PECULIAR PROPERTIES OF SYSTEMS UNDER SECONDARY RESONANCES WITH AN EXTERNAL PERTURBATION

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This report is about the dynamics of the Hamiltonian systems with two degrees of freedom, in the presence of a low-frequency disturbance. It is proved that a low-frequency disturbance induces a stochastic instability. Besides, a low-frequency disturbance can stimulate the development of a parametric instability. This instability is a new mechanism of transformation of low-frequency oscillation energy to a energy of high-frequency oscillation. As examples of the parametric instability development in distributed systems, we have examined the possibility of the parametric amplification of the x-ray radiation emission in crystals and the possibility of the parametric amplification of the waves propagating in gyrotropic media.

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INTRODUCTION

As it is known, the secondary resonance's can essentially influence dynamics of Hamiltonian systems only if the disturbance amplitude could be large enough (see, for example, [1]). Really, when the disturbance is small ($\varepsilon << 1$), the scale of the effects conditioned by the secondary resonance's also small, being proportional to $1/(1/\varepsilon)!$ (e.g., the secondary nonlinear resonance width). Below it is to be demonstrated that if there exists an external low-frequency disturbance, the influence of the secondary resonance's locked with it could be much more essential.

1. STATEMENT OF THE PROBLEM AND GENERAL EQUATIONS

Let us investigate dynamics of the following system with two degree of freedom, characterized by the Hamiltonian

$$H = H_0(J) + \varepsilon \cdot H_1(J, \theta, t) \qquad (1)$$

We now regard the disturbance as a periodic one over the angular variable. We also suppose that it can be presented as a series:

$$H_1 = \sum_{k,m} h_{k,m}(J,t) \cdot \exp(in\theta).$$

Here $n \cdot \theta = k\theta_1 + m\theta_2$.

Let us consider that the resonance relations $s_1 \cdot \omega_1 = s_2 \cdot \omega_2$ are justified for the undisturbed systems (here s_k denotes integer numbers; $\omega_k = \partial H_0 / \partial J_k$). According to the resonance theory of disturbances, one can apply the following course-of-value function in order to make a transition to new canonical variables:

$$W(\theta, I) = (s_1\theta_1 - s_2\theta_2) I_1 + I_2\theta_2 \ . \qquad (2)$$

It is worth emphasizing that the course-of-value function is explicitly independent of time. Making use of this function, one gets the following expression for the new Hamiltonian:

$$\overline{H}(I, \Psi) = \overline{H}_0(I) + \varepsilon \cdot \sum_{k,m} h_{k,m}(I,t) \cdot \exp\left[i\left(\frac{k}{s_1}\Psi_1 + \left(k \cdot \frac{s_2}{s_1} + m \right) \cdot \Psi_2 \right) \right] \qquad (3)$$

Here I, Ψ denote new canonical variables. The new angular variable $\Psi_1 = s_1\theta_1 - s_2\theta_2$ is changing slowly, while the angular variable $\Psi_2 = \theta_2$ keeps on changing quickly. Let us average the Hamiltonian (3) over the fast angular variable. We also suppose that the coefficients in the expansion of the disturbance are characterized by the following symmetry: $h_{-k,m} = h_{k,-m}$ i.e., the disturbance is real. Thus, the averaged Hamiltonian takes the form:

$$\overline{H} = H_0(I) + \varepsilon \cdot \sum_m h_{-ms_1, ms_2} \cdot 2 \cdot \cos(m\Psi_1) \qquad (4)$$

2. DYNAMICS OF NONDEGENERATE SYSTEMS

Let us suppose that I_0 designates the values of actions, under which the resonance condition is precisely true. Limiting ourselves just to the dominant terms in (4), we get the following simple expression for the Hamiltonian:

$$\overline{H} = \left(\frac{\partial^2 \overline{H}_0}{\partial I_1^2} \right)_{(\vec{I}_0)} \cdot \frac{1}{2} \cdot (\Delta I)^2 + 2\varepsilon \cdot h_{-s_1,s_2}(\vec{I}_0, t) \cdot \cos\Psi_1 . \qquad (5)$$

Here it is taken into account that I_2 is an integral. Hence $\Delta I = I_1 - I_{10}$. The Hamiltonian (5) is the standard one with the time dependent coefficients. Dynamics of the system, characterized by the Hamiltonian (5), is describable by the following equation of the mathematical pendulum:

$$\ddot{\Psi}_1 + \omega_B^2 \cdot \sin\Psi_1 = 0 . \qquad (6)$$

Here $\omega_B^2 = -2\varepsilon \cdot h_{-s_1,s_2}(\vec{I}_0, t) \cdot \left(\frac{\partial^2 \overline{H}_0}{\partial I_1^2} \right)_{(\vec{I}_0)}$.

For simplicity, let us choose the temporal dependence of the disturbance coefficients in the form

$h = h(I_0)\left(1 + \mu \cdot \cos\Omega t\right); \quad h \equiv h_{-s_1, s_2}$. In this case (6) may be presented as:

$$\ddot{\Psi} + \sin\Psi = \frac{\mu}{2}\left[\sin\left(\Psi + \Omega_1 \tau\right) + \sin\left(\Psi - \Omega_1 \tau\right)\right] \tag{7}$$

with $\Psi \equiv \Psi_1; \tau = \omega_B t; \Omega_1 = \Omega / \omega_B$.

Dynamics of the system, describable by Eq. (7), is characterized by three nonlinear resonances. The half-width of the primary one is equal to 2 $(\Delta\dot{\Psi} = 2)$, whereas the half-widths of the two other make $2\sqrt{\mu}$. The distance between these nonlinear resonances is Ω_1. Consequently, if the condition $\Omega_1 < 2\left(1 + \sqrt{\mu}\right)$ is true, the system dynamics becomes chaotic. In particular, this condition indicates the following: if the frequency of the external low-frequency disturbance is on the order of the bound system beat frequency or smaller than the bounce frequency, the disturbed system dynamics is always chaotic, notwithstanding the value of the disturbance amplitude. Surely, one should keep in mind that the chaotic motion characteristics (e.g., the diffusion coefficient) essentially depend on the disturbance amplitude.

It is worth mentioning another important specificity of dynamics of the disturbed system small oscillations. As one can see, equation (6) describes dynamics of the linear pendulum when the oscillation amplitudes are small. If the disturbance frequency is so that the condition for the parametric resonance is satisfied $\Omega = 2\omega_B$, the external low-frequency disturbance induces the parametric instability. In this case, the oscillation amplitudes exponentially increase with the increment $\sim\mu$.

3. DYNAMICS OF DEGENERATED SYSTEMS

Above we have investigated the nondegenerated case, i.e. the second derivative with respect to the undisturbed Hamiltonian is regarded as nonzero. This condition corresponds to the undisturbed frequency dependence on the magnitude of the action. We now examine dynamics of the degenerate system. For such system $\left(\partial^2 \overline{H}_0 / \partial I^2\right)_{(I_0)} = 0$. As it is known, changes in both the magnitude of the action and the angular variable are equally small under the influence of a small disturbance. Let us expand the expression for the Hamiltonian (4) in the close vicinity to the points that correspond to the stationary ones in the absence of the explicit temporal dependence. The expansion is made not only in the action variable but in the angular variable as well. The Hamiltonian derived corresponds to the Hamiltonian of the linear pendulum. It is handy to write the equation of motion of this pendulum as:

$$\dot{\varphi} = A(t) \cdot j; \qquad \dot{j} = B(t) \cdot \varphi . \tag{8}$$

Here $\varphi \equiv \Psi - \Psi_0$ is a small deviation of the angular variable from the stationary state; $j \equiv I_1 - I_{10}$ is a small deviation of the action variable from the value that corresponds to the precise resonance;

$$A = \left[\partial^2 h_{00} / \partial I_1^2 + 2 \cdot \partial^2 h_{-s_1, s_2} / \partial I_1^2\right];$$

$$B = 2h_{-s_1, s_2}.$$

In particular, Eq. (8) indicates the following: if the ratio of the coefficients A and B are time-independent, the system (8) is characterized by an additional integral: $\varphi^2 - c \cdot j^2 = \text{const}$. In this case equation (8) is completely integrable. There do not develop any parametric instability. If the ratio of A to B is time-dependent, than the system (8) is equivalent to the ordinary differential equation of the second order with the coefficients that vary in time:

$$\ddot{z} + \left[-AB + \left(\dot{A}/2A\right)^2 + \ddot{A}/2A\right] \cdot z = 0, \tag{9}$$

here $z = \Psi / \sqrt{A}$.

As equation (9) indicates, there can exist the parametric instability under certain values of the disturbance parameters.

4. COUPLED LINEAR OSCILLATORS

Linear systems make an important example of degenerate systems. For instance, let us examine a system that consists of two coupled linear oscillators. The system Hamiltonian has the form:

$$\begin{aligned} H = {} & \frac{1}{2} \cdot \sum_{k=1}^{2}\left[a_k \cdot p_k^2 + b_k \cdot q_k^2\right] + \\ & + \frac{\varepsilon}{2} \cdot \sum_{k=1}^{2}\left[\alpha_k(t) \cdot p_k^2 + \beta_k(t) \cdot q_k^2\right] + \\ & + \varepsilon \cdot \left[c(t) \cdot p_1 \cdot p_2 + d(t) \cdot q_1 \cdot q_2\right]. \end{aligned} \tag{10}$$

In (10) the first sum describes the system of two independent oscillators. The second sum describes their small disturbance. The third addendum describes the small coupling between the oscillators. It is easy to show that in the system of the ordinary differential equations of the (8)-type, which corresponds to the Hamiltonian (10), the ratio of the coefficients A to B is independent of time. Thus, the system of coupled linear oscillators, describable by the Hamiltonian (10), acquires intricate temporal dynamics under the influence of the time-dependent disturbance. However, the conditions for the parametric instability are unrealizable under the action of an external low-frequency disturbance. Whether always it so? Below we shall see, that for non-stationary systems with two degree of freedoms with imperfect connections it not so. Besides in the distributed systems also it is possible to realize the conditions for development of parametrical instability. Let's illustrate the formulated statement on the elementary example. This example can be a system of two connected identical oscillators, the connection between which depends on time and is unmutual. Set of equations which describes dynamics of such oscillators is possible to write as:

$$\ddot{x}_1 + x_1 = \mu_1(t) \cdot x_2$$

$$\ddot{x}_2 + x_2 = \mu_2(t)\cdot x_1 . \tag{11}$$

If $(\mu_1/\mu_2) \neq 1$ the system (11) cannot have Hamiltonian (10) (Hamiltonian for a system (11) in general case can be written in an extended phase space). The reason of absence of a Hamiltonian consists that the virtual work of connections is not equal to zero, i.e. the connection in this case are imperfect. By a physical example of a system, which can be circumscribed by a set of equations (11), is dynamics of fields of two connected identical resonators. The connections between these resonators are different. Such connections can be realized, for example, with the help of channels, which have gyrotropic insertions. Let coupling coefficients look like $\mu_i = \alpha_i + \beta_i \cdot \cos(\gamma\cdot t)$, where α_i, β_i - constants. Than, if condition of parametrical instability $\gamma = 2\Omega$ $(\Omega = \sqrt{\alpha_1 \cdot \alpha_2}\,/\,2)$ is fulfilled, system (11) has solution $a_1 \sim a_2 \sim \exp(\Phi\, t)$, where $\Phi = (\beta_1/2\cdot\alpha_1)$.

5. SYSTEMS WITH AN INFINITE NUMBER OF THE DEGREES OF FREEDOM

Thus, in the linear system with two degrees of freedom does not exist the mechanism for the parametric pumping of the low-frequency oscillation energy to the energy of high-frequency motions. For obtaining the energy pumping in this system, an element of nonlinearity is required. At the same time, for the most interesting cases (e.g., the stimulation of the X-ray radiation emission), the magnitudes of nonlinearity are vanishingly small. Seemingly, the systems with nonideal constraints possess the necessary properties. However, these systems are rather rare. The system with a large number of the degrees of freedom must possess the necessary properties. Therefore, it is of practical interest to determine the conditions for this parametric amplification in the system with a large number of the degrees of freedom. Below we will demonstrate that there does really take place the amplification of this kind in distributed systems.

To prove the existence of this possibility, let us examine a medium where occurs interaction between two wave processes. For instance, one can talk about the three-wave interaction in the approximation of a constant value of the amplitude of one of the waves (the pumping wave amplitude). Besides, we also suppose that the medium parameters are slowly changing in space and in time. Specific examples will be given below. The equations that describe evolution of the wave amplitudes may be presented as:

$$\alpha_0 \frac{\partial A_0}{\partial z} + \mu_0 \frac{\partial A_0}{\partial \tau} = \frac{1}{2i}\left[Q_0 A_0 + \frac{q_0}{2} A_1\right]$$
$$\alpha_1 \frac{\partial A_1}{\partial z} + \mu_1 \frac{\partial A_1}{\partial \tau} = \frac{1}{2i}\left[Q_1 A_1 + \frac{q_0}{2} A_0\right]. \tag{12}$$

Here A_i denote the slowly-varying amplitudes of the waves that interact with one another; q_i designate the constants that determine the wave coupling; $Q_i = Q_i(z,\tau)$ prescribed functions, slowly varying in time and space. Description of a rather large number of physical processes is reducible to the system (12).

First, let us determine the most general conditions under which the system (12) can possess parametrically-increasing solutions. For this purpose, let us rewrite the system (12) in partial derivatives in the form of the system of characteristic equations:

$$\frac{dA_0}{dz} = \frac{1}{2i\alpha_0}\left[Q_0 \cdot A_0 + \frac{q_0}{2} A_1\right]; \quad \frac{d\tau}{dz} = \frac{\mu_0}{\alpha_0};$$
$$\frac{dA_1}{dz} = \frac{1}{2i\alpha_1}\left[Q_1 \cdot A_1 + \frac{q_1}{2} A_0\right]; \quad \frac{d\tau}{dz} = \frac{\mu_1}{\alpha_1}. \tag{13}$$

In (13) the first pair of equations is equivalent to the first equation in the system (12), whereas the second pair is equivalent to the second equation in equation (12). As it is easy to see, if the functions Q_0 and Q_1 coincide and if the characteristics are equal $(\mu_0/\alpha_0 = \mu_1/\alpha_1)$, the system (13) is completely integrable. As well as in the system with two degrees of freedom, in Eq.(13) does not exist any parametric instability. In this case, the presence of low-frequency disturbances results just in a certain complication in dynamics of the amplitudes A_i. It is also evident that even if one of this conditions (either the equality of the characteristics or coincidence of the functions Q_0 and Q_1) is not fulfilled, generally speaking, the system equation (12) (or 13)) can possess the solution that describes the parametric amplification.

Let us now proceed to a more detail determination of the conditions for the parametric amplification. Let us suppose that the coefficients Q_i are periodic functions of the time τ and coordinate Z. We choose this dependence in the form $Q_i = \varepsilon_i \cdot \cos(K z - \tau)$. Hence, one can look for the solution to the system (12) in the following form:

$$A_i = \sum a_{i,n} \cdot \exp(in\tau) . \tag{14}$$

In order to find the Fourier amplitudes $a_{i,n}$, we substitute the solution (14) into (12). Thus, one gets the following infinite coupled system of ordinary differential equations of the first order:

$$\alpha_0 \frac{\partial a_{0,n}}{\partial z} + in\mu_0 \cdot a_{0,n} = \frac{q_0}{4i} a_{1,n} +$$
$$+ \frac{\varepsilon_0}{4i}\left[a_{0,n+1}\cdot \exp(iK z) + a_{0,n-1}\cdot\exp(-iK z)\right]$$
$$\alpha_1 \frac{\partial a_{1,n}}{\partial z} + in\mu_1 \cdot a_{1,n} = \frac{q_1}{4i} a_{0,n} +$$
$$+ \frac{\varepsilon_1}{4i}\left[a_{1,n+1}\cdot \exp(iK z) + a_{1,n-1}\cdot\exp(-iK z)\right]. \tag{15}$$

This system useful to rewrite as

$$v_n'' + \lambda_n^2 v_n = \varepsilon_0 F_n . \tag{16}$$

Here

$$v_n = a_{0,n}\cdot \exp(iR_n z/2),$$

$$B_n = \left[\frac{q_1 q_0}{16 \cdot \alpha_0 \alpha_1} - n^2 \frac{\mu_0 \mu_1}{\alpha_0 \alpha_1} \right],$$

$$R_n = n \left[\frac{\mu_0}{\alpha_0} + \frac{\mu_1}{\alpha_1} \right],$$

$$\lambda_n^2 = B_n + R_n^2 / 4,$$

$$D = \left[1 + \left(\varepsilon_1 \cdot \alpha_0 \right) / \left(\varepsilon_0 \cdot \alpha_1 \right) \right],$$

$$F_n = \frac{1}{4i\alpha_0} \left\{ \begin{array}{l} v_{n+1} \cdot \exp\left(iK z - iR_{n+1} z/2 \right) \cdot \\ \cdot \left[C_n^+ - \frac{i}{2} D R_{n+1} \right] + v_{n-1} \cdot \\ \cdot \exp\left(- iK z - iR_{n-1} z/2 \right) \cdot \\ \cdot \left[C_n^- - \frac{i}{2} D R_{n-1} \right] + \\ + D \left[\begin{array}{l} v'_{n+1} \cdot \exp\left(iK z - iR_{n+1} z/2 \right) + \\ v'_{n-1} \cdot \exp\left(- iK z - iR_{n-1} z/2 \right) \end{array} \right] \end{array} \right\},$$

$$C_n^{\pm} = i \left[\pm K + n \frac{\mu_1}{\alpha_1} + \frac{\varepsilon_1 \alpha_0}{\varepsilon_0 \alpha_1} \frac{(n \pm 1)}{\alpha_0} \mu_0 \right].$$

The system (16) describes an infinite system of linear coupled oscillators. Their coupling is conditioned by the presence of a small ($\varepsilon_i << 1$) slow and periodic change in the parameters of the medium where takes place the wave interaction. We now look for the solution to the system (16) in the following form:

$$v_n = w_n(z) \cdot \exp\left(i\lambda_n z \right) + \\ + \varsigma_n(z) \cdot \exp\left(- i\lambda_n z \right) + \varepsilon_0 V(z) \quad (17)$$

Let us substitute (17) into system equation (16), imposing the condition of the absence of driving resonant forces in the equation for the function $V(z)$. If the resonance conditions

$$K = \lambda_n - \lambda_{n+1} - \frac{1}{2}\left(R_n - R_{n+1} \right) \quad (18)$$

are satisfied, than w_n are joint with w_{n+1} and ς_n are joint with ς_{n+1}. The equations for each of the amplitudes w_n or ς_n are independent of the amplitudes characterized by other numbers. For instance, to determine the function w_n, one can derive the following ordinary differential equation of the second order with constant coefficients:

$$w''_n + G \cdot w_n = 0. \quad (19)$$

Here

$$G \equiv \frac{(-1) \cdot \varepsilon_0^2}{64 \cdot \alpha_0^2 \cdot \lambda_n \lambda_{n+1}} \cdot \\ \cdot \left[C_n^+ - \frac{i}{2} D \cdot \left(R_{n+1} - 2\lambda_{n+1} \right) \right] \cdot \\ \cdot \left[C_{n+1}^- - \frac{i}{2} D \cdot \left(R_n - 2\lambda_n \right) \right].$$

As equation (19) indicates, when the system parameters are so that G<0, than the functions w_n will exponentially increase. The field Fourier amplitudes $a_{i,n}$ will be increased too.

Let us suppose that another resonance condition

$$K = \lambda_n + \lambda_{n+1} - \frac{1}{2}\left(R_n - R_{n+1} \right) \quad (20)$$

is fulfilled. Then the condition of the absence of the resonant driving terms on the right of the equation for V(z) lead to the joint between the functions w_n and ς_{n+1}. As well as in the previous resonance case, for determining the function w_n, one can derive the ordinary differential equation of the second order, analogous with (19):

$$w''_n + G_1 \cdot w_n = 0. \quad (21)$$

Here

$$G_1 \equiv \frac{(+1) \cdot \varepsilon_0^2}{64 \cdot \alpha_0^2 \cdot \lambda_n \lambda_{n+1}} \cdot \\ \cdot \left[C_n^+ - \frac{i}{2} D \cdot \left(R_{n+1} + 2\lambda_{n+1} \right) \right] \cdot \\ \cdot \left[C_{n+1}^- - \frac{i}{2} D \cdot \left(R_n - 2\lambda_n \right) \right].$$

As well as in the previous resonance case if $G_1 < 0$ there takes place amplification of the radiation emitted.

6. PARAMETRIC AMPLIFICATION OF THE X-RAYS IN CRYSTAL

Our goal is to demonstrate that the process of scattering of X-rays by the perfect crystals is reducible to the case described above when the crystal susceptibility is modulated by a low-frequency disturbance.

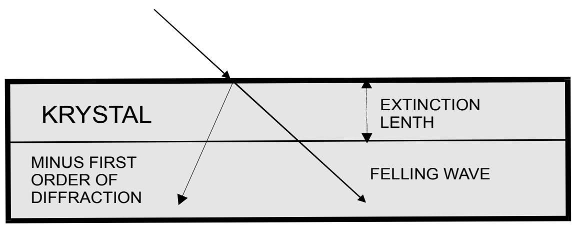

Fig. 1.

We now suppose that the crystal is located in the lower half-space z>0 (see Fig. 1) and its susceptibility is describable by the formula:

$$\chi = \chi_0 + q\cos(\kappa \cdot r) + \varepsilon \cdot \cos(K \cdot z - \Omega t) \quad (22)$$

Here κ denotes the vector of the inverse grate of the crystal; $q \sim \chi_0 >> \varepsilon$ - designates the degree of the spatially-temporal periodic inhomogeneity, induced by an external source; $K << |\kappa|$.

For describing the process of scattering of X-rays by this crystal, let us limit ourselves to the framework of the two-wave dynamic theory of diffraction. In this case, the field in the crystal can be presented as

$$\vec{E} = \sum_{j=0}^{1} A_j(\vec{r},t)\cdot \exp(-i\vec{k}_j\vec{r} + i\omega t) \qquad (23)$$

Here $k_1 = k_0 + \kappa$; $\quad k_1^2 = k_0^2 = \omega^2/c^2$.

Now let us consider the incident radiation to be completely unlimited in the direction transverse to z-axis, i.e. we neglect the effects, conditioned by bounded crystal and incident radiation in the transverse direction. Besides, we suppose that $\Omega << \omega$. Thus, changes in amplitudes of the interacting waves depend only on time and z-coordinate. Maxwell's equations readily yield the averaged equations that describe dynamics of changes in these amplitudes. In the dimensionless variables $\tau = \Omega t; z = k_0 z = \omega z/c$, these equations can be written in the following form:

$$\begin{aligned} &\alpha_0 \frac{\partial A_0}{\partial z} + \mu \frac{\partial A_0}{\partial \tau} = \frac{1}{2i}(Q\cdot A_0 + \frac{q}{2}A_1), \\ &\alpha_1 \frac{\partial A_1}{\partial z} + \mu \frac{\partial A_1}{\partial \tau} = \frac{1}{2i}[(Q+2\delta)A_1 + \frac{q}{2}A_0], \end{aligned} \qquad (24)$$

here

$$\mu \equiv \frac{\Omega}{\omega} \sim q;$$

$$Q \equiv \chi_0 + \varepsilon\cdot\cos(K z - \tau);$$

$$\alpha_i \equiv \frac{k_{iz}}{k_0} = \cos\theta_i;$$

$$k_o^2 - k_1^2 = 2\delta\cdot k_0^2 .$$

If $\mu = \varepsilon = 0$, the system equation (24) takes the form of the well-known one (e.g., see [2]), which possesses pendular solutions. We also suppose that the Bragg resonance conditions are precisely satisfied i.e., there is now Bregg detuning ($\delta = 0$). Besides, the wave numbers are chosen so that it corresponds to the wave propagation not in vacuum (as in (23)) but in the medium characterized by the susceptibility χ_0, that is, $k_1^2 = k_0^2 = \omega^2/c^2\cdot(1-\chi_0)$. In this case, the system (24) completely coincides with the system (12), where one must equate: $\mu_0 = \mu_1 = \mu$, $Q_0 = Q_1 = Q$; $q_0 = q_1 = q$.

7. WAVE PROPAGATION IN GYROTROPIC MEDIA

As it is known, in generally case, E and $H-$ waves are coupled in boundless gyrotropic media. The total electromagnetic wave field in these media possesses all components of the electric and magnetic fields. We now examine a gyrotropic half-space, on the boundary of which, for instance, the E-wave field (E_x, H_y, E_z) is determinate. As the wave is propagating deep into the gyrotropic medium, the E-wave field energy completely transfers into the H-wave energy (H_x, E_y, H_z) (see Fig. 2). Looking like rotation of the plane of the wave polarization, this energy transformation represents the Faraday effect. Thus, the Faraday effect can be presented as a result of dynamics of two coupled waves. It is worth mentioning that the resonance of the interaction between E and H-waves plays the part of the primary resonance in this model. In addition, if the components of the tensor of the dielectric or magnetic susceptibility are periodic functions of the coordinate and time, in this system secondary resonances can effectively indicate themselves. Moreover, it is possible to show, that dynamics of amplitudes Å and Í waves is described by a set of equations (12) and one can use external low-frequency wave for amplification high-frequency waves.

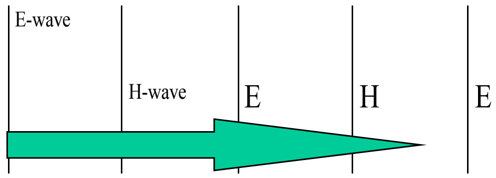

Fig. 2

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