ANOMALOUS DIFFUSION OF THE COSMIC RAYS IN THE FRACTAL GALAXY

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We consider the problem of the cosmic ray spectrum formation assuming that cosmic rays are produced by galactic sources. The fractional diffusion equation proposed in our recent papers is used to describe the cosmic rays propagation in interstellar medium. We show that in the framework of this approach it is possible to explain the locally observed basic feature of the cosmic rays in the energy region $10^{10}\div10^{20}$ eV: distinction in spectral exponent of protons and other nuclei, mass composition variation, "knee" problem, flattering of the primary spectrum at $E\geq10^{18}\div10^{19}$ eV.

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1. INTRODUCTION

Numerous hypotheses on the propagation and acceleration mechanisms of galactic cosmic rays and their sources have been proposed to explain the steepening of the all-particle spectrum at $3\cdot10^{15}$ eV (the "knee") and the flattering of the spectrum around 10^{19} eV (the "ankle") (see, for example, the reviews by Erlykin (1995), Kalmykov and Khristiansen (1995), Ptuskin (1997), Cronin (2001), Olinto (2001) and Berezinsky et al. (1990)). However, in spite of considerable theoretical and experimental efforts, a self-consistent model, which can explain these features in the energy spectrum and the mass composition variations was not developed.

The aim of the present paper is to formulate a new mechanism of cosmic rays spectrum formation that explains the locally observed basic features of the cosmic rays in the energy region $10^{10}\div10^{20}$ eV. Our research was motivated by recent advances in the field of the fractional diffusion of cosmic rays (Lagutin et al., 2001b). In that paper the authors made an important discovery related to the nature of the "knee", namely, that the "knee" in primary cosmic rays spectrum is due to fractal structure of the interstellar medium (ISM). In other words, under natural physical assumption about the diffusivity $D(E)\sim E^{\delta}$ and the source function $S(E)\sim E^{-p}$, with δ, p being the constants, the "knee" appears when the free paths of particles between galactic inhomogeneties may be anomalously large. These paths (the so called "Lévy flights") are distributed according to inverse power law $p(\mathbf{r}) \propto Ar^{-3-\alpha}$, $r\rightarrow\infty$, $\alpha<2$ being an intrinsic property of the fractal structures.

In what follow, we remind our reader of the fractional diffusion equation used for describing of the cosmic rays propagation in the fractal ISM (§2), discuss the mechanism of the cosmic rays spectrum formation (§3). Our results are summarized in (§4).

2. FRACTIONAL DIFFUSION OF COSMIC RAYS

We assume that after injection by a source the particle propagating through the fractal ISM can be in one of two states: a state of "Lévy flights" or a state of rest ─ state of motion in a magnetic inhomogeneity being a trap of the returned type. We suppose that the particle can spend anomalously long time in a trap. The time distribution, adopted here, has a tail of power-law type $q(t) \propto B\, t^{-\beta-1}$, $t\rightarrow\infty$ with $\beta<1$ (the so called "Lévy trapping time"). Diffusion in model under consideration is a process in which the particle state changes successively at random moments in time. The equation for Green's function $G(\mathbf{r},t,E;E_0)$ describing particle diffusion without energy losses and nuclear interactions under condition that the particle started from origin $\mathbf{r}_0=0$ at the time $t_0=0$ with energy E_0 has the form (Lagutin and Uchaikin (2001))

$$\frac{\partial G}{\partial t} = -D(E,\alpha,\beta)D_{0+}^{1-\beta}(-\Delta)^{\alpha/2}G(\vec{r},t,E;E_0) + \delta(r)\delta(t)\delta(E-E_0). \quad (1)$$

Here D_{0+}^{μ} denotes the Riemann-Liouville fractional derivative (Samko et al. (1987))

$$D_{0+}^{\mu}f(t) \equiv \frac{1}{\Gamma(1-\mu)}\frac{d}{dt}\int_0^t (t-\tau)^{-\mu}f(\tau)d\tau,\ \mu<1,$$

$(-\Delta)^{\alpha/2}$ – the fractional Laplacian (Samko et al. (1987))

$$(-\Delta)^{\alpha/2}f(x) = \frac{1}{d_{m,l}(\alpha)}\int_{R^m}\frac{\Delta_y^l f(x)}{|y|^{m+\alpha}}dy,$$

where $l>\alpha$, $x\in R^m$, $y\in R^m$,

$$\Delta_y^l f(x) = \sum_{k=0}^{l}(-1)^k\binom{l}{k}f(x-ky)$$

and

$$d_{m,l}(\nu) = \int_{R^m}(1-e^{iy})^l|y|^{-m-\nu}dy.$$

The anomalous diffusivity $D(E,\alpha,\beta)$ is determined by the constants A and B in the asymptotic behaviour for "Lévy flights" (A) and "Lévy waiting time" (B) distributions:

$D(E, \alpha,\beta) \propto A(E,\alpha) / B(E, \beta)$.

Taking into account that both the free path and the probability to stay in trap during the time interval t for particle with charge Z and mass number A depend on particle magnetic rigidity R, we accept $A \propto R^{\delta_L}$, $B \propto R^{-\delta_T}$ so that $D=(\upsilon/c)D_0(\alpha,\beta)R^{\delta}$ with $\delta = \delta_L+\delta_T$.

The solution of the equation (1) with zero boundary conditions at infinity found in (Lagutin and Uchaikin, 2001) is

$$G(r,E,t;E_0) = \delta(E - E_0)(D(E_0,\alpha,\beta)t^{\beta})^{-3/\alpha} \times \Psi_3^{(\alpha,\beta)}\left(|\vec{r}|\left(D(E_0,\alpha,\beta)t^{\beta}\right)^{-1/\alpha}\right) \quad (2)$$

where

$$\Psi_3^{(\alpha,\beta)}(r) = \int_0^{\infty} q_3^{(\alpha)}(r\tau^{\beta})q_1^{(\beta,1)}(\tau)\tau^{3\beta/\alpha}d\tau . \quad (3)$$

Here $q_3^{(\alpha)}(r) = (2\pi)^{-3}\int \exp(-i\vec{k}r - |k|^{\alpha})dk$ is the density of three-dimensional spherically-symmetrical stable distribution with characteristic exponent $\alpha \le 2$ (Zolotarev et al. (1999); Uchaikin and Zolotarev (1999)) and $q_1^{(\beta,1)}(t)$ is one-sided stable distribution with characteristic exponent β (Zolotarev, 1983):

$$q_1^{(\beta,1)}(t) = (2\pi i)^{-1}\int_S \exp(\lambda t - \lambda^{\beta})d\lambda .$$

Using Green's function (2) we can find the cosmic ray concentration for point impulse source

$$S(r,t,E) = S_0E^{-p}\delta(r)\Theta(T - t)\Theta(t),$$

$$\Theta(\tau) = \begin{cases} 1, \tau > 0, \\ 0, \tau > 0. \end{cases}$$

We have

$$N(\vec{r},t,E) = \frac{S_0E^{-p}}{(D(E,\alpha,\beta))^{3/\alpha}}\int_{\max[0,t-T]}^{t} \tau^{-3\beta/\alpha} \times \Psi_3^{(\alpha,\beta)}\left(|\vec{r}|\left(D(E,\alpha,\beta)t^{\beta}\right)^{-1/\alpha}\right). \quad (4)$$

In our paper (Lagutin et al., 2001d), the main parameters of the model (p, δ, β, D_0, α) were been evaluated from experimental data. It has been shown that under condition $p \approx 2.9$, $\delta \approx 0.27$, $D_0 \approx (1 \div 4)\cdot 10^3$ $pc^{1.7}y^{0.8}$ in the case $\alpha=1.7$, $\beta=0.8$ the model can explain the different values of spectral exponent of protons and other nuclei, mass composition variations at $E \sim 10^2 \div 10^5$ GeV/nucleon, the steepening of the all-particle spectrum.

3. COSMIC RAYS SPECTRUM

The physical arguments and the calculations indicate that the bulk of observed cosmic rays with energy $10^8 \div 10^{10}$ eV is formed by numerous distant (r>1 kpc) sources. It means that the contribution of these sources to the observed flux may be evaluated in the framework of the steady-state approach. Based on our results (Lagutin et al., 2001a) we present the flux of the particles of type i from all distant sources in the form

$$J_G^i(r > 1kpc) = \upsilon_i C_{0i}E^{-p-\delta/\beta} , \quad (5)$$

where υ_i is a particle velocity, C_{0i} is a constant to evaluate from experimental data by fit.

The contribution of the nearby (r≤1 kpc) relatively young ($t \le 10^5$ y) sources defines the spectrum in the high energy region and, as has been shown in (Lagutin et al. (2000), Lagutin and Uchaikin, 2001; Lagutin et al., 2001d, Lagutin et al. (2002)) provides the “knee”. We present this component in the form

$$J_L^i(r \le 1kpc) = \frac{\upsilon_i}{4\pi}\sum_j N(\vec{r_j},t_j,E), \quad (6)$$

where ($\mathbf{r}_j$, t_j) are the coordinate and the age of the source (j), $N_i(\mathbf{r}_j, t_j, E)$ is the concentration given by (4). List of the supernova remnants used in our calculations is given in (Lagutin et al. (2001c))

The similar separation of the flux into two components with significantly different properties is frequently used in the studies of cosmic rays (see, for example, Atoyan et al., (1995) and references herein). However, the presence of the large free paths of the particles (“Lévy flights”) in our model leads us to introduce a third component. This third component is formed by the particles which pass a distance between an acceleration site of a source and solar system without scattering. The flux of non-scattered particles J_{NS}^i is determined by the injected flux ($\propto S_{0i}E^{-p}$) and the “Lévy flight” probability p (>r).

Taking into account that for the particle with energy E p (>r)~A(E,α)r^{α}~E^{δ_L} , we have

$$J_{NS}^i = C_{1i}^0 E^{-p+\delta_L} . \quad (7)$$

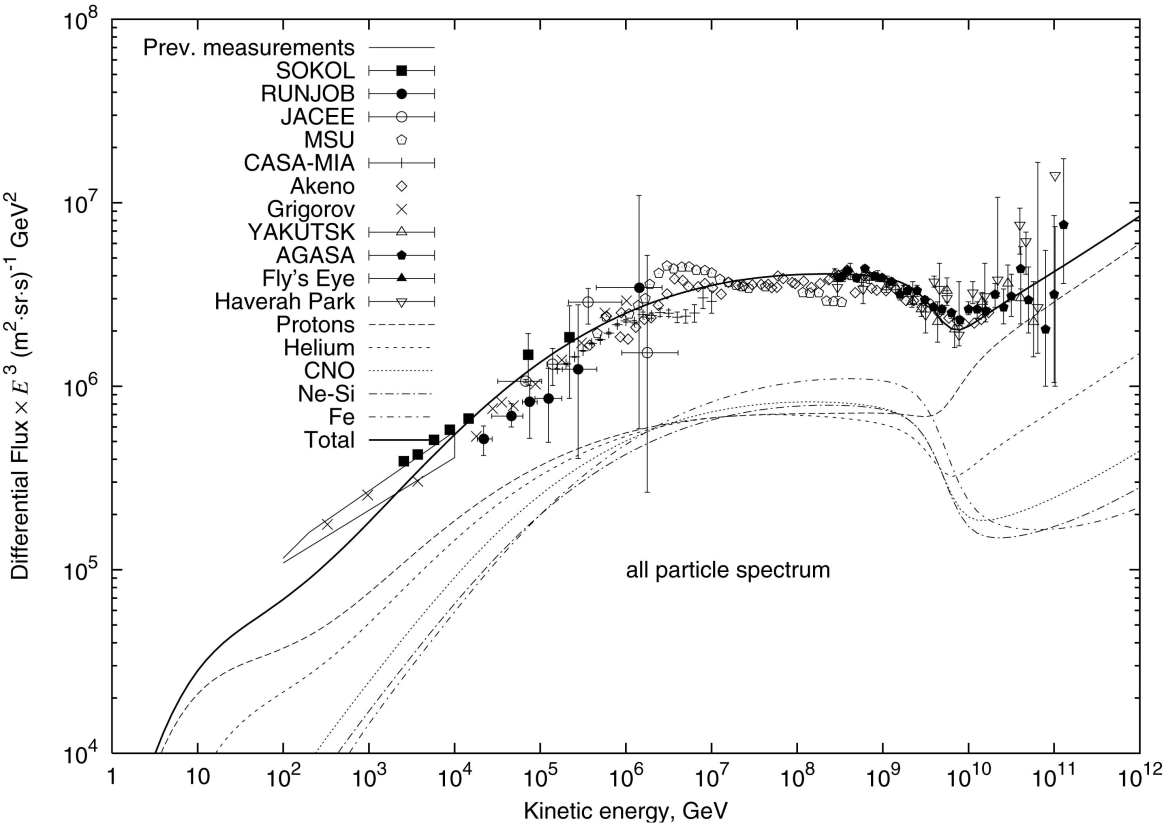
We assume that this component defines the spectrum in the ultrahigh energy region $E \ge 10^{18}$ eV and provides the flattering of the spectrum. In other words, in our model the “ankle” in primary cosmic rays spectrum is also due to the “Lévy flights” of the cosmic ray particles.

The steepening of the all-particle spectrum in the energy region $10^{18} \le E \le 10^{19}$ eV, indicating that the ISM becomes more transparent for cosmic rays particles with energy $E > 10^{18}$ eV, supports such an interpretation.

Thus, the differential flux J_i of the particles of the type i due to all sources of Galaxy may be presented in the form

$$J_i(E) = \upsilon_i C_{0i}E^{-p-\delta/\beta} + \frac{\upsilon_i}{4\pi}\sum_{j(r \le 1kpc)} N_i(\vec{r_j},t_j,E) + C_{1i}^0 E^{-p+\delta_L} . \quad (8)$$

The results of our calculations are given in figure and table. The model of Axford and Gleeson (1968) with parameter φ=750 MV have been used to describe the solar modulation.



Comparison of our calculations of spectra with experimental data. Grigirov – Grigorov et al. (1970), SOKOL – Ivanenko et al. (1990, 1993), MSU – Fomin et al. (1991), JACEE – Asakimory et al. (1998), CASA-MIA – Glasmacher et al. (1999), AKENO – Yoshida et al. (1995), RUNJOB – Apanasenko et al. (2001), YAKUTSK, AGASA, Fly's Eye, Haverah Park – from Cronin (1999).

Mass composition of cosmic rays in anomalous diffusion model

E, GeV/part.	H	He	CNO	Ne-Si	Fe	<ln A>	<A>
1.E+02	0.54	0.31	0.08	0.05	0.02	0.88	5.28
3.E+02	0.46	0.29	0.11	0.08	0.06	1.17	8.18
1.E+03	0.41	0.28	0.13	0.09	0.08	1.36	10.11
3.E+03	0.37	0.27	0.15	0.11	0.09	1.50	11.39
1.E+04	0.34	0.27	0.17	0.12	0.11	1.63	12.67
3.E+04	0.31	0.25	0.18	0.13	0.13	1.76	14.14
1.E+05	0.27	0.24	0.19	0.15	0.15	1.90	15.71
3.E+05	0.25	0.23	0.20	0.16	0.17	2.03	17.28
1.E+06	0.22	0.21	0.20	0.17	0.19	2.14	18.76
3.E+06	0.20	0.20	0.20	0.18	0.21	2.24	20.08
1.E+07	0.19	0.19	0.20	0.18	0.23	2.32	21.21
3.E+07	0.18	0.18	0.20	0.19	0.25	2.38	22.15
1.E+08	0.17	0.17	0.20	0.19	0.26	2.43	22.86
3.E+08	0.17	0.16	0.20	0.19	0.27	2.45	23.31
1.E+09	0.18	0.16	0.19	0.19	0.27	2.44	23.46
3.E+09	0.23	0.15	0.18	0.18	0.26	2.30	22.35
1.E+10	0.55	0.17	0.09	0.08	0.11	1.18	10.71
3.E+10	0.65	0.17	0.07	0.05	0.06	0.80	6.63
1.E+11	0.68	0.18	0.06	0.04	0.04	0.69	5.41
3.E+11	0.70	0.18	0.05	0.04	0.03	0.63	4.77

4. SUMMARY

We have considered the problem of the cosmic rays spectrum formation assuming that cosmic rays are produced by the galactic sources. The fractional diffusion equation proposed in our recent papers have been used to describe the cosmic rays propagation in ISM.

We showed that:

- The main features in the primary cosmic rays spectrum (the "knee", the "ankle") and mass composition variations are the consequence of the anomalous diffusion of the particles in the fractal ISM;
- The exponent of the cosmic rays spectrum in a source turns out to be equal to 2.9;
- Mass composition of the particles in a source is: p≈72%, He≈18%, CNO≈5%, (Ne-Si)≈3%, Fe≈2%.

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