TO KINETICS OF ELECTROMAGNETIC FIELD IN MEDIUM TAKING INTO ACCOUNT FLUCTUATIONS

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Kinetic equation for statistical operator of electromagnetic field in equilibrium medium consisting of charged particles has been obtained in quasirelativistic approximation. Interpretation of the results in terms of photon in medium is given. Spectrum of the photons and amplitude of decay one photon into two have been calculated. Obtained kinetic equation can be applied to studying fluctuations of the field.

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INTRODUCTION

Problem of fluctuation influence on nonequilibrium processes is the leading topic of investigation in area of statistical physics. Present paper is dedicated to kinetics of electromagnetic field (photon gas) in equilibrium medium of charged particles with taking into account all the fluctuations of the field. With this purpose kinetic equation for statistical operator of the field, that gives the most complete its description, will be built. The electromagnetic interaction is considered as a weak one but it is implied that this interaction fast forms an equilibrium bath (system b) from all charged particles of the system. Photon subsystem (system s) will remain in nonequilibrium state and slow tend to equilibrium.

1. BASIC KINETIC EQUATION FOR SYSTEM IN BATH

We start with general theory of nonequilibrium processes in a system s weak interacting with equilibrium system b. Space of states of s+b system is $\mathrm{H_s} \otimes \mathrm{H_b}$ where $\mathrm{H_s}$, $\mathrm{H_b}$ are spaces of states of s and b. Statistical operator $\rho(t)$ of s+b satisfies the quantum Liouville equation

$$\dot{\rho}(t) = -\frac{i}{\hbar}\left[\hat{H},\rho(t)\right] \equiv \mathrm{L}\rho(t)$$

$$\hat{H} = \hat{H}_s + \hat{H}_b + \hat{H}_{sb}, \quad \mathcal{L} = \mathcal{L}_s + \mathcal{L}_b + \mathcal{L}_{sb} \tag{1}$$

with obvious notations for contributions to Hamilton operator $\hat{H}$ and Liouville operator $\mathcal{L}$. Statistical operator $\rho_s(t)$ of the system s is defined by the formulae

$$\mathrm{Sp}\rho(t)\hat{\mathrm{A}}_\mathrm{s} = \mathrm{Sp_s}\rho_s(t)\hat{\mathrm{A}}_\mathrm{s} \quad \rho_s(t) = \widetilde{\mathrm{S}}\mathrm{p_b}\rho(t), \tag{2}$$

where operations Sp, $\mathrm{Sp_s}$ $\widetilde{\mathrm{S}}\mathrm{p_b}$ are introduced by the relations

$$\mathrm{Sp}\hat{\mathrm{A}} = \sum_{n\alpha} \langle n\alpha|\hat{\mathrm{A}}|n\alpha\rangle,$$

$$\mathrm{Sp_s}\hat{\mathrm{A}}_\mathrm{s} = \sum_{\alpha} \rangle\alpha|\hat{\mathrm{A}}_\mathrm{s}|\alpha\langle,$$

$$\widetilde{\mathrm{S}}\mathrm{p_b}\hat{\mathrm{A}} = \sum_{\alpha nn'} \langle n\alpha|\hat{\mathrm{A}}|n'\alpha\rangle|n\rangle\langle n'| \tag{3}$$

We use notations $\hat{\mathrm{A}}_\mathrm{s}$, $\hat{\mathrm{A}}_\mathrm{b}$, $\hat{\mathrm{A}}$ for arbitrary operators in $\mathrm{H_s}$, $\mathrm{H_b}$, $\mathrm{H_s}\otimes\mathrm{H_b}$; $\{|n\rangle\}$, $\{|\alpha\rangle\}$, $\{|n\alpha\rangle\}$ are bases in spaces $\mathrm{H_s}$, $\mathrm{H_b}$, $\mathrm{H_s}\otimes\mathrm{H_b}$ ($|n\alpha\rangle = |n\rangle|\alpha\rangle$). Operations Sp, $\mathrm{Sp_s}$ $\widetilde{\mathrm{S}}\mathrm{p_b}$ and their analogs $\mathrm{Sp_b}$, $\widetilde{\mathrm{S}}\mathrm{p_s}$ have simple and useful properties.

$$\mathrm{Sp}\hat{\mathrm{A}} = \mathrm{Sp}_s\widetilde{\mathrm{S}}\mathrm{p}_b\hat{\mathrm{A}}, \quad \widetilde{\mathrm{S}}\mathrm{p}_b[\hat{\mathrm{A}}_b,\hat{\mathrm{A}}] = 0,$$

$$\widetilde{\mathrm{S}}\mathrm{p}_b\hat{\mathrm{A}}_s\hat{\mathrm{A}} = \hat{\mathrm{A}}_s\widetilde{\mathrm{S}}\mathrm{p}_b\hat{\mathrm{A}}, \quad \widetilde{\mathrm{S}}\mathrm{p}_b\hat{\mathrm{A}}\hat{\mathrm{A}}_s = (\widetilde{\mathrm{S}}\mathrm{p}_b\hat{\mathrm{A}})\hat{\mathrm{A}}_s$$

$$\widetilde{\mathrm{S}}\mathrm{p}_b\hat{\mathrm{A}}_s\hat{\mathrm{A}}_b = \hat{\mathrm{A}}_s\mathrm{Sp}_b\hat{\mathrm{A}}_b.$$

Equilibrium bath is described by equilibrium statistical operator w_b ($[H_b,\mathrm{w}_b] = 0$, $\mathrm{Sp_b w}_b = 1$). In general situation interaction $\hat{H}_{sb}$ can lead to evolution of the bath but we will neglect of this effect. We will consider description of the system s+b by statistical operator $\rho_s(t)$ and constant parameters of the bath using method of the reduced description of nonequilibrium states [1]. This method is based on the functional hypothesis

$$\rho(t) \xrightarrow[t>>\tau_0]{} \rho(\rho_s(t,\rho_0))$$

$$(\rho_s(t) \xrightarrow[t>>\tau_0]{} \rho_s(t,\rho_0), \quad \rho_0 \equiv \rho(t=0),$$

$$\widetilde{\mathrm{S}}\mathrm{p_b}\rho(\rho_s) = \rho_s) \tag{4}$$

(here τ_0 is a characteristic time, $\rho(\rho_s)$ is a function which does not depend on t and ρ_0). The similar problem was studied in paper [2] but there inconvenient for our purpose technique was proposed.

Statistical operator of the system $\rho(\rho_s(t,\rho_0))$ satisfies the Liouville equation (1) at $t >> \tau_0$ and can be defined for $0 \le t \lesssim \tau_0$ with the help of this equation. Finally we obtain the following equation for $\rho_s(t, \rho_0)$

$$\dot{\rho}_s(t, \rho_0) = L_s(\rho_s(t, \rho_0)) \quad (t \ge 0);$$

$$(L_s(\rho_s) \equiv \mathcal{L}_s \rho_s + \widetilde{L}(\rho_s),$$

$$\widetilde{L}_s(\rho_s) \equiv \widetilde{\mathrm{S}}\mathrm{p}_s \mathcal{L}_{sb} \rho(\rho_s)) \tag{5}$$

that will be called by us the basic kinetic equation. Then the Liouville equation in terms of $\rho(\rho_s)$ takes the form

$$\frac{\partial \rho(\rho_s)}{\partial \rho_s} L_s(\rho_s) = \mathcal{L}\rho(\rho_s),$$

where derivative at the left is defined an evident way and is a linear operator in the space H_s. Statistical operator $\rho(\rho_s)$ satisfies the following obvious condition of the complete correlation weakening

$$e^{t\mathcal{L}_0} \rho(\rho_s) \xrightarrow[t \to +\infty]{} e^{t\mathcal{L}_0} \rho_s \mathrm{w}_b$$

($\mathcal{L}_0 \equiv \mathcal{L}_s + \mathcal{L}_b$). It means that evolution of the system s+b by absence of interaction s and b breaks correlations between s and b. This formula leads to the following relation

$$e^{t\mathcal{L}_0} \rho(e^{-t\mathcal{L}_b} \rho_s) - \rho_s \mathrm{w}_b \xrightarrow[t \to +\infty]{} 0 \tag{6}$$

that we consider as a boundary condition to the Liouville equation for statistical operator $\rho(\rho_s)$. Now using standard approach of the reduced description method [1] it is easy to obtain the following integral equation for operator $\rho(\rho_s)$

$$\rho(\rho_s) = \rho_s w_b + \int_0^{+\infty} d\tau e^{t\mathcal{L}_0} \Big\{ \mathcal{L}_{sb} \rho(\rho_s) - \frac{\partial \rho(\rho_s)}{\partial \rho_s} \widetilde{L}_s(\rho_s) \Big\}_{\rho_s \to e^{-\tau\mathcal{L}_s} \rho_s} . \tag{7}$$

This equation with expression (5) for $\widetilde{L}_s(\rho_s)$ can be solved in perturbation theory in interaction $\hat{H}_{sb}$ with respect to $\rho(\rho_s)$ and $\widetilde{L}_s(\rho_s)$. in such a way we get the basic kinetic equation (5) for statistical operator $\rho_s(t, \rho_s)$ in the form

$$\dot{\rho}_s = \frac{i}{\hbar}[\rho_s, \hat{H}_s] + \frac{1}{\hbar}(\rho_s \hat{\mathrm{A}} + \hat{\mathrm{A}}^+ \rho_s) + + \sum_i (\hat{\mathrm{B}}_i^+ \rho_s \hat{\mathrm{C}}_i + \hat{\mathrm{C}}_i^+ \rho_s \hat{\mathrm{B}}_i) \tag{8}$$

where $\hat{\mathrm{A}}, \hat{\mathrm{B}}_i, \hat{\mathrm{C}}_i$ are some operators in H_s do not depending on t and ρ_0. Contributions to (8) with $\hat{\mathrm{A}}$ allows us to introduce an effective Hamilton operator $\hat{H}_{s,\mathrm{ef}}$ of the system s

$$\hat{H}_{s,\mathrm{ef}} = \hat{H}_s + \frac{i}{2}(\hat{\mathrm{A}}^+ - \hat{\mathrm{A}}) \tag{9}$$

2. BASIC KINETIC EQUATION FOR ELECTROMAGNETIC FIELD IN EQUILIBRIUM MEDIUM

Let us turn now to our system from photons and several components of charged particles. The Hamilton operator of this system in quasirelativistic approximation is chosen in standard notations in the form [1]

$$\hat{H}_s = \sum_{k\alpha} \hbar\omega_k c^+_{k\alpha} c_{k\alpha} ;$$

$$\hat{H}_{sb} = \hat{H}_1 + \hat{H}_2, \quad \hat{H}_1 = -\frac{1}{c}\int_V dx \hat{j}_n(x) \hat{A}_n(x),$$

$$\hat{H}_2 = \frac{1}{2c^2}\int_V dx \hat{A}^2(x) \hat{\chi}(x) \tag{10}$$

with

$$\hat{j}_n(x) = \sum_a \hat{j}_{an}(x), \quad \hat{\chi}(x) = \sum_a \frac{e_a}{m_a} \hat{\rho}_a(x)$$

$$(e_a = z_a e,\ e > 0);$$

$$\hat{A}_n(x) = c\sum_{k\alpha} \left(\frac{2\pi\hbar}{\omega_k V}\right)^{1/2} (c_{k\alpha} + c^+_{-k,\alpha}) e_{k\alpha n} e^{ikx} = = \frac{1}{\sqrt{V}} \sum_k \hat{A}_{nk} e^{ikx} \tag{11}$$

Here $\hat{\rho}_a(x), \hat{j}_{an}(x)$ are charge and current densities of particles of a-th component, $\hat{A}_n(x)$ is operator of the vector potential of electromagnetic field, e is the elementary charge (we use the Coulomb gauge of potentials of the field). In this approximation Hamilton operator $\hat{H}_b$ takes into account only Coulomb interaction between particles of the bath. We use equilibrium statistical operator of the medium in its usual form

$$\mathrm{w}_b = \exp\beta \Big\{ \Omega - \hat{H}_b + \sum_a \mu_a \hat{N}_a \Big\},$$

where $\hat{N}_a$ is number of particles operator of a-th component. We consider the elementary charge e as a formal small perturbation theory parameter and therefore estimate $\hat{H}_1$, $\hat{H}_2$ as contributions to $\hat{H}$ of order e, e^2 respectively (operators $\hat{\rho}_a(x), \hat{j}_{an}(x)$ are proportional to e). Despite of dependence of Hamilton operator $\hat{H}_b$ on e we do not expand related to w_s values in e series.

Using Eqs. (5,7) we obtain the right hand side $L_s(\rho_s)$ of the basic kinetic equation (5)

$$L_s(\rho_s) = \frac{i}{\hbar}\left[\rho_s, \hat{H}_s + \frac{\chi}{2c^2}\sum_k \hat{A}^+_{nk}\hat{A}_{nk}\right] +$$

$$+\left(\frac{1}{c^2\hbar^2}\int_{-\infty}^{0} d\tau \mathrm{I}_{nl}(k,\tau)[\hat{A}_{lk}, \rho_s \hat{A}^+_{nk}(\tau)] + h.c.\right) +$$

$$+ O(e^3), \tag{12}$$

where $\mathrm{I}_{nl}(k,t)$ is Fourier transformed current correlation function

$$\mathrm{I}_{nl}(x,t) \equiv \mathrm{Sp}_b \mathrm{w}_b \hat{j}_n(x,t)\hat{j}_l(0)$$

with

$$\hat{j}_n(x,t) = e^{-t\mathcal{L}_b}\hat{j}_n(x), \quad \hat{A}_{nk}(t) = e^{-t\mathcal{L}_s}\hat{A}_{nk},$$

$$\chi \equiv \sum_a \frac{e_a \rho_a}{m_a}, \quad \rho_a \equiv \mathrm{Sp}_b \mathrm{w}_b \hat{\rho}_a(0). \tag{13}$$

Time and space Fourier transformed correlation function $\mathrm{I}_{nl}(k,\omega)$ can be expressed through external conductivity $\tilde{\sigma}_{nl}(k,\omega)$

$$\mathrm{I}_{nl}(k,\omega) = 2\hbar\omega\left(1 - e^{-\beta\hbar\omega}\right)^{-1}\mathrm{Re}\tilde{\sigma}_{nl}(k,\omega) \tag{14}$$

(we define Fourier transformation following [1]). External conductivity $\tilde{\sigma}_{nl}(k,\omega)$ defines electromagnetic reaction of the system and is related with current Green function by the formula [1]

$$G^{(+)}_{nl}(k,\omega) = \chi\delta_{nl} - i\omega\tilde{\sigma}_{nl}(k,\omega). \tag{15}$$

Entering here retarded Green function is defined according to expression

$$G^{(+)}_{nl}(x,t) \equiv -\frac{i}{\hbar}\theta(t)\mathrm{Sp}_b \mathrm{w}_b[\hat{j}_n(x,t), \hat{j}_l(0)]$$

and allows to calculate the correlation function $\mathrm{I}_{nl}(k,\omega)$ using famous from Green function theory formula

$$G^{(+)}_{ml}(k,\omega) - G^{(+)}_{lm}(k,\omega)^* =$$

$$= \frac{i}{\hbar}\mathrm{I}_{ml}(k,\omega)\left(e^{-\beta\hbar\omega} - 1\right).$$

In present work we study an isotropic system for which tensor $\tilde{\sigma}_{nl}(k,\omega)$ has the structure

$$\tilde{\sigma}_{nl}(k,\omega) = \hat{k}_n\hat{k}_l\tilde{\sigma}_l(k^2,\omega) +$$

$$+\left(\delta_{nl} - \hat{k}_n\hat{k}_l\right)\tilde{\sigma}_t(k^2,\omega),$$

$$\tilde{\sigma}_{nl}(k,\omega)^* = \tilde{\sigma}_{nl}(k,-\omega), \tag{16}$$

where scalar functions $\tilde{\sigma}_l(k^2,\omega)$, $\tilde{\sigma}_t(k^2,\omega)$ are longitudinal and transverse external conductivities ($\hat{k}_n \equiv k_n/|k|$). Taking into account the first formula from (16) and Eq. (15) we obtain relation (14). Instead of external conductivities one can use dielectric permittivities $\tilde{\varepsilon}_l(k^2,\omega)$, $\tilde{\varepsilon}_t(k^2,\omega)$ connected with $\tilde{\sigma}_l(k^2,\omega)$, $\tilde{\sigma}_t(k^2,\omega)$ by the relations

$$\tilde{\sigma}_l = \frac{i\omega}{4\pi}\frac{1-\varepsilon_l}{\varepsilon_l}, \tilde{\sigma}_t = \frac{i\omega}{4\pi}\frac{(1-\varepsilon_t)(\omega^2-\omega_k^2)}{\omega^2\varepsilon_t - \omega_k^2} \tag{17}$$

After some calculation with account to (14), (16) we can rewrite the right hand side of the basic kinetic equation (12) in the form

$$L_s(\rho_s) = \frac{i}{\hbar}[\rho_s, \hat{H}_s] +$$

$$+\frac{i}{\hbar}\left[\rho_s, \sum_{k\alpha}\frac{\pi\chi\hbar}{\omega_k}\left(c^+_{k\alpha} + c_{-k,\alpha}\right)\left(c_{k\alpha} + c^+_{-k,\alpha}\right)\right] +$$

$$+\frac{i}{\hbar}\left(\sum_{k\alpha}\frac{1}{\omega_k}\left[a_k c^+_{k\alpha} + b_k c_{-k,\alpha}, \rho_s\left(c_{k\alpha} + c^+_{-k,\alpha}\right)\right] -\right.$$

$$\left. - h.c.\right) + O(e^3) \tag{18}$$

with coefficients a_k, b_k defined by formulae

$$a_k = \int_{-\infty}^{+\infty} d\omega\, \frac{\mathrm{Re}\tilde{\sigma}_t(k^2,\omega)}{\omega - \omega_k + i0}\frac{2\hbar\omega}{1-e^{-\beta\hbar\omega}}$$

$$b_k = \int_{-\infty}^{+\infty} d\omega \frac{\mathrm{Re}\tilde{\sigma}_t(k^2,\omega)}{\omega + \omega_k + i0} \frac{2\hbar\omega}{1 - e^{-\beta\hbar\omega}} \quad (19)$$

According to Eqs. (17) $\tilde{\sigma}_t(k^2, \pm\omega_k) = 0$ and integrands in Eqs. (19) do not have poles at real ω. In our case for calculation of a_k, b_k one have to use function $\tilde{\sigma}_t(k^2,\omega)$ for Coulomb plasma and the rule for avoiding the poles in Eqs. (19) must be kept. In the same approximation the effective Hamilton operator for photons in the medium is given according Eq. (9) by the formula

$$\hat{H}_{s,\mathrm{ef}} = \sum_{k\alpha} \{\alpha_k c^+_{k\alpha} c_{k\alpha} + \\ + \frac{1}{2}\beta_k (c_{k\alpha} c_{-k,\alpha} + c^+_{k\alpha} c^+_{-k,\alpha})\} + O(e^3) \quad (20)$$

with

$$\alpha_k = \hbar\omega_k + \beta_k,\ \beta_k = \frac{\hbar}{2\omega_k}(4\pi\chi + s_k),$$

$$s_k \equiv \int_{-\infty}^{+\infty} d\omega \frac{4\omega^2 \mathrm{Re}\tilde{\sigma}_t(k^2,\omega)}{(\omega + i0)^2 - \omega_k^2}. \quad (21)$$

Hamilton operator $\hat{H}_{s,\mathrm{ef}}$ can be diagonalized with the help of Bogolyubov transformation

$$c_{k\alpha} = \tilde{c}_{k\alpha}\,\mathrm{ch}\varphi_k + \tilde{c}^+_{-k,\alpha}\,\mathrm{sh}\varphi_k$$

$$(\mathrm{th}\varphi_k = -\frac{\beta_k}{\alpha_k + \varepsilon_k},\ \varepsilon_k = \sqrt{\alpha_k^2 - \beta_k^2}) \quad (22)$$

that gives

$$\hat{H}_{s,\mathrm{ef}} = \sum_{k\alpha} \varepsilon_k \tilde{c}^+_{k\alpha} \tilde{c}_{k\alpha} + E_0^{(2)} + O(e^3),$$

$$\varepsilon_k = \hbar\sqrt{\omega_p^2 + \omega_k^2 + s_k},\ E_0^{(2)} \equiv \sum_k (\varepsilon_k - \alpha_k);$$

$$\omega_p = \sqrt{4\pi\chi}. \quad (23)$$

Here $\tilde{c}^+_{k\alpha}, \tilde{c}_{k\alpha}$ are creation and annihilation operators and ε_k is spectrum of photons in the medium, ω_p is the plasma frequency.

One can consider above introduced photons in the medium as quanta corresponding to transverse electromagnetic waves. Dispersion law $\tilde{\omega}_k$ for such waves can be found from relations [1]

$$z_k^2 \varepsilon_t(k^2,\omega) - \omega_k^2 = 0,$$

$$\tilde{\omega}_k = \mathrm{Re}\,z_k,\ \gamma_k = -\mathrm{Im}\,z_k, \quad (24)$$

where γ_k is damping coefficient of the waves. Exact expression for photon energy in the medium is given by the formula $\varepsilon_k = \hbar\tilde{\omega}_k$. According to Eqs. (17,24) the transverse conductivity $\tilde{\sigma}_t(k^2,\omega)$ has a pole at $\omega = z_k$.

Using Eqs. (5,7) one can find contribution $L_s^{(3)}(\rho_s)$ to the right hand side of the basic kinetic equation that has third order in e. The corresponding contribution $\hat{H}^{(3)}_{s,\mathrm{ef}}$ to the effective Hamilton operator $\hat{H}_{s,\mathrm{ef}}$ has the structure

$$\hat{H}^{(3)}_{s,\mathrm{ef}} = \sum_{123} \{\Phi_1(123)\tilde{c}^+_1 \tilde{c}^+_2 \tilde{c}^+_3 + \\ + \Phi_2(12,3)\tilde{c}^+_1 \tilde{c}^+_2 \tilde{c}_3 + h.c.\} \quad (25)$$

($\tilde{c}_i \equiv \tilde{c}_{k_i\alpha_i}$ and so on) and in the leading order of the perturbation theory describes the decay one photon into two and the reversal process. Amplitudes $\Phi_1(123), \Phi_2(12,3)$ are expressed through correlation functions

$$\mathrm{Sp}_b \mathrm{w}_b \hat{j}_n(x,t)\hat{\chi}(0),$$

$$\mathrm{Sp}_b \mathrm{w}_b \hat{j}_n(x,t)\hat{j}_l(x',t')\hat{j}_m(0)$$

that define nonlinear external conductivity of the Coulomb plasma. The properties of the photon spectrum ε_k and amplitudes $\Phi_1(123), \Phi_2(12,3)$ will be discussed in another place.

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