METHOD OF REDUCED DESCRIPTION IN THEORY OF LONG WAVE NONEQUILIBRIUM FLUCTUATIONS

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The regular method for construction of kinetic equations of long-wave fluctuation theory is developed in a microscopic approach on the base of generalization of the kinetic Bogolyubov theory. The transition to the hydrodynamic theory of long-wave fluctuations is investigated in detail. The derived hydrodynamic equations describe a turbulent liquid state. The concept of nonequilibrium entropy for fluctuating systems is introduced. The H-theorem is proved.

PACS: 05.20.y, 05.20.Dd, 05.40.+j, 82.20.M

INTRODUCTION

It is well known [1-3] that for the evolution process of a system at times $t \gg \tau_r$ (a hydrodynamic evolution stage; τ_r is the relaxation time) the states in which the correlation radius of many-particles distribution functions increases with time (long-wave fluctuations) inevitably appear. In this connection it is of great interest to construct a long fluctuation kinetic theory which must underlie a long fluctuation hydrodynamic theory just as the usual kinetic theory underlies hydrodynamics.

A set of works (see, for example [4,5], etc.) deals with the long fluctuation kinetic theory. In the works the main attention was focused on the derivation of the specific equations for a one-particle and a binary correlation function in a certain approximation. In such an approach the basic ideas of the kinetic Bogolyubov theory [6] (the functional hypothesis; the boundary conditions problem representing a formulation of the functional hypothesis in zeroth-order of perturbation theory; the principle of spatial weakening correlations) used to be lost. Besides, the ideas did not seem to work in the long fluctuation theory. One of the basic objects of the present work is to show that the long fluctuation kinetic theory not only agrees with the general ideas of the kinetic Bogolyubov theory but needs the latter for its specific applications.

1. METHOD OF REDUCED DESCRIPTION OF LONG NON-EQUILIBRIUM FLUCTUATIONS

In studying the kinetics of long-wave fluctuations it is necessary to deal with systems having a large correlation radius increasing with time so that the assumption forming the basis for [6] about rapid decays of correlations at $|\mathbf{x}_i - \mathbf{x}_j| \geq r_0$ (r_0 is a particle interaction radius) is not fulfilled. Therefore, for such systems the formalism of [6] has to be modified in a certain way.

With the purpose of formulation of modified functional hypothesis we introduce smoothed s-particle distribution functions $\underline{f}_S(x_1,...,x_S;t)$ which are obtained from usual many-particle distribution functions $f_S(x_1,...,x_S;t)$ in going to the asymptotic domain $|\mathbf{x}_i - \mathbf{x}_j| \geq r_0$,

$$f_S(x_1,...,x_s;t) \xrightarrow[|\mathbf{x}_i - \mathbf{x}_j| >> r_0]{} \underline{f}_S(x_1,...,x_s;t) \equiv$$
$$\equiv Pf_S(x_1,...,x_s;t) \qquad (1.1)$$

(here $x_i \equiv (\mathbf{x}_i, \mathbf{p}_i)$ is the coordinate of the phase point of the i-th particle, i = 1,2,...,S; P is a symbol of the smoothing operation).

We explain the concept of smoothing operation in detail. If the initial many-particle distribution functions have been smooth (on the scale of r_0) functions of x_i (i = 1,2,...,S), then on account of the temporal evolution the functions $f_S(x_1,...,x_S;t)$ will have at $|\mathbf{x}_i - \mathbf{x}_j| < r_0$, (i,j = 1,…,S) a complex irregular character displaying irregular properties on scale r_0 of the potential energy of the S-particle interaction. The trend of this irregular dependence can be shown by way of example of the function f(x) (x stands for $|\mathbf{x}_i-\mathbf{x}_j|$) having two spatial scales of variation, $f(x) = f(x/r_0, x/L)$ (r_0 is the characteristic microscopic scale of the variation of f(x) on small distances, $L \gg r_0$ is a characteristic macroscopic scale of the variation on large distances). It is the function which one deals with solving the BBGYK equation chain (). Then the smoothing operation of the function f(x) is defined by the formula

$$\underline{f}(x) = Pf(x) \equiv f\left(\infty, \frac{x}{L}\right), \quad P^2 = 1 \qquad (1.2)$$

According to the reduced description method of Bogolyubov [6] we will consider that a system state is completely described by smoothed many-particle distribution functions at times $t \gg \tau_0$ ($\tau_0 \approx r_0/v$ is the characterization time; v is the average particle velocity). It means that the exact many-particle distribution functions f_S will be dependent on time and initial many-particle distribution functions only by smoothed many-particle distribution functions at $t \gg \tau_0$,

$$f_S(x_1,...,x_S;t) \xrightarrow[t>>\tau_0]{} $$
$$\xrightarrow[t>>\tau_0]{} f_S\left(x_1,...,x_S;\underline{f}_1(t),\underline{f}_2(t),...\right). \qquad (1.3)$$

Thus although the distribution functions $f_S(x_1,...,x_S;t)$ do depend, generally speaking, on the initial many-particle distribution functions $f_S(x_1,...,x_S;0)$, at times well in excess of τ_0 the dependence is simplified and contained only functions $\underline{f}_1(t)$, $\underline{f}_2(t)$, ..., the functionals of which become the quantities of f_S.

In this sense the functionals (1.3) are universal and independent of the pattern of initial conditions for many-particle distribution functions.

The considered functionals (1.3) in accordance with (1.1) have to satisfy the following relationship:

$$f_S\left(x_1,...,x_S;\underline{f}_1(t),\underline{f}_2(t),...\right) \xrightarrow[|\mathbf{x}_i-\mathbf{x}_j|>>r_0]{}$$
$$\xrightarrow[|\mathbf{x}_i-\mathbf{x}_j|>>r_0]{} Pf_S\left(x_1,...,x_S;\underline{f}_1(t),\underline{f}_2(t),...\right) \equiv$$
$$\equiv \underline{f}_S(x_1,...,x_S;t) \qquad (1.4)$$

We will study system dynamics having proceeded from the BBGYK equation chain for the many-particle distribution functions

$$\frac{\partial f_S}{\partial t} = -i\Lambda_S f_S + R_S f_{S+1}, \quad \Lambda_S = \Lambda_S^0 + \Lambda_S^1 \qquad (1.5)$$

Here operators Λ_S and R_S are defined by formulae

$$-i\Lambda_S^0 f_S = \left\{H_S^0, f_S\right\} \equiv \sum_{1\le i\le S} \frac{\mathbf{p}_i}{m}\frac{\partial f_S}{\partial \mathbf{x}_i}, \qquad (1.6)$$

$$-i\Lambda_S^1 f_S = \left\{V_S, f_S\right\} \equiv$$
$$\equiv \sum_{1\le i<j\le S} \frac{\partial V(\mathbf{x}_i-\mathbf{x}_j)}{\partial \mathbf{x}_i}\left(\frac{\partial}{\partial \mathbf{p}_i} - \frac{\partial}{\partial \mathbf{p}_j}\right),$$

$$R_S f_{S+1} = \int dx_{S+1}\left\{\sum_{1\le i\le S} V_{i,S+1}, f_{S+1}\right\} \equiv$$
$$\equiv \sum_{1\le i\le S+1} \int \frac{\partial V(\mathbf{x}_i-\mathbf{x}_{S+1})}{\partial \mathbf{x}_i}\frac{\partial f_{S+1}}{\partial \mathbf{p}_i}$$

the operator R_S transforms a function of a phase space of S+1 particles in one of a phase space of s particle), where $H_S^0=\sum_{1\le l\le S}H(x_l)$, $V=\sum_{1\le l<j\le S}V(\mathbf{x}_l-\mathbf{x}_j)$, $H(x_l)=p_l^2/2m$ is the free particle Hamiltonian and $V_{lj}=V(\mathbf{x}_l-\mathbf{x}_j)$ is the Hamiltonian of pair interaction between particles (the symbol {,} denotes Poisson's brackets).

It is easy to obtain an equation of motion for the smoothed distribution functions $\underline{f}_S$ from the equation chain for many-particle distribution functions. For this purpose< going in (1.5) to the asymptotic region $|\mathbf{x}_i-\mathbf{x}_j|>>r_0$ (i,j = 1,...,S) and allowing for

$$V(\mathbf{x}_i-\mathbf{x}_j) \xrightarrow[|\mathbf{x}_i-\mathbf{x}_j|\to\infty]{} 0,$$

we have accordingly (1.2)

$$\frac{\partial}{\partial t}\underline{f}_S = \mathrm{L}_S^0 + L_S \equiv \mathrm{L}_S,$$
$$\mathrm{L}_S^0 = -i\Lambda_S^0 \underline{f}_S, \quad L_S = PR_S f_{S+1}, \qquad (1.7)$$

(we took into account that $P\{V_S,f_S\}=0$).

A further problem will be to find a solution of the equation chain (1.5) in the form $f_S=f_S(x_1,...,x_S;\underline{f}_1,\underline{f}_2,...)$ not researching the initial evolution stage. To obtain a unique solution of the problem we formulate for the functionals (1.3) boundary conditions which in accord with (1.4), (1.6) take the form

$$S_S^0(\tau)f_S(x_1,...,x_S;S_1^0(-\tau)\underline{f}_1(t),S_2^0(-\tau)\underline{f}_2(t),...)$$
$$\xrightarrow[\tau\to\infty]{} \underline{f}_S(x_1,...,x_S;t). \qquad (1.8)$$

Here $S_S^0(\tau)=\exp(i\tau\Lambda_S^0)$.

To obtain the equation of motion for the parameters $\underline{f}_S(t)$ of reduced description it is necessary (see (1.7), (1.8)) to find an apparent form of the functionals $f_S(x_1,...,x_S;\underline{f}_1,\underline{f}_2,...)$ in a certain approximation, i.e. to solve Eq. (1.5) with allowance for (1.3), (1.8). In such cases the usual ways of calculation are iterations over a weak interaction (for an arbitrary particle density) or a low density of particles (for an arbitrary interaction between particles provided that the interaction does not lead to the production of bound states of particles).

2. GENERAL KINETIC EQUATION OF LONG FLUCTUATION THEORY

Introduce the generating functional of smoothed many-particle distribution functions

$$F(u;\underline{f}) = 1 + \qquad (2.1)$$
$$+\sum_{S=1}^{\infty}\frac{1}{S!}\int dx_1...\int dx_S u(x_1)...u(x_S)\underline{f}_S(x_1,...,x_S).$$

A functional G(u;g) connected with the functional $F(u;\underline{f})$ by the relationship

$$F(u,\underline{g}) = exp\left[G(u,\underline{g})\right] \qquad (2.2)$$

is the generating functional of smoothed correlation functions $\underline{g}_S(x_1,...,x_S)$,

$$G(u;\underline{g}) = \qquad (2.3)$$
$$= \sum_{S=1}^{\infty}\frac{1}{S!}\int dx_1...\int dx_S u(x_1)...u(x_S)\underline{g}_S(x_1,...,x_S),$$

where $g_1 \equiv f$. Represent the generating functional $G(u;\underline{g})$ in the form

$$G(u;\underline{g}) = \int dx u(x) f(x) + \mathrm{G}(u;\underline{g}), \qquad (2.4)$$

where $\mathrm{G}(u;g)$ is the generating functional of the proper correlation function

$$\mathrm{G}(u;\underline{g}) = \qquad (2.5)$$
$$= \sum_{S=2}^{\infty}\frac{1}{S!}\int dx_1...\int dx_S u(x_1)...u(x_S)\underline{g}_S(x_1,...,x_S).$$

The kinetic equation for the one-particle distribution function f(x) and generating functional $\mathrm{G}(u;g)$ is to be produced in the form

$$\frac{\partial f(x)}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f(x)}{\partial \mathbf{x}} = exp\left[\mathrm{G}\left(\frac{\delta}{\delta f};\underline{g}\right)\right]L(x;f), \qquad (2.6a)$$

$$\frac{\partial \mathrm{G}(u;\underline{g})}{\partial t} + \int dx u(x)\frac{\mathbf{p}}{m}\frac{\partial}{\partial \mathbf{x}}\frac{\delta \mathrm{G}(u;\underline{g})}{\delta u(x)} =$$
$$= \left\{exp\left[\mathrm{G}\left(u+\frac{\delta}{\delta f};\underline{g}\right) - \mathrm{G}(u;\underline{g})\right] - exp\left[\mathrm{G}\left(\frac{\delta}{\delta f};\underline{g}\right)\right]\right\}$$
$$\times \int dx u(x) L(x;f), \qquad (2.6b)$$

where $G(\delta/\delta f;g)$ is generating functional $G(u;g)$ in which an operation of functional differentiation over $\underline{f}(x)$ is substituted instead of a functional argument u(x). It is the equations (2.6) which are general equations of the theory of long nonequilibrium fluctuations. As is known, a usual kinetic equation for the one-particle distribution function takes the form

$$\frac{\partial f(x)}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f(x)}{\partial \mathbf{x}} = L(x;f). \quad (2.7)$$

We see that the dynamics of long nonequilibrium fluctuations is determined by the functional L(x;f) - collision integral of Bogolyubov's kinetic theory. It agrees with Onsager's principle according to which macroscopic (long) fluctuations evolve in time with laws of macroscopic physics; such a law of macroscopic physics in case of kinetic theory is Eq. (2.7).

The second of Eqs. (2.6) admits the solution $G=0$. With that, the equations becomes (2.7). However, the solution $G=0$ corresponds to the very special initial conditions $g_S|_{t=0}=0$ (S=2,3...). If at the initial moment the correlation functions $g_2(x_1,x_2)=f(x_1)f(x_2)\xi(x_1,x_2),...$, are small in comparison with $f(x_1)f(x_2)$ ($|\xi(x_1,x_2)|<<1$), then the one-particle distribution function f(x) exponentially decays up to times $\tau_0=\tau_r\ln[1/|\xi(x_1,x_2)|]$ ($\tau_r=l/v$ is relaxation time, l is a mean free path) according to the standard kinetic equation (2.7).The general kinetic equations (2.6) are decisive at $t\geq\tau_0$.

We note that the functional hypothesis in the form (1.3)was decisive for the construction of the long fluctuation theory. However, for the construction of the usual kinetic theory, in which only the one-particle distribution function is a parameter of the reduced description, the functional hypothesis is formulated as

$$f_S(x_1,...,x_S;t) \xrightarrow[t>>\tau_0]{} f_S(x_1,...,x_S;f(x',t)), \quad (2.8)$$

where $f_S(x_1,...,x_S;f)$ are universal functionals of one-particle distribution functions. In our opinion the functional hypothesis (1.3) describes a general situation corresponding to arbitrary initial conditions while the hypothesis in the form (2.8) is not valid in the general case and corresponds to very special initial conditions which are described by the universal functionals $f_S(x_1,...,x_S;f)$ where an arbitrary initial one-particle distribution function f(x;0) figures as f(x):

$$f_S(x_1,...,x_S;0) = f_S(x_1,...,x_S;f(x';0)).$$

It should be noted as well that stationary solutions of Eqs. (2.6) for a statistical equilibrium state take the form

$$\underline{g}_S = 0, \quad S = 2,3,..., \quad f = f_0.$$

It means there are no long fluctuations in the statistical equilibrium state. Short-wave fluctuations in the equilibrium state are determined by the functionals $f_S(x_1,...,x_S;f)$ in which the Maxwell distribution f_0 has to be substituted for a functional argument f(x). As is shown in [8], many-particle distribution functions obtained by such way completely coincide with the many-particle distribution Gibbs' functions.

3. GENERAL HYDRODYNAMIC EQUATION OF LONG FLUCTUATION THEORY

Smoothed hydrodynamic averages of products of the additive motion integral,

$$\zeta_{\alpha_1...\alpha_S}(\mathbf{x}_1,...,\mathbf{x}_S;t) = \quad (3.1)$$

$$= \int d\mathbf{p}_1...\int d\mathbf{p}_S\zeta_{\alpha_1}(\mathbf{p}_1)...\zeta_{\alpha_S}(\mathbf{p}_S)\underline{f}_S(x_1,...,x_S;t),$$

will be denoted by $\zeta_a(t)$ (here $\zeta_\alpha(\mathbf{p})$ (α=0,i,4;i=1,2,3) are additive integral of motion; $\zeta_0(\mathbf{p})=p^2/2m$ is an energy, $\zeta_i(\mathbf{p})=p_i$ is a momentum, $\zeta_4(\mathbf{p})=m$ is a particle mass). It is easy to see that the generating functional $F(v;\zeta_a)$ of the smoothed hydrodynamic averages $\zeta_{\alpha_1...\alpha_S}(\mathbf{x}_1,...,\mathbf{x}_S)$ is connected with the generating functional $F(u;\underline{f})$ of the smoothed many-particle distribution function $\underline{f}_S$ by the formula

$$F(v;\zeta_a) = F(u;\underline{f})\Big|_{u(x)=v_\alpha\zeta_\alpha(\mathbf{p})} = \quad (3.2)$$

$$1+\sum_{S=1}^{\infty}\frac{1}{S!}\int d\mathbf{x}_1...\int d\mathbf{x}_S v_{\alpha_1}(\mathbf{x}_1)...v_{\alpha_S}(\mathbf{x}_S)\times$$

$$\times\zeta_{\alpha_1...\alpha_S}(\mathbf{x}_1,...,\mathbf{x}_S;t)$$

(it has extended the summation over repeated index "α"). The functional $G(v;\xi_a)$ connected with the functional $F(v;\zeta_a)$ by the relationship

$$F(v;\varsigma_a) = \exp[G(v;\xi_a)] \quad (3.3)$$

is the generating functional of the hydrodynamic correlation functions $\xi_{\alpha_1...\alpha_S}(\mathbf{x}_1,...,\mathbf{x}_S)$,

$$G(v;\xi_\alpha) = \sum_{S=1}^{\infty}\frac{1}{S!}\int d\mathbf{x}_1...\int d\mathbf{x}_S v_{\alpha_1}(\mathbf{x}_1)...v_{\alpha_S}(\mathbf{x}_S)\times$$

$$\times\xi_{\alpha_1...\alpha_S}(\mathbf{x}_1,...,\mathbf{x}_S;t), \quad (3.4)$$

which will be denoted by $\xi_a(t)$ ($\xi_\alpha(\mathbf{x})\equiv\zeta_\alpha(\mathbf{x})=\int d\mathbf{p}\zeta_\alpha(\mathbf{p})f_1(x)$ are densities of additive motion integrals). The generating functional $G(u;g)$ of the proper correlation function $\xi_{\alpha_1...\alpha_S}(\mathbf{x}_1,...,\mathbf{x}_S;t)$, $S\geq2$ similarly to (2.5) have the form

$$G(v;\xi_\alpha) = \sum_{S=2}^{\infty}\frac{1}{S!}\int d\mathbf{x}_1...\int d\mathbf{x}_S v_{\alpha_1}(\mathbf{x}_1)...v_{\alpha_S}(\mathbf{x}_S)\times$$

$$\times\xi_{\alpha_1...\alpha_S}(\mathbf{x}_1,...,\mathbf{x}_S;t), \quad (3.5)$$

To obtain the closed equation of motion for the generating functional $F(v;\zeta_a)$ of hydrodynamic averages (or equations of motions for the densities of additive motion integrals $\zeta_\alpha(\mathbf{x},t)$ and generating functional $G(u;g)$ of the proper correlation function $\xi_{\alpha_1...\alpha_S}(\mathbf{x}_1,...,\mathbf{x}_S;t)$) it is necessary to find a solution of Eqs.(2.6) in a hydrodynamic approximation. With this purpose we used the functional hypothesis

$$\underline{f}_S(x_1,...,x_S;t) \xrightarrow[t>>\tau_r]{} \underline{f}_S(x_1,...,x_S;\zeta_a(t)),$$

or

$$F(u;\underline{f}(t)) \xrightarrow[t>>\tau_r]{} F(u;\underline{f}(\zeta_a(t))), \quad (3.6)$$

which has a simple physical sense: according to the method of reduced description it is considered that at a moment $t>>\tau_r$ (τ_r is a relaxation time) for the hydrodynamic stage of evolution a system state is completely described by densities $\zeta_\alpha(\mathbf{x},t)$ of additive motion integrals and smoothed correlation functions of hydrodynamic parameters $\xi_{\alpha 1\ldots\alpha S}(\mathbf{x}_1,\ldots,\mathbf{x}_S;t)$) at the same moment.

For the hydrodynamic stage of evolution of the system the specific sizes of spatial inhomogeneities (over all spatial coordinates of correlation functions) are large in comparison with a mean free particle path, which develops the theory of perturbations over spatial gradients of reduced description parameters.

Perturbation theory is developed to solve equations for kinetics of long-wave fluctuations. This theory is analogous to the Chapman-Enscog procedure which is used to derive equations of usual (nonfluctuating) hydrodynamics proceeding from usual kinetic equation. As a result the following general equations for fluctuation hydrodynamics can be derived

$$\frac{\partial \zeta_\alpha(\mathbf{x},t)}{\partial t} = exp\left[G\left(\frac{\delta}{\delta\zeta};\xi_a\right)\right]T_\alpha(\mathbf{x};\zeta), \qquad (3.7)$$

$$\frac{\partial G(v;\xi_a)}{\partial t} = \left\{exp\left[G\left(v+\frac{\delta}{\delta\zeta};\xi_\alpha\right)-G(v;\xi_\alpha)\right]-\right.$$

$$\left.-\,exp\left[G\left(\frac{\delta}{\delta\zeta};\xi_a\right)\right]\right\}\int d\mathbf{x}v_\alpha(\mathbf{x})T_\alpha(\mathbf{x};\zeta), \qquad (3.8)$$

where

$$T_\alpha(x;\ \zeta) \equiv -\frac{\partial\zeta_{\alpha k}(\zeta(x))}{\partial x_k} + \frac{\partial}{\partial x_i}\eta_{\alpha\gamma;\,ik}(\zeta(x))\frac{\partial\zeta_\gamma(x)}{\partial x_k}$$

(3.8)

In the expression (3.8) the current densities of hydrodynamic parameters $\zeta_{\alpha k}$ are determined by the formula

$$\zeta_{\alpha k}(\zeta(\mathbf{x})) = \int d\mathbf{p}\frac{p_k}{m}\zeta_\alpha(\mathbf{p})f_0(x),$$

where $f_0(x)$ is local-equilibrium Maxwell distribution and the quantity $\eta_{\alpha\gamma;ik}(\zeta(\mathbf{x}))$ determines dissipative kinetic coefficients (see in connection with this, for example, [7]).

We emphasize that at the fluctuation hydrodynamic stage of evolution the system dynamics is determined by the unique quantity $T_\alpha(\mathbf{x};t)$ (see (3.8)) describing the usual (without fluctuations) hydrodynamics of a viscous liquid

$$\frac{\partial \zeta_\alpha(\mathbf{x},t)}{\partial t} = T_\alpha(\mathbf{x};\zeta), \qquad (3.9)$$

as well as that at fluctuation kinetic stage of evolution the dynamics of long-wave fluctuations has been determined by the unique quantity L(x;f) which is a functional of Bogolyubov's theory (see (2.6), (2.7)).

In conclusion let us note the following. In case of usual (without fluctuations) hydrodynamics an entropy density s($\mathbf{x}$) is determined by the hydrodynamic parameters $\zeta_\alpha(\mathbf{x},t)$ ($s(\mathbf{x})\equiv s(\zeta(\mathbf{x},t))$) and satisfy the equation

$$\frac{\partial s(\mathbf{x},\zeta)}{\partial t} = -\frac{\partial s_k(\mathbf{x};\zeta)}{\partial x_k} + I(\mathbf{x};\zeta), \qquad (3.10)$$

where $s_k(\mathbf{x};\zeta)$ is an entropy current density and $I(\mathbf{x};\zeta)$ is an entropy production, and what is more $I(\mathbf{x};\zeta)\geq 0$ since the H-theorem is true in usual hydrodynamics (see for instance [7]). It is shown [8], that at the fluctuation hydrodynamic stage of evolution equation for entropy has the form

$$\frac{\partial s(\mathbf{x},\xi_a)}{\partial t} = -\frac{\partial s_k(\mathbf{x};\xi_a)}{\partial x_k} + I(\mathbf{x};\xi_a), \qquad (3.11)$$

where

$$s(\mathbf{x},\xi_a) = exp\left[G\left(\frac{\delta}{\delta\zeta};\xi_a\right)\right]s(\mathbf{x},\zeta), \qquad (3.12)$$

$$s_k(\mathbf{x},\xi_a) = exp\left[G\left(\frac{\delta}{\delta\zeta};\xi_a\right)\right]s_k(\mathbf{x},\zeta),$$

$$I(\mathbf{x},\xi_a) = exp\left[G\left(\frac{\delta}{\delta\zeta};\xi_a\right)\right]I(\mathbf{x},\zeta),$$

and what is more by force of positivity of $I(\mathbf{x};\zeta)$ the entropy production $I(\mathbf{x};\xi)$ is positive too, $I(\mathbf{x};\xi)\geq 0$, at given fluctuations. Similarly the H-theorem is to be proved for fluctuation kinetics.

ACKNOWLEDGMENT

The authors acknowledge INTAS for the partial financial support (Project № 00-00577).

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