[\hat{P}_i,\hat{P}_k\right]\right]\hat{\Delta}(x) = 0.$$

These formulas lead to an admissible structure of residual and spatial symmetry generators

$$\hat{T} \equiv al_i\hat{L}_i + b_\alpha\hat{S}_\alpha + d_i\left(\hat{P}_i - pl_i\hat{N}\right), \tag{3.11}$$

$$\hat{P}_k \equiv \hat{P}_k - pl_k\hat{N} - q_{k\alpha}\hat{S}_\alpha - Al_kl_j\hat{L}_j.$$

Let us consider a special case of $q_{k\alpha} = 0$ and $A = 0$ [12]. The requirement of spatial symmetry

$$\left[\hat{w},\hat{P}_k\right] = 0, \quad \hat{P}_k \equiv \hat{P}_k - p_k\hat{N}, \tag{3.12}$$

means that a macroscopically great number of particles can be in the state with impulse p. The symmetry of the equilibrium state relative to rotations in configurational and spin spaces is not violated and is determined by the formulas

$$\left[\hat{w},\hat{L}_k\right] = 0, \quad \left[\hat{w},\hat{S}_\alpha\right] = 0. \tag{3.13}$$

Relations (3.12) allow one to find the coordinate dependence of the order parameter in the nonhomogeneous equilibrium state

$$\Delta(x) = Sp\hat{w}\hat{\Delta}(x) = \eta(Y,p)\exp 2i\varphi(x),$$

$$\varphi(x) = px + \varphi(0). \tag{3.14}$$

4. EQUILIBRIUM. VECTOR ORDER PARAMETER OF FERMI SUPERFLUID

The vector order parameter is given by formulae [13]

$$\hat{\Delta}_\alpha(x) = \frac{i}{2}\hat{\psi}_{a\mu}(x)(\tau_2)_{aa'}(\sigma_2\sigma_\alpha)_{\mu\mu'}\hat{\psi}_{a'\mu'}(x). \tag{4.1}$$

According with this definition we have

$$\left[\hat{N}_a,\hat{\Delta}_\alpha(x)\right] = -\hat{\Delta}_\alpha(x), \qquad a = 1,2,$$

$$i\left[\hat{S}_\alpha,\hat{\Delta}_\beta(x)\right] = -\varepsilon_{\alpha\beta\gamma}\hat{\Delta}_\gamma(x),$$

$$i\left[\hat{\mathcal{P}}_k, \hat{\Delta}_\alpha(x)\right] = -\nabla_k \hat{\Delta}_\alpha(x),\qquad(4.2)$$
$$i\left[\hat{\mathcal{L}}_k, \hat{\Delta}_\alpha(x)\right] = -\varepsilon_{kjl} x_j \nabla_l \hat{\Delta}_\alpha(x).$$

Residual symmetry generator has the form

$$\hat{T} \equiv a_i \hat{\mathcal{L}}_i + b_\alpha \hat{S}_\alpha + c_a \hat{N}_a .$$

Therefore we get equation

$$\left[i\varepsilon_{\alpha\beta\gamma} b_\gamma - (c_1 + c_2)\delta_{\alpha\beta}\right]\Delta_\beta = 0 .$$

Then we can write

$$(c_1 + c_2)\left[(c_1 + c_2)^2 - b^2\right] = 0 .$$

Possible equilibrium structures of vector order parameter and residual symmetry generators are presented on Table 1:

Table 1

Residual symmetry Generator	**Order Parameter** Δ_α
$a_i \hat{\mathcal{L}}_i + b d_\alpha \hat{S}_\alpha + 2c_1\left(\hat{N}_1 - \hat{N}_2\right)$	Ad_α
$a_i \hat{\mathcal{L}}_i + b\left(d_\alpha \hat{S}_\alpha \pm \frac{1}{2}\left(\hat{N}_1 + \hat{N}_2\right)\right) + 2c_1\left(\hat{N}_1 - \hat{N}_2\right)$	$B(e_\alpha \pm if_\alpha)$

5. HOMOGENEOUS EQUILIBRIUM STATES OF SUPERFLUID ^{3}HE

The order parameter of a superfluid fluid with triplet pairing contains the spin index $\alpha = 1,2,3$ corersponding to the spin angular momentum $s = 1$, and the vectorial index $k = 1,2,3$ relevant, by virtue of Pauli's exclusion principle, to the orbital momentum $l = 1$. As an order parameter operator $\hat{\Delta}_{\alpha k}(x)$ it is convenient to choose [12]

$$\hat{\Delta}_{\alpha k}(x) \equiv \hat{\psi}(x)\sigma_2\sigma_\alpha \nabla_k \hat{\psi}(x) - \nabla_k \hat{\psi}(x)\sigma_2\sigma_\alpha \hat{\psi}(x)\qquad(5.1)$$

Here σ_α – are the Pauli matrices. According to this definition, the following equalities are valid:

$$i\left[\hat{S}_\alpha, \hat{\Delta}_{\beta i}(x)\right] = -\varepsilon_{\alpha\beta\gamma}\hat{\Delta}_{\gamma i}(x),$$
$$\left[\hat{N}, \hat{\Delta}_{\beta i}(x)\right] = -2\hat{\Delta}_{\beta i}(x),\qquad(5.2)$$
$$i\left[\hat{\mathcal{P}}_k, \hat{\Delta}_{\alpha i}(x)\right] = -\nabla_k \hat{\Delta}_{\alpha i}(x),$$
$$i\left[\hat{\mathcal{L}}_k, \hat{\Delta}_{\alpha i}(x)\right] = -\varepsilon_{kjl} x_j \nabla_l \hat{\Delta}_{\alpha i}(x) - \varepsilon_{kil}\hat{\Delta}_{\alpha l}(x).$$

The operator violating the symmetry of equilibrium state represents a linear functional of the order parameter operator

$$\hat{F} = \int d^3x\left(\hat{\Delta}_{\alpha k}(x) f_{k\alpha}(x) + h.c.\right)\qquad(5.3)$$

The quasiaverage value of the order parameter is the function of thermodynamic parameters and the functional of $f_{k\alpha}(x)$

$$\Delta_{\alpha k}(x) = Sp\hat{w}\hat{\Delta}_{\alpha k}(x) = \Delta_{\alpha k}(Y, f(x))\qquad(5.4)$$

By virtue of algebra (5.2) and symmetry relations (4.8)-(4.10) we obtain the equality, defining the equilibrium structure of the order parameter

$$\left(a_i\varepsilon_{ikj}\delta_{\gamma\beta} + b_\alpha \varepsilon_{\alpha\beta\gamma}\delta_{kj} + 2ic\delta_{\gamma\beta}\delta_{kj}\right)\Delta_{\beta j} = 0 .$$

The nonzero solution for the order parameter is provided by the following condition:

$$\det\left|a_i\varepsilon_{ikj}\delta_{\gamma\beta} + b_\alpha \varepsilon_{\alpha\beta\gamma}\delta_{kj} + 2ic\delta_{\beta\gamma}\delta_{kj}\right| = 0.$$

So, we have

$$2ic\left(a^2 - 4c^2\right)\left(b^2 - 4c^2\right)\left[(a-b)^2 - 4c^2\right]\times$$
$$\times\left[(a+b)^2 - 4c^2\right] = 0.$$

The results of classification by using this equation are presented in Table 2. There are described 12 anisotropic phases and one isotropic phase of superfluid ^{3}He homogeneous states.

6. NONUNIFORM EQUILIBRIUM STATES OF SUPERFLUID PHASES ^{3}He

First we shall consider the spatial symmetry subgroups, the generator of which consists of two operators.

Case I: The spatial symmetry generator is

$$\hat{P}_k \equiv \hat{\mathcal{P}}_k - p_k \hat{N} .\qquad(6.1)$$

From this definition and taking into account algebra (5.2)we shall obtain the following equation for the order parameter

$$\nabla_i \Delta_{\beta k}(x) = 2ip_i \Delta_{\beta k}(x) .\qquad(6.2)$$

Its solution has the form

$$\Delta_{\beta k}(x) = e^{2i\varphi(x)}\underline{\Delta}_{\beta k}(0),\quad \varphi(x) = \varphi + px,\qquad(6.3)$$

here $\underline{\Delta}_{\beta k}(0)$ is the homogeneous part of the order parameter independent of the coordinate. The conditions of nonviolated symmetry and the spatial symmetry lead to the equation for $\underline{\Delta}_{\beta k}(0)$ and the constraint of parameters,

$$a_k\varepsilon_{kil}\underline{\Delta}_{\beta l} + b_\alpha \varepsilon_{\alpha\beta\gamma}\underline{\Delta}_{\gamma i} + 2i\underline{c}\underline{\Delta}_{\beta i} = 0,$$
$$a \times p = 0,\qquad(6.4)$$

where $\underline{c} \equiv c + pd$. For the homogeneous part of the order parameter $\underline{\Delta}_{\beta k}(0)$ (6.3), the procedure of classification considered above is valid.

Case II: The operator of spatial symmetry looks like

$$\hat{P}_k \equiv \hat{\mathcal{P}}_k - q_{k\alpha}\hat{S}_\alpha .\qquad(6.5)$$

The requirement of spatial symmetry results in the equation for the order parameter

$$\nabla_i \Delta_{\beta k}(x) = q_{i\alpha}\varepsilon_{\alpha\beta\gamma}\Delta_{\gamma k}(x) .\qquad(6.6)$$

The Jacobi identity for the operators $\hat{w}, \hat{P}_i, \hat{P}_k$ allows one to establish the structure of parameter $q_{i\alpha}$:

$$q_{i\alpha} = q_i n_\alpha .\qquad(6.7)$$

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***Table 2.** Classification of possible equilibrium states with tensor order parameters*

Residual symmetry generator	m_s	m_l	Order parameter	Phase
$\hat{L}_i + R_{i\alpha}\hat{S}_\alpha$	-	-	$\Delta R_{\alpha i}$	B
$\vec{l}\hat{\vec{L}} - \frac{m_l}{2}\hat{N}$ $\vec{d}\hat{\vec{S}} - \frac{m_s}{2}\hat{N}$	± 1 0 ± 1 0	0 ± 1 ± 1 0	$\Delta d_\alpha (m_k \mp i n_k)$ $\Delta (e_\alpha \mp i f_\alpha) l_k$ $\Delta (e_\alpha \mp i f_\alpha)(m_k \mp i n_k)$ $\Delta d_\alpha l_k$	A β A_1 Polar
$\vec{d}\hat{\vec{S}} - 2m_l \lvert m_s \rvert \vec{l}\hat{\vec{L}} - \frac{1}{2} m_s \hat{N}$	0, ± 1 ± 1 0	0 ± 1 ± 1	$d_\alpha (A m_k + B n_k + C l_k)$ $A(m_k \mp i n_k)(e_\alpha \mp i f_\alpha) + B d_\alpha (m_k \mp i n_k)$ $(e_\alpha \mp i f_\alpha)(A m_k + B n_k + C l_k)$	- A_1+A -
$\vec{l}\hat{\vec{L}} - 2m_s \lvert m_l \rvert \vec{d}\hat{\vec{S}} - \frac{1}{2} m_l \hat{N}$	0 ± 1 ± 1	0,± 1 ± 1 0	$(A e_\alpha + B f_\alpha + C d_\alpha) l_k$ $A(m_k \mp i n_k)(e_\alpha \mp i f_\alpha) + B l_k (e_\alpha \mp i f_\alpha)$ $(A e_\alpha + B f_\alpha + C d_\alpha)(m_k \mp i n_k)$	- A_1+β A_2
$\vec{l}\hat{\vec{L}} + \vec{d}\hat{\vec{S}} - \frac{m_l + m_s}{2}\hat{N}$	0,± 1 0 ± 1 ± 1	0,∓ 1 ± 1 0 ± 1	$e_\alpha (A m_k + B n_k) + f_\alpha (-B m_k + A n_k) + C d_\alpha l_k$ $A l_k (e_\alpha \mp i f_\alpha) + B d_\alpha (m_k \mp i n_k)$ $\Delta (e_\alpha \mp i f_\alpha)(m_k \mp i n_k)$	ς ε A_1

***Note.** A,B,C are arbitrary complex numbers*

Here q_k is the magnetic spiral vector, n_α is the axis of anisotropy in the spin space. The equilibrium value of the order parameter with this spatial symmetry has the form

$$\Delta_{\beta k}(x) = a_{\beta\gamma}(n\theta(x))\underline{\Delta}_{\gamma k}(0),$$

$$\theta(x) = \theta + qx, \tag{6.8}$$

where $a_{\beta\gamma}$ is the orthogonal matrix of spin rotation. The requirement of residual symmetry (4.9) with due regard for spatial symmetry (6.5) allows us to obtain the equation for the homogeneous part of the order parameter

$$a_k \varepsilon_{kil} \underline{\Delta}_{\beta l} + \underline{b}_\alpha \varepsilon_{\alpha\beta\gamma} \underline{\Delta}_{\gamma i} + 2ic\underline{\Delta}_{\beta i} = 0,$$

$$\underline{b}_\alpha \equiv b_\alpha + dqn_\alpha$$

and the constraint of the symmetry parameters

$$b \times n = 0, \quad a_j \varepsilon_{jmn} q_n = 0.$$

Case III: The spatial symmetry is defined by the equality

$$\hat{P}_k \equiv \hat{P}_k - t_{kj}\hat{L}_j. \tag{6.9}$$

The requirements of spatial and residual symmetries (6.9) (3.5), the Jacobi identities with due regard for the algebra (4.2) lead to the admissible structure of the matrix t_{ij}:

$$t_{ik} = t l_i l_k. \quad 6.10)$$

The nonhomogeneous part of order parameter is written as

$$\Delta_{\gamma i}(x) = a_{ik}(l\psi(x))\underline{\Delta}_{\gamma k}(0). \qquad (6.11)$$

Here $a_{ik}(l\psi(x))$ is the orthogonal matrix of rotation around the axis l in configurational space by an angle $\psi(x) = \psi + tlx$. This solution describes the helicoidal structure. The parameter $2\pi t^{-1}$ determines a pitch of a helicoid, whose direction is given by the unit vector l. The condition of residual symmetry with account of (6.9),(6.11) leads to the equation for the homogeneous part of order parameter

$$\underline{a}_k \varepsilon_{kil} \underline{\Delta}_{\beta l} + b_\alpha \varepsilon_{\alpha\beta\gamma} \underline{\Delta}_{\gamma i} + 2ic\underline{\Delta}_{\beta i} = 0,$$

$$\underline{a}_i \equiv a_i + tl_i l d$$

and the constraint of the parameters of symmetry

$$\underline{a} \times l = 0.$$

Case IV: The spatial symmetry operator looks like

$$\hat{P}_k \equiv \hat{\mathrm{P}}_k - p_k \hat{N} - q_k n_\alpha \hat{S}_\alpha - tl_j l_k \hat{\mathrm{L}}_j. \qquad (6.12)$$

The structure of the order parameter has the form

$$\Delta_{\beta i}(x) = e^{2i\varphi(x)} a_{\beta\gamma}(\vec{n}\theta(x)) a_{ik}(l\psi(x)) \underline{\Delta}_{\gamma k}(0)$$

The equation for the homogeneous part of the order parameter $\underline{\Delta}_{\gamma k}(0)$ is as follows

$$\underline{a}_i \varepsilon_{ikl} \underline{\Delta}_{\beta l}(0) + \underline{b}_\alpha \varepsilon_{\alpha\beta\gamma} \underline{\Delta}_{\gamma l}(0) + 2i\underline{c}\underline{\Delta}_{\beta k}(0) = 0,$$

$$\underline{a}_i \equiv a_i + tl_i l d, \quad \underline{b}_\alpha \equiv b_\alpha + dq n_\alpha,$$

$$\underline{c} \equiv c + pd.$$

The restrictions on the parameters a_i, b_α, c, d_i of the generator $\hat{T}$ and the parameters $p_k, q_k, n_\alpha, t, l_k$ of the spatial symmetry operator $\hat{P}_k$ result in the collinearity of vectors $p, q, l, \underline{a}$, and also of b, n.

CONCLUSION

It has been demonstrated that nonhomogeneous structures of order parameter can be presented as a product of the nonhomogeneous part of the order parameter dependent on spatial coordinates by the homogeneous part. In the general case, the nonhomogeneous part is the product of orthogonal matrices of rotation in spin and configuration spaces by the oscillation phase term. For the renormalized homogeneous part of order parameter, the traditional procedure of classification is valid.

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