PRECISION METHOD FOR THE DETERMINATION OF NEUTRINO MIXING ANGLE

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Based on the properties of the cascade statistics of reactor antineutrinos the effective method of neutrino oscillations searching is offered. The determination of physical parameters of this statistics, i.e., the average number of fissions and the average number of antineutrinos per fission, does not require a priori knowledge of geometry and characteristics of the detector, the reactor power and composition of nuclear fuel.

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1. INTRODUCTION

The hypothesis of massive neutrino mixing stimulated the experiments on searching oscillations of reactor electron antineutrinos $\tilde{\nu}_e$ [1]. The oscillation effect $\tilde{\nu}_e \to \nu_\chi$ can be manifested as spectrum deformation and change of $\tilde{\nu}_e$ flux from distances R according to the dependence [2]:

$$I = I_0\left[1 - \tfrac{1}{2}\sin^2 2\vartheta\left(1-\cos 2\pi R/L\right)\right], \quad (1)$$

where I_0 is the intensity in absence of oscillations; ϑ is the mixing angle; L=2.5E_ν/Δ^2 is the length of oscillations, m; E_ν is the neutrino energy, MeV; Δ^2=m$_1^2$ – m$_2^2$ is the squares of masses difference, eV^2.

The inverse β–decay reaction was used for this purpose in series of papers:

$$\tilde{\nu}_e + p \to e^+ + n. \quad (2)$$

In the oscillation absence the counting rate of detector is connected to reactor heat-generation power W by relation:

$$\langle n_\nu\rangle = \frac{\langle W\rangle}{\langle E_f\rangle}\cdot\frac{\gamma\varepsilon_0}{4\pi\langle R\rangle^2}\cdot N_p\cdot\Sigma_\nu, \quad (3)$$

where

$$\sum{}_\nu = M_\nu\langle\sigma_{\nu p}\rangle,\quad M_\nu = \int_{E_{nop}}^{E_{max}}\rho(E_\nu)dE_\nu,$$

$$\langle\sigma_{\nu p}\rangle = \int_{E_{nop}}^{E_{max}}\sigma_{\nu p}(E_\nu)\rho(E_\nu)dE_\nu \Bigg/ \int_{E_{nop}}^{E_{max}}\rho(E_\nu)dE_\nu; \quad (4)$$

$\langle E_\nu\rangle=\sum(\alpha_i\cdot E_{\nu i})$ is the average energy absorbed in reactor per fission at given fuel composition; α_i is contribution from i-th isotope (i=5,9,8,1) in total fission cross-section, which depends on mode of spectrum $\rho(E_\nu)$ determination [3]; $(4\pi\langle R\rangle^2)^{-1}$ is the effective solid angle with allowance for real distribution of energy-gene-ration in reactor core volume; N_p and $\gamma\varepsilon_0$ are the detector characteristics (number of hydrogen atoms in a target and detection efficiency with allowance for the part of detected neutrons γ corresponding to the reaction (2)); Σ_ν and $\langle\sigma_{\nu p}\rangle$ are neutrino reaction cross-sections, and their dimensions are cm^2/fission and cm^2/ν-particles accordingly; $\Sigma_\nu=\Sigma(\alpha_i\cdot\Sigma_{\nu i})$ at given fuel composition; M_ν is number of electron antineutrinos per fission; $\rho(E_\nu)=\Sigma(\alpha_i\cdot\rho_{\nu i})$ is antineutrino energy spectrum (MeV^{-1}·fission^{-1}) emitted by fission-products of all fuel components (actinides); $\sigma_{\nu p}(E_\nu)$ is the interaction cross-section of monoenergetic (with an energy E_ν) antineutrinos with allowance for recoil, weak magnetism and radiation corrections.

The cascade type of antineutrino statistics makes it possible to modify Eq. (3) by the following way. It is obvious, that the statistics of the reactor antineutrinos is formed due to two-cascade stochastic process. Primary random process (number of fissions $\langle\lambda\rangle$) generates secondary random process (β-decays chain or number of antineutrinos per fission $\langle\varepsilon\rangle$). Then due to well-known Burgess theorem the mathematical expectation $\langle n_\nu\rangle$ and variance var(n_ν) of antineutrinos connected by such relations [4]:

$$\langle n_\nu\rangle = \langle\lambda\rangle\langle\varepsilon\rangle, \quad (5)$$

$$\mathrm{var}(n_\nu) = \langle\lambda\rangle\mathrm{var}(\varepsilon) + \mathrm{var}(\lambda)\langle\varepsilon\rangle^2, \quad (6)$$

Substituting Eq. (5) in Eq. (3) and using the ratio of two counting rates in the same antineutrino flux we have

$$\frac{\langle\varepsilon\rangle_1}{\langle\varepsilon\rangle_2} = \frac{M_{\nu 1}}{M_{\nu 2}}, \quad (7)$$

where $\langle\varepsilon\rangle_1$ and $\langle\varepsilon\rangle_2$ can be determined either theoretically or experimentally depending on experiment strategy (one-detector or two-detector measurement scheme).

But in any case at the large distances from reactor Eq.(1) with allowance for Eq.(7) can be re-written in form:

$$\frac{\langle\varepsilon\rangle_1}{\langle\varepsilon\rangle_2} = 1 - (1/2)\sin^2(2\vartheta). \quad (8)$$

Eq. (8) is interesting for two nontrivial reasons. Firstly, the ratio of average numbers of antineutrinos (being statistically fine sensitive value) does not depend neither on geometry and properties of the detector nor from reactor power and isotope composition of nuclear fuel. Secondly, it is obvious, that for determination of <ε> by solution of the system of momental equations (as Eqs. (5)-(6)) the a priori information about reactor antineutrino statistics type is extremely necessary.

The determination of the type of reactor antineutrino statistics, which properties could become base for the development of precision method for determination of neutrinos mixing angle, was the main purpose of our research.

2. STATISTICS OF REACTOR ELECTRON ANTINEUTRINO PRODUCTION. THEORY AND EXPERIMENT

We suppose that the statistics of reactor antineutrinos is formed due to two-cascade stochastic process, in which the primary and secondary random processes are Poisson. Then, by virtue of obvious equality of expectation <*n*·> and variance *var(n)* of each of random processes, Eqs. (5)-(6) have following concrete form:

$$\langle n \rangle = \langle \lambda \rangle \langle \varepsilon \rangle, \quad (9)$$

$$\mathrm{var}(n) = \langle n \rangle (1 + \langle \varepsilon \rangle), \quad (10)$$

where sampling average <*n*>, variance *var (n)* of antineutrinos are determined experimentally.

Using the method of generating functions it is easy to found the probability density distribution of antineutrino random number:

$$p(n) = \sum_{k=0}^{\infty} \left[\frac{(k\langle \varepsilon \rangle)^n \exp(-k\langle \varepsilon \rangle)}{n!} \cdot \frac{\langle \lambda \rangle^k \exp(-\langle \lambda \rangle)}{k!} \right], \quad (11)$$

which exactly coincides with Neyman type A two-parametrical distribution [4].

It is easy to show also that Neyman type A distribution is the particular case of Saleh-Teich distribution [5] and with <*n*>→ ∞ it goes to Gauss distribution [4]. But in any case it keeps possibility for the determination of very important parameters of our distribution, i.e., average number of fissions <λ> and average number of antineutrinos per fission, by the system of momental equations (9)-(10).

Despite the abundance of publications devoted to the reactor antineutrino detection we could find only one experiment (Fig. 1a) [6] containing data, which suffice for plotting the experimental distribution of detected reactor antineutrinos (Fig. 1b). Due to small sample of measurements corresponding to one reactor campaign (approximately 40-90 measurements [6,7]) Fig.1b reflects only qualitative goodness of fit of the experimental distribution with Neyman distribution, which asymptotically transforms to Gauss distribution. Let us note that the Neyman's statistics works well under conditions of low event intensity but large sample of the measurements [4]. In this case the oscillatory nature of Neyman cascade distribution can be exhibited (Fig. 2).

If the statistics of reactor electron antineutrinos is described by Neyman distribution, the average number of electron antineutrinos per fission should be approximately identical, i.e. <ε> ≈ *const*, for reactors of different heat power but with same fuel composition. Moreover, only a third of 6-7 antineutrinos produced by fissioned nucleus has an energy above 1.8 MeV (threshold of the reaction (2)) therefore rough estimation gives value <ε>≈2-2.3. The obtaining quantitative estimation of this value was the main purpose of indirect check goodness of fit of the experimental reactor electron antineutrino distribution with Neyman distribution. We used experimental data with same type nuclear reactors (Rovno, Ukraine and Bugey, France [6,7]) for 1984 -1995 period.

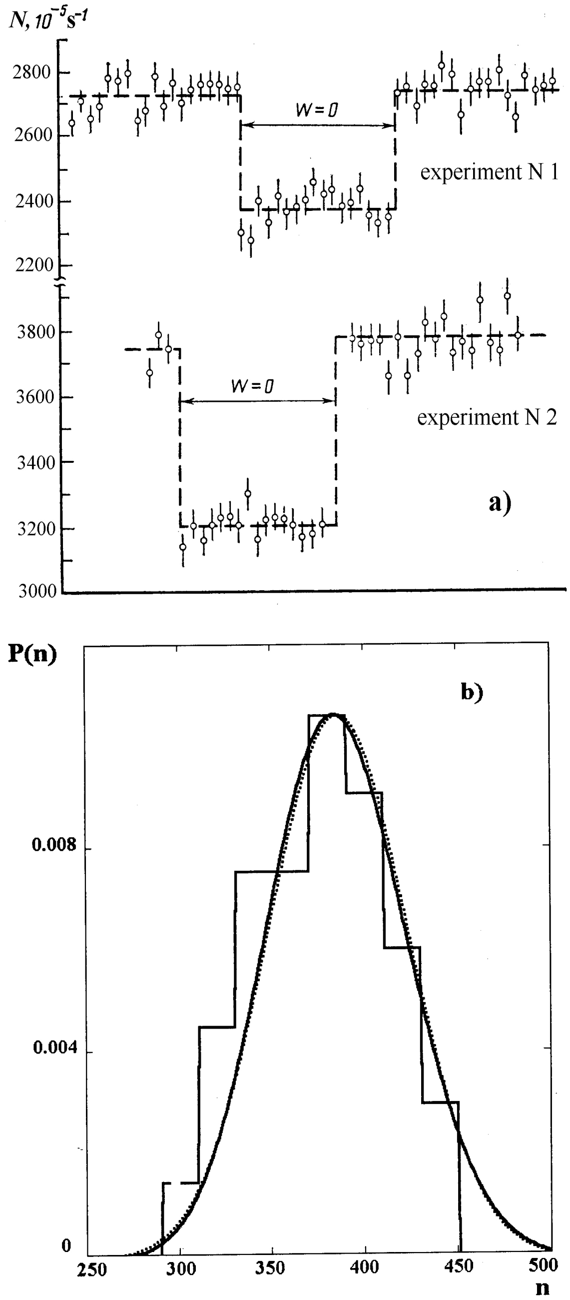

Fig. 1. Counting rate of integral detector (for 10^5 s). a) Experiments №1 and №2 [6] with operating and stopped reactor; b) Neyman (—) and Gauss (···) distributions of reactor antineutrinos in experiment №1

The results of these experiments handling are presented in Table 1. The analysis of data shows that, firstly, average number of antineutrinos per fission in averaged fuel, not only approximately equal in different experiments with the same reactor (Rovno, Ukraine [6]) but has also (with allowance for an averaging) physically acceptable value $\langle\varepsilon\rangle \approx 2.66$. Secondly, the average number of antineutrinos in different experiments with the same type reactors having the different power [6,7] coincides to within 5%. It confirms the known supposition that the same type reactors have small differences of nuclear fuel composition.

Table 1. *Experimental and calculated parameters of reactor antineutrino statistics*

Parameters	Rovno NPP [6]		Bugey-5 [7]
	Experiment N 1	Experiment N 2	
N_p	$1.152\cdot10^{28}$	$1.591\cdot10^{28}$	$4.953\cdot10^{28} \pm 0{,}5$ %
ε_0	0.540	0.568	0.549 ± 0.3 %
W, MW	1379	1371	2735 ± 0.6 %
R, m	18	17.96	14.882 ± 0.3 %
N	33*	17**	88.47
T, s	10^5	10^5	$8.6\cdot10^4$
$\langle n_\nu\rangle$	386	561.5	3022 ± 11 ± 12 ± 12
$var(n)$	1409.34	2062.5	10704.87***
$\langle\varepsilon\rangle$	2.65 ± 0.37	2.67 ± 0.51	2.54 ± 0.21
$\langle\lambda\rangle$	145.66	210.3	1189.76

* from sample of experimental data N 1 four record measurements were excepted.

** from sample of experimental data N 2 two record measurements were excepted.

*** was determined from expression: $var(n)/N = \delta^2$, where δ is statistical error of value $\langle n_\nu\rangle$ equal to ± 11

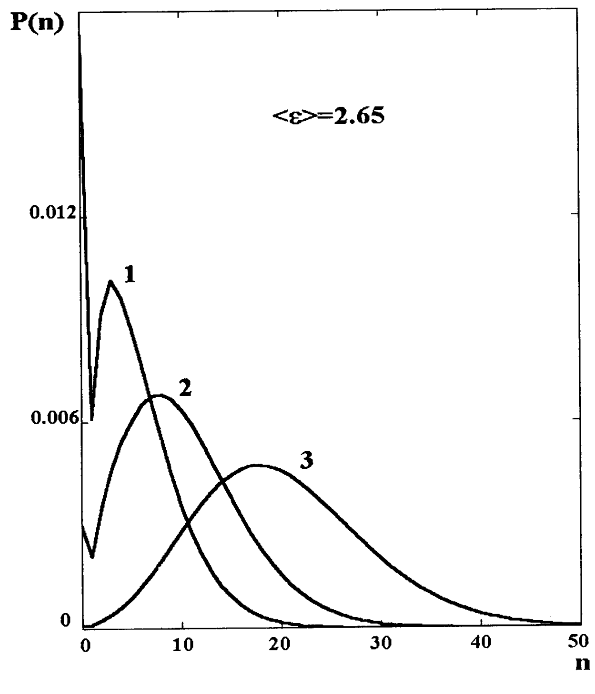

Fig. 2. *Simulation of Neyman distribution with <ε>=2.65 and different <n>: 1 - 5; 2 – 10; 3 - 20*

Let us adduce the relation for estimation of value $\langle\varepsilon\rangle$ relative error. From Eq. (10) follows that with passage from differentials to increments:

$$\frac{\Delta[\varepsilon]}{\varepsilon} = \frac{\Delta[\mathrm{var}(n)]}{\mathrm{var}(n) - \langle n\rangle} + \frac{\Delta[\langle n\rangle]}{\langle n\rangle}\frac{\mathrm{var}(n)}{\mathrm{var}(n) - \langle n\rangle}.$$

Then, using the estimation of sampling mean-square error of random value $\langle n\rangle$ (by central limit theorem) and error of sampling variance $var(n)$ (by Bessel approximation formula), we get

$$\frac{\Delta[\varepsilon]}{\varepsilon} = \frac{\mathrm{var}(n)}{\mathrm{var}(n) - \langle n\rangle}\left[\left(\frac{2}{N-1}\right)^{1/2} + \frac{1}{\langle n\rangle}\left(\frac{\mathrm{var}(n)}{N}\right)^{1/2}\right].$$

3. DISCUSSION AND CONCLUSIONS

On the base of all totality of known now experimental data for the first time it is shown that the reactor antineutrino statistics is described with a high accuracy by two-cascade Neyman type A distribution. Nontrivial properties of the moments of this distribution make it possible to determinate the important parameters of electron antineutrino source, i.e., average number of fissions $\langle\lambda\rangle$ and average number of antineutrinos per fission $\langle\varepsilon\rangle$. Let us consider below the obvious consequences of the cascade-stochastic properties of reactor antineutrinos statistics, which can essentially intensify the experimental possibilities and the quality of the researches of fundamental tasks in neutrino physics.

Firstly, the knowledge of value $\langle\lambda\rangle$ makes it possible to determine such important characteristics of electron antineutrino source as normalised antineutrino energy spectrum $\rho(E_\nu)$, which can be obtained by a normalization of the calculated antineutrino spectrum $N(E_\nu)$ on average number of fissions $\langle\lambda\rangle$:

$$\rho(E_\nu) = \frac{1}{\langle\lambda\rangle} N(E_\nu) . \qquad (12)$$

Here calculated antineutrino spectrum $N(E_\nu)$ is obtained by solution of an integral equation relative to «true» positron spectrum $N(T_e)$:

$$N(E_e) = \int N(T_e)\cdot \Re(T_e, E_e) dT_e \qquad (13)$$

with a consequent shift of obtained spectrum $N(T_e)$ on value of energy threshold of the reaction (2), which connects a positron kinetic energy T_e to antineutrino energy E_ν by the following relation

$$E_\nu = T_e + 1.804 + r_n$$

where 1.804 MeV is threshold of reaction (2), r_n (<< E_ν) is average recoil energy transmitted to neutron; $N(E_e)$ is an observable positron spectrum obtained, for instance, by spectrometry method [3], E_e is an energy detected by spectrometer; $\Re(T_e,E_e)$ is the spectrometer response function [3].

Secondly, such way of $\rho(E_\nu)$ determination, in it turn, enables to determine the inverse β– decay reaction cross-section:

$$\Sigma_\nu = \int dE_\nu \rho(E_\nu)\cdot \sigma_{\nu p}(E_\nu) . \qquad (14)$$

Here we note that at observance of certain known conditions this way can be used also in some dynamic neutrino experiments.

Thirdly, taking in consideration physical equivalence of values $<\varepsilon>$ and integral M_ν (Eq. (4)), we obtain new method of the determination of parameters α_i (describing the relative contribution from i-th isotope, for instance, i = 5; 9; 8; 1, in total fission cross-section) based on the following obvious system of equations:

$$\begin{cases} \rho(E_\nu) = \sum \alpha_i \rho_i, \\ \langle\varepsilon\rangle = \sum \alpha_i M_{\nu i}, \\ \Sigma_\nu = \sum \alpha_i \int dE_\nu \rho_i(E_\nu)\cdot\sigma_{\nu p}(E_\nu), \\ \sum \alpha_i = 1. \end{cases} \qquad (15)$$

Here as partial antineutrino energy spectra $\rho_i(E_\nu)$ is expedient using so-called "converted" antineutrino spectra [2]. Let us note, that further the role of these spectra will increase as in the field of applied researches (for instance, for neutrino diagnostics of inside-reactor processes and fuel-containing masses [3,7]) and in the fundamental researches (in particular, at the analysis of the reaction (2) for the determination of an axial constant of the weak charged current of nucleons and reactor antineutrino polarization [7]).

It is obvious, that the cascade-stochastic approach to α_i parameters determination has all necessary properties of an independent and absolute method. This is very actual method for remote on-line diagnostics of basic parameters of reactor core (starting from the determination of current heat power and heat-generation up to the dynamics of concentration of everyone actinide component of nuclear fuel and daughter fission products during reactor operation). In the elementary case of the reactor power determination it looks like this

$$W = \sum \langle E_f\rangle_i \cdot C_i = \sum \langle E_f\rangle_i \cdot \alpha_i \langle\lambda\rangle . \qquad (16)$$

The universality of the offered method for experimental determination of restrictions on the neutrino mixing parameters consists in obtaining more fine additional information about physical nature of the compound statistics of reactor antineutrinos. It, in turn, allows using the values reflecting higher degree approximation to the investigated process dynamics. For instance, taking into account the properties of Neyman statistics moments (9)-(10) and Eqs. (4), (15) in case of one-detector measurement scheme Eq. (8) can be modified like this

$$\frac{\langle\varepsilon\rangle_{\exp}}{\langle\varepsilon\rangle_{theor}} = \frac{[\mathrm{var}(n_\nu)/\langle n_\nu\rangle] - 1}{\sum \alpha_i \int \rho_i(E_\nu) dE_\nu} = 1 - 1/2\sin^2(2\vartheta) . \qquad (17)$$

Eq. (17) makes it possible to measure probable periodic changes of electron antineutrino intensity (due to detection only $\tilde{\nu}_e$ from the reaction (2)) by simple but effective procedure of the determination of statistical characteristics $<\varepsilon>_{exp}$, i.e., average number of electron antineutrinos per fission. Here $<\varepsilon>_{theor}$ is the same value but in oscillations absence. In this case indeterminacy of experiment due to the geometry and characteristics of the detector and also parameters of reactor (as source of antineutrinos) are excluded practically completely.

Finally, we note that our conclusions based on the found regularities, which describe type, structure and properties of reactor antineutrino distribution, by virtue of importance of the considered problem require the additional confirmation in the special test experiments.

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