

PART 2

ANOMALOUS DIFFUSION, FRACTALS, AND CHAOS

SPECTRA OF COSMIC RAY ELECTRONS AND POSITRONS IN THE GALAXY

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A new study of the cosmic ray electron and positron spectra is presented, using an anomalous diffusion model to describe the particles propagation in the Galaxy. The parameters defining the anomalous diffusion have been recently determined from the study of nuclei propagation. The computed electron and positron spectra under assumption that positrons, as well as electrons, are accelerated by a galactic source, are in a good agreement with the measurements. The source spectral index, found from experimental data, in this approach turns out to be equal to 2.95 for electrons and positrons. The predicted positron fraction $e^+/(e^++e^-)$ in high energy region $E \approx 10^2 \div 10^3$ GeV is ~ 0.06 .

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1. INTRODUCTION

Observations of non-thermal radiation of the Galaxy stimulated investigations of propagation of cosmic ray electrons through the interstellar medium. Since basic paper [1], the problem of calculation of electron spectrum was considered in series of papers (see, for example, [2-10]). The normal diffusion equation for concentration of the electrons with energy E , $N(\mathbf{r}, t, E)$, generated by sources distribution with density function $S(\mathbf{r}, t, E)$,

$$\frac{\partial N}{\partial t} = D \Delta N(\mathbf{r}, t, E) + \frac{\partial}{\partial E} (b(E)N(\mathbf{r}, t, E)) + S(\mathbf{r}, t, E), \quad (1)$$

has been used to study the electron energy spectrum modifications in the interstellar medium (ISM). In the equation (1) D is the diffusivity, $b(E)$ describes the energy-loss rate of electrons.

Recently, in the papers [11-13], new view of the cosmic ray propagation problem was presented. It has been shown that the "knee" in the primary cosmic ray spectrum is due to large free paths (the so called "Lévy flights") of cosmic rays particles between magnetic domains - traps of the returned type. As the "Lévy flights" distributed according to inverse power law $\propto Ar^{-3-\alpha}$, $r \rightarrow \infty$, $\alpha < 2$, is an intrinsic property of fractal structures, in the fractal-like medium the normal diffusion equation (1) certainly does not hold.

Based on this argument in [14] an anomalous diffusion (superdiffusion) model for describing of electrons transport in the fractal-like ISM was proposed. This superdiffusion equation for concentration of the electrons has been presented in the form

$$\frac{\partial N}{\partial t} = -D(E, \alpha)(-\Delta)^{\alpha/2} N(\mathbf{r}, t, E) + \frac{\partial}{\partial E} (b(E)N(\mathbf{r}, t, E) + S(\mathbf{r}, t, E)), \quad (2)$$

where $D(E, \alpha)$ is the anomalous diffusivity and $(-\Delta)^{\alpha/2}$ is the fractional Laplacian (called "Riss operator" [15]).

The solution of superdiffusion equation (2) in the case of point impulse source with inverse power spectrum and the behavior of energy spectrum of electrons in high energy region were found.

The main goal of this paper is to calculate the spectra of electrons and positrons from sub-GeV to TeV energies in the framework of anomalous diffusion model. We don't use the assumption made in [14] that the mean time of particle staying in a trap $\langle \tau \rangle$ is finite. In this paper, similarly to [13], we suppose that a particle can spend anomalously a long time in a trap. An anomalously long time means that

$$\langle \tau \rangle = \int_0^{\infty} dt \tau q(\tau) = \infty,$$

so the distribution of particles staying in traps, $q(\tau)$, has a tail of power law type $\propto B t^{\beta-1}$, $t \rightarrow \infty$ with $\beta < 1$ (the so called "Lévy trapping time").

2. FLUX OF ELECTRONS FROM POINT SOURCE

The flux of electrons, $J(\mathbf{r}, t, E)$, is related to the source $S(\mathbf{r}_0, t_0, E_0)$ by the propagator $G(\mathbf{r}, t, E; \mathbf{r}_0, t_0)$:

$$J(\mathbf{r}, t, E) = \int_{\mathbf{r}_0}^{\mathbf{r}} dr_0 \int_E^{\infty} dE_0 \int_{-\infty}^t dt_0 G(\mathbf{r}, t, E; \mathbf{r}_0, t_0) \times S(\mathbf{r}_0, t_0, E_0) \delta(t - t_0 - \tau). \quad (3)$$

Here

$$\tau = \int_E^{E_0} \frac{dE'}{b(E')}, \quad (4)$$

$\delta(t - t_0 - \tau)$ reflects the law of energy conservation in the continuous losses approach.

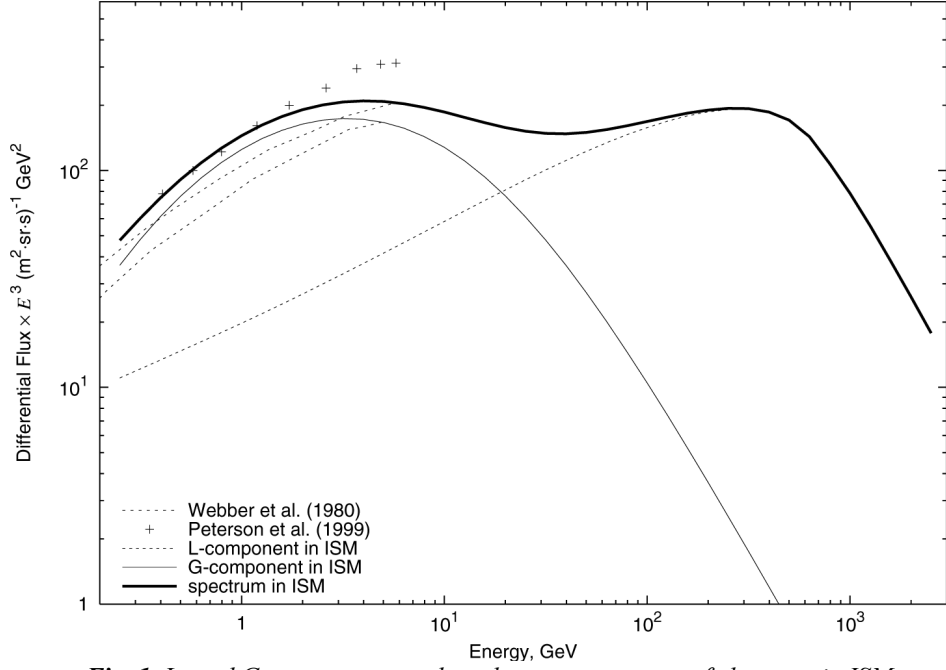


Fig. 1. L- and G-components and total energy spectrum of electrons in ISM

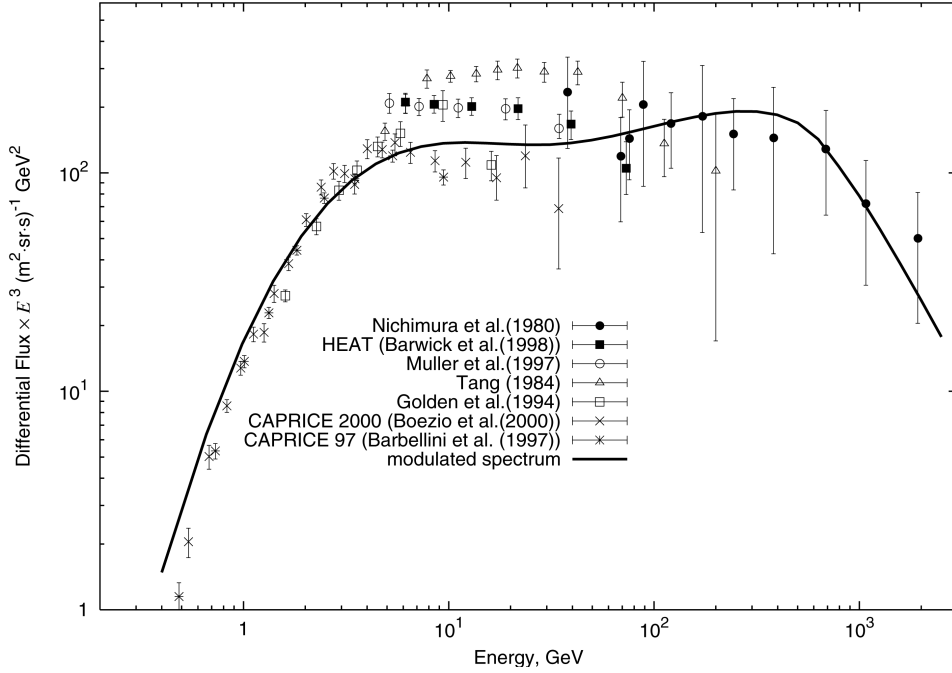


Fig. 2. Modulated energy spectrum of electrons near the solar system

The propagator in the anomalous diffusion model under consideration has the form [13]

$$G(\vec{r}, t, E; \vec{r}_0, t_0) = \frac{c}{4\pi} (D(E, \alpha, \beta) t^\beta)^{-3/\alpha} \times \Psi_3^{(\alpha, \beta)}(|r| (D(E, \alpha, \beta) t^\beta)^{-1/\alpha}), \quad (5)$$

where

$$\Psi_3^{(\alpha, \beta)}(r) = \int_0^\infty q_3^{(\alpha)}(r\tau^\beta) q_1^{(\beta, 1)}(\tau) \tau^{3\beta/\alpha} d\tau. \quad (6)$$

Here $q_3^{(\alpha)}(r)$ is the density of three-dimensional spherically-symmetrical stable distribution with characteristic exponent $\alpha < 2$ ([16]) and $q_1^{(\beta, 1)}(\tau)$ is one-sided stable distribution with characteristic exponent β [17]. The parameters α, β are determined by the fractional structure of ISM and by trapping mechanism correspondingly, the anomalous diffusivity $D(E, \alpha, \beta)$ - by the constants A and B in the asymptotic behaviour for “Lévy flights” (A) and “Lévy waiting time” (B) distributions:

$$D(E, \alpha, \beta) = A(E, \alpha) / B(E, \beta).$$

The energy-loss rate of relativistic electrons is described by the equation (14) from [9]

$$-\frac{dE}{dt} = b(E) = b_0 + b_1 E + b_2 E^2 \approx b_2 (E + E_1)(E + E_2), \quad (7)$$

where $b_0 = 3.06 \cdot 10^{-16} \text{ n (GeV s}^{-1})$ is for the ionization losses of the electrons in ISM with number density $n \text{ (cm}^{-3})$, $b_1 E$ with $b_1 = 10^{-15} \text{ n (s}^{-1})$ corresponds to the bremsstrahlung energy losses, and $b_2 E^2$ with $b_2 = 1.38 \cdot 10^{-16} \text{ (GeV s)}^{-1}$ represents synchrotron and inverse Compton losses (for $B \approx 5 \mu\text{G}$ and $\omega \approx 1 \text{ (eV/cm}^3)$), $E_1 \approx b_0/b_1$, $E_2 \approx b_1/b_2$. Using (7), the solution of the equation (4) relative to E can be presented in the form [9]:

$$E_0(\tau) = \frac{E + E_1}{1 - (1 - e^{-b_1 \tau})(E + E_2)/(E_2 - E_1)} - E_1.$$

Taking into account that $b_1 \tau \leq 3.15 \cdot 10^{-8} \text{ n } 10^5 = 3.15 \text{ n } 10^{-3} \ll 1$, we derive from the later equation

$$E_0(\tau) = \frac{E + E_1}{1 - b_1 \tau (E + E_2)/(E_2 - E_1)} - E_1. \quad (8)$$

With help of the equations (3), (5), (8) it's easy to calculate the flux of electrons for the sources interesting for astrophysics. For example, for point impulse source

$$S(r, t, E) = S_0 E^{-p} \delta(r) \theta(T - t) \theta(t),$$

$$\theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0, \end{cases}$$

we have

$$J_T(r, t, E) = S_0 \int_{\max\{0, t-T\}}^{\min\{t, 1/b_2(E+E_2)\}} dt E_0^{-p}(\tau) \times (\lambda(E, \tau) \tau^{\beta-1})^{-1/\alpha} (1 - b_2 \tau (E + E_2))^{-2} \times \Psi_3^{(\alpha, \beta)}(r(\lambda(E, \tau) \tau^{\beta-1})^{-1/\alpha}), \quad (9)$$

where

$$\lambda(E, t) = \int_E^{E_0(t)} \frac{D(E', \alpha, \beta)}{b(E')} dE'.$$

It should be noted that in the case $\beta = 1$, the equation (9) comes to the solution, obtained earlier in [14]. If $\alpha = 2$, $\beta = 1$, we have the standard solution [1].

3. ENERGY SPECTRUM OF ELECTRONS

The flux J of electrons due to all sources of Galaxy can be separated into two components:

$$J = \int_0^{1 \text{ kpc}} \dots + \int_{1 \text{ kpc}}^R \dots = J_L(r \leq 1 \text{ kpc}) + J_G(r > 1 \text{ kpc}). \quad (10)$$

The similar separation is frequently used in the studies of cosmic rays (see, for example, [9] and references therein).

The first component (L) in (10) describes the contribution of the nearby sources (at distance $r \leq 1 \text{ kpc}$) to observed flux J . The second component (G) is the contribution of the distant sources ($r > 1 \text{ kpc}$) to J .

The nearby sources used in our calculations are presented in Table 1. Based on result (9) we suppose

$$J_L = \sum_{r \leq 1 \text{ kpc}} J_T(r_i, t_i, E), \quad (11)$$

where injection time $T \approx 10^4 \div 10^5 \text{ y}$.

Table 1. List of the nearby supernova remnants

Name	R, pc	t, 10 ⁵	Source
Lopus Loop	400	0.38	[7]
Monoceros	600	0.46	[7]
Vela	400	0.11	[7]
Cyg. Loop	600	0.35	[7]
CTB 13	600	0.32	[7]
S 149	700	0.43	[7]
STB 72	700	0.32	[7]
CTB 1	900	0.47	[7]
HB 21	800	0.23	[7]
HB 9	800	0.27	[7]
Monogem	300	0.86	[18]
Geminga	400	3.4	[18]
Loop I	100	2.0	[18]
Loop II	175	4.0	[18]
Loop III	200	4.0	[18]
Loop IV	210	4.0	[18]

The second component (G) is evaluated under assumption that the distant sources ($r > 1 \text{ kpc}$) are distributed uniformly both in space and time in the Galaxy.

The parameters defining the anomalous diffusivity and used in our calculations have been recently derived from the study of nuclei propagation [12]: $\alpha = 1.7$; $\beta = 0.8$; $D(E, \alpha, \beta) = D_0(E/1 \text{ GeV})^\delta$ with $D_0 \approx (1 \div 4) \cdot 10^{-3} \text{ pc}^2 \text{ y}^{-1}$ and $\delta = 0.27$. Only one parameter p defining injection spectrum of electrons in the sources is found by fit. Extensive calculations show that the best fit of experimental data may be get at $p \approx 2.95$.

The spectra of L- and G- components and the total spectrum in ISM are demonstrated in Fig. 1. The experimental energy spectrum of synchrotron electrons in ISM obtained in [19,20] is presented in this figure.

To describe the influence of the solar modulation, the force model of [21] is used:

$$J_{\text{mod}}(E) = \frac{E^2 - (m_e c^2)^2}{[E + \Phi(t)]^2 - (m_e c^2)^2} J[E + \Phi(t)].$$

The modulated spectrum near solar system under condition that $\Phi(t) = 600 \text{ MeV}$ is shown in Fig. 2. We conclude from Fig. 1 and Fig. 2 that experimental data have a good agreement with our theoretical calculations.

4. ENERGY SPECTRUM OF POSITRONS AND POSITRON FRACTION

It is commonly believed that the cosmic ray positrons are a secondary component resulting from the decay of π^+ , κ^+ produced in the nuclear interactions of cosmic rays with the ISM. Previous calculations of the

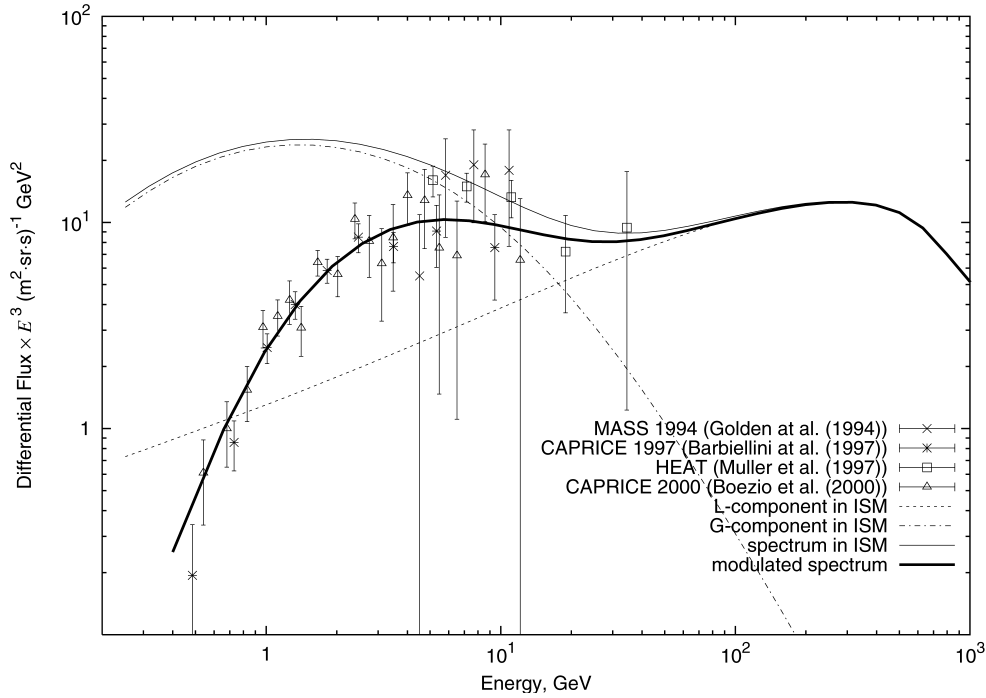


Fig. 3. Modulated energy spectrum of positrons near the solar system

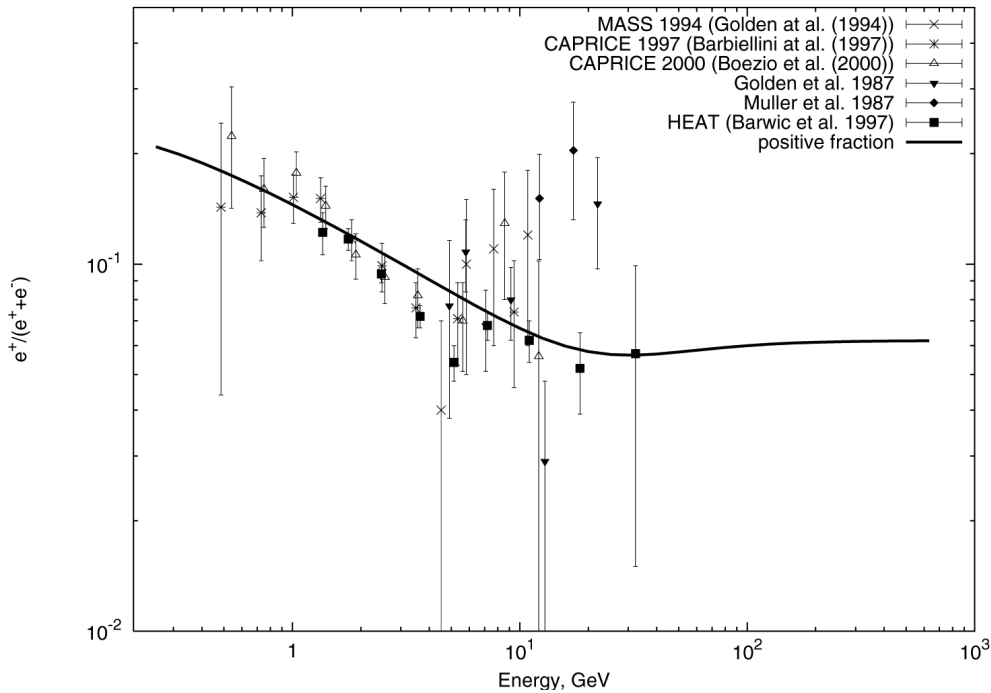


Fig. 4. Positron fraction $e^+/(e^++e^-)$ near the solar system

secondary positron flux made in [10,22] have shown that the predicted positron fraction is in good agreement with the measurements up to 10 GeV, beyond which the observed flux is higher than that calculated. To reproduce the positron observations above 10 GeV either a primary positron component or a harder interstellar nucleon spectrum is required [9,10,22]. As the harder interstellar nucleon spectrum is not consistent with direct proton measurements, in this paper we suppose that high-energy

positron excess is due to contribution of a primary positron component.

We demonstrate in Fig. 3 and Fig. 4 that a primary positron component as large as 6% of the primary electron spectrum allows us to reproduce the observed positron spectrum as well as the positron fraction.

5. CONCLUSION

We have carried out a new study of the cosmic ray electron and positron spectra using an anomalous

diffusion model to describe the particles propagation in the Galaxy. The parameters defining the anomalous diffusion have been recently determined from the study of nuclei propagation. The computed electron and positron spectra under assumption that positrons, as well as electrons, are accelerated by a galactic source, are in a good agreement with the measurements.

We have shown that the sources of high-energy ($E \geq 100$ GeV) electrons and positrons, observed in the solar system are relatively young local sources ($r \leq 200$ pc, $t \sim 105$ y), injecting particles during the time $T \sim 104 \div 105$ y. The behavior of spectra in the low-energy region is defined by distant ($r \geq 1$ kpc) sources.

The source spectral index, found from experimental data, in this approach turns out to be equal to 2.95 for electrons and positrons. The proximity of this exponent to one obtained earlier [12] for nuclei components ($p \approx 2.9$) can indicate the same mechanism of particles acceleration. The predicted positron fraction $e^+/(e^++e^-)$ in high energy region $E \approx 10^2 \div 10^3$ GeV is ~ 0.06 .

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