LONG-WAVELENGTH TRANSITION RADIATION BY RELATIVISTIC ELECTRONS IN A "PRE-WAVE ZONE"

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The spectral-angular density of electromagnetic energy flux through the small detector in the "pre-wave zone" caused by relativistic electron in "forward" direction in the case of normal electron transition through a thin metallic transverse-bounded target is investigated. We show that detected intensity can be very differ from the spectral-angular density of radiation as well it is measured in the "wave zone" as it is measured in "pre-wave zone" by infinite detector. This distortion is depended from the detector placing in the "pre-wave zone" and from the ratio between transversal size of target and effective transversal diameter of radiation formation region.

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1. INTRODUCTION

The transition radiation (TR) from relativistic electrons is widely used in last years in various experimental investigations. For example, the long-wavelength TR is explored for diagnostic of electron's beams and as a source of quasi-monochromatic long-wavelength waves [1-4]. The thin metallic plates are usually used for production the long-wavelength transition radiation from the ultra-short relativistic electron's bunches. Transition radiation in the "forward" direction is formed over long distance for the case of high-energy electrons and millimeter waverange. This distance ("coherence length" or "pre-wave zone") is determined in the order of value by the relation $L \approx 2\gamma^2\lambda$, where γ is the Lorentz-factor of electron and λ is the radiated wave length. It can reach the tens of meters for relativistic electrons and millimeter waverange. So, if the detector is placed in the "pre-wave zone", the strong interference between own electrons fields and radiation fields is observed [5,6]. This well-known effect was investigated theoretically and experimentally for case when both detector and metallic plate have infinite diameter.

Recently the investigation of new effects connected with finite diameter of targets and detector was presented. So, the effect of targets diameter influence on the TR intensity was investigated [7-9]. The influence of detector diameter on the TR intensity was studied in [10] for "backward" TR in small-angle approximation. It has been shown in [7-10] that the TR intensity is suppressed in compare with classical case of infinite diameter of target and detector, when the target or detector diameter is less or equal to $\gamma\lambda$. Such effects can be very important for analysis of modern experiments concerned to the millimeter and sub-millimeter TR of relativistic ultra-short electron bunches.

In the present paper we will investigate the long-wavelength "forward" TR from relativistic electron on a thin metallic transversally bounded target. We will analyze the spectral-angular distribution of electromagnetic energy flux through the small detector within "formation zone".

2. THE TRANSITION RADIATION AND OWN ELECTRON ELECTROMAGNETIC FIELD

We will consider the case of normally electron passage through the center of thin metallic disc with radius a . We assume that disk is fairly thin, i. e. the thickness of target is less than radiated wavelength λ , but much more than "penetration length" of electron field into a metal. Let's the electron is moved along the OZ axis. We will interest by the energy flux through the small plate (detector), which placed at the distance r from the target center. Firstly, we should calculate the electromagnetic fields, which are formed after electron passage through the target. The summary electric field in forward (concerning electrons passage) direction is consisted of the own electron field $\mathbf{E}^{(e)}(\mathbf{r},t)$ and TR field $\mathbf{E}'(\mathbf{r},t)$:

$$\mathbf{E}^{+}(\mathbf{r},t) = \mathbf{E}^{(e)}(\mathbf{r},t) + \mathbf{E}'(\mathbf{r},t) . \quad (1)$$

The Fourier component of the $\mathbf{E}^{(e)}(\mathbf{r},t)$ field with respect to time is determined by the expression

$$\mathbf{E}^{(e)}_{\omega}(\mathbf{r}) = \frac{2e\omega}{v\gamma}\left(\frac{\boldsymbol{\rho}}{\rho}\frac{c}{v}K_1\left(\frac{\omega\rho}{v\gamma}\right) - i\frac{\mathbf{v}}{v}\frac{1}{\gamma}K_0\left(\frac{\omega\rho}{v\gamma}\right)\right)\exp\left(iz\frac{\omega}{v}\right), \quad (2)$$

where e - is an electron's charge, $v = |\mathbf{v}|$, $\boldsymbol{\rho}$ is transversal coordinate ($\mathbf{r} = \boldsymbol{\rho} + z \cdot \mathbf{e_z}$, $\mathbf{e_z}$ is unit vector of OZ axis), K_0 and K_1 are McDonald's functions of zero and first kind.

In the relativistic case (when $\gamma >> 1$) the electron's own field can be considered as transversal. It can be represented with accuracy of γ^{-1} by expression

$$\mathbf{E}_\omega^{(e)}(\mathbf{r}) \approx \frac{\boldsymbol{\rho}}{\rho}\frac{e\omega}{v^2\gamma}2K_1\left(\frac{\omega\rho}{v\gamma}\right)\exp\left(iz\frac{\omega}{v}\right). \quad (3)$$

We assume that target is an ideal thin metallic screen, therefore the part of own electron field which fall into target is fully reflected and the part which missed the target is diffracted onto space $z > 0$. So, we can write for summary electric field and TR field the next conditions

$$\mathbf{E}_\omega^{(e)}(\mathbf{r}) + \mathbf{E}'_\omega(\mathbf{r}) = \Theta(\rho - a)\mathbf{E}_\omega^{(e)}(\mathbf{r}), \; z = 0,$$
$$\mathbf{E}'_\omega(\mathbf{r}) = -\Theta(a - \rho)\mathbf{E}_\omega^{(e)}(\mathbf{r}), \; z = 0, \quad (4)$$

where $\Theta(x)$ - is a Heaviside step function: $\Theta(x) = 1$, if $x \geq 0$, and $\Theta(x) = 0$, if $x < 0$. The "forward" radiation field $\mathbf{E}'(\mathbf{r},t)$ is propagating in positive direction of OZ axis as a wave packet of free electromagnetic waves

Now, by using the equations (3) and (5) we obtain the expression for the "forward" and "backward" radiation fields in the point $\mathbf{r} = \boldsymbol{\rho} + z \cdot \mathbf{e_z}$

$$\mathbf{E}'_\omega(\mathbf{r}) = -\frac{2e}{v}\frac{\boldsymbol{\rho}}{\rho}\int_0^\infty \chi^2 d\chi \frac{J_1(\chi\rho)\cdot F(\chi a, \frac{\omega a}{v\gamma})}{\chi^2 + \left(\omega/v\gamma\right)^2} \times \exp\left(\pm iz\sqrt{\left(\omega/c\right)^2 - \chi^2}\right), \quad (5)$$

where

$$F = \left(\chi^2 + \left(\omega/v\gamma\right)^2\right)\frac{\omega}{v\gamma\chi}\int_0^a \rho\, d\rho J_1(\chi\rho)K_1(\omega\rho/v\gamma). \quad (6)$$

The J_1 is Bessel function of first kind. The formula (5) is describing the "forward" TR field which appeared after relativistic electron passage through a thin metallic disk. This expression is correct under all distances greater than radiated wavelength from the target.

3. SPECTRAL-ANGULAR DENSITY OF ELECTROMAGNETIC ENERGY FLUX

Now let's consider the electromagnetic energy flux that crossed over the entire observation time through the small detecting plate that is perpendicular to the vector $\mathbf{r}$ and placed at the distance r from the target. We should calculate the value of Poynting vector. The flux of the Poynting vector is given by

$$S = \frac{c}{4\pi}\int(\mathbf{E}\times\mathbf{H})\mathbf{n}\, d^2\sigma\, dt, \quad (7)$$

where $\mathbf{n} = \mathbf{r}/r$ and $d^2\sigma$ is an elementary area with normal vector $\mathbf{n}$. Here the integration is over the surface of detecting plate and the time of observation. We express the field $\mathbf{H}$ through $\mathbf{E}$ and expand these fields into Fourier integrals by time. After that, we can write summary energy flux in the "forward" direction on a unit of frequency and on the unit of solid angle by expression

$$\frac{dS}{d\omega\, d\Omega} = \frac{cr^2}{4\pi^2}\left|\mathbf{E}_\omega^{(e)}(\boldsymbol{\rho},z) + \mathbf{E}'_\omega(\boldsymbol{\rho},z)\right|^2, \quad \omega > 0. \quad (8)$$

Here $d\Omega = \sin\vartheta\, d\vartheta\, d\varphi$ (ϑ and φ is the polar and the azimuthal angles). We can calculate the transition radiation intensity and energy flux through the detecting plate by using the equations (5). So, the spectral-angular density of electromagnetic flux is followed

$$\frac{dS}{d\omega\, d\Omega} = \frac{e^2}{\beta^4 c\pi^2}y^2\cos\vartheta \cdot \left|S^{(e)} - S^{(rad)}\right|^2, \quad (9)$$

where

$$S^{(e)} = \gamma^{-1}K_1\left(y\sin\vartheta\,\gamma^{-1}\right)\exp\left[iq\beta^{-1}\right],$$

$$S^{(rad)} = \beta\int_0^1 x^2 dx \frac{J_1(xy\sin\vartheta)\cdot F(x\gamma, u)}{x^2 + \gamma^{-2}}\exp\left(iq\sqrt{1 - x^2}\right),$$

$$F(x\gamma, u) = \frac{(x\gamma)^2 + 1}{x\gamma}\int_0^u w\, dw K_1(w)J_1(x\gamma w),$$

$$y = \omega \cdot r \cdot c^{-1}, \; q = y\cos\vartheta, \; u = \omega \cdot a \cdot (c\cdot\gamma)^{-1}, \beta = v \cdot c^{-1}.$$

The expression (9) describes the energy flux in "forward" direction through the small detecting plate that is seemed on the solid angle $d\Omega$ and placed into point $\mathbf{r} = \boldsymbol{\rho} \pm z \cdot \mathbf{e_z}$. In other words, it is a spatial distribution of electromagnetic energy flux for waves with the frequency ω. It is obviously that spectral-angular density of electromagnetic energy flux is composed from the term of energy flux of own electron field ("electron's term"), from the term of energy flux of TR field ("radiation term") and the interference term. The terms of equation (9) depends on distance from the target and on target's diameter. Further we will analyze the angular distribution of spectral-angular density of "forward" electromagnetic energy flux for various distances from the target.

4. DISCUSSION

Let's consider the angular distribution of electromagnetic energy flux (9).

While the transversal size of target is essentially greatly than transversal diameter of formation zone ($u >> 1$), the function F is near to unit. This case corresponds to the problem of transition radiation by relativistic electron on the infinite plate. So, for the distances $z > \gamma\lambda$ we can to simplify the formula (9)

$$\frac{dS}{d\omega\, d\Omega} = \frac{e^2}{\beta^2 c\pi^2 \gamma^2} y^2 \cos\vartheta \left| S_0^{(e)} + S_0^{(rad)} \right|^2 , \qquad (10)$$

where

$$S_0^{(e)} = \beta^{-1} K_1\left(y \sin\vartheta\, \gamma^{-1}\right),$$

$$S_0^{(rad)} = \int_0^\infty u^2 du \frac{J_1(u y \gamma^{-1} \sin\vartheta)}{u^2+1} \exp\left(-i \frac{y\cos\vartheta}{2\gamma^2} u^2\right).$$

The term of radiation field energy flux in the equation (10) depend on the value $\frac{y\cos\vartheta}{2\gamma^2}$. It is the ratio between the longitudinal detector-target distance and the longitudinal "coherence length" ("pre-wave zone") $l = 2\gamma^2 c/\omega$. Such dependence for $S_0^{(red)}$ from parameter z/l is caused by the small diameter of detector. Let's explain this fact in detail. Toward this end we consider the spectral-angular density of electromagnetic energy "forward" flux in case when the detector is a transversally-infinite plate. It represented by the following expression [7]

$$\frac{dS_\infty}{d\omega\, d\Omega} = \frac{e^2}{\pi^2 c} \frac{\sin^2\vartheta}{\sin^2\vartheta + \gamma^{-2}} \left| S_\infty^{(e)} - S_\infty^{(rad)} \right|^2 , \qquad (11)$$

where

$$S_\infty^{(e)} = \exp(iz\omega/v), \quad S_\infty^{(rad)} = \exp(iz\cos\vartheta\, \omega/c).$$

The module of $S_\infty^{(rad)}$ doesn't depend on target-detector distance, whereas the module of $yS_0^{(rad)}$ sufficiently depends on this distance. Let's note that the spectral-angular density of TR does not depend from the distance to the target in the classical case when the detector is an infinite plate. Following reason caused this situation. The fields of TR are formed by the radiation from the target surface with $\lambda\gamma$ effective diameter. So, the interference between radiation from various elementary radiators on the target's surface is appeared. Thanks to this interference the term $S_0^{(rad)}$ depends on the target-detector distance. Within the limits of "pre-wave zone" (i.e., when $z \le 2\gamma^2\lambda$) only some part of whole radiated field is fall into detector. The radiation source is not dot-like in this case. In the "wave zone" (i.e., when $z >> 2\gamma^2\lambda$) the module of $yS_0^{(rad)}$ doesn't depends from the target-detector distance. The radiation source is seemed as a dot-like source in this case. The above-considered effect occurred when the detector is small plate and it is absents when the detector is an infinite plate.

The whole spectral-angular distribution (10) has also the target-detector distance dependence that is caused by the interference term. It is the interference between own electron field and TR fields, which is sufficient within the "coherence length" $l = 2\gamma^2 c/\omega$.

The graphs on Fig. 1 show the angular distribution of electromagnetic energy flux (10) under various distances from the target. The formula (10) showed that the detector is placed closely to the electron's trajectory (i.e., when $\rho \le \lambda\gamma$) the own electron's field gives the main contribution to the summary electromagnetic energy flux. This situation corresponds to the small-angle region on Fig. 1. If the detector is placed far from the electron's trajectory (i.e., when $\rho \ge \lambda\gamma$) the main contribution in the whole electromagnetic energy flux is caused by the TR fields. This case corresponds to the great-angle region on Fig. 1.

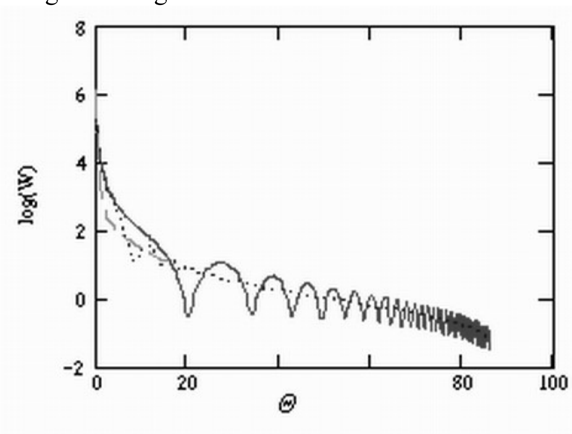

***Fig. 1**. Angular distribution of spectral-angular density of electromagnetic energy flux in "forward" direction for electron with $\gamma = 30$. Solid curve corresponds case when $y = 50$, dotted curve - $y = 300$ and dashed curve - $y = 3000$. W(Θ)=cπ²e⁻²(dS/dωdΩ) and Θ is marked in a degree scale*

The angular distribution on Fig. 1 have showed that if the detector is placed in the "pre-wave zone" ($y = 50$, $y = 300$), the summary electromagnetic flux is strongly oscillate in the large-angles region. These oscillations are caused by the interference both between own electron's field and TR fields, and radiation fields from the various elementary radiators on the target's surface. Both this interference effects are decreased under increasing of target-detector distance. So, within the "wave zone" ($y = 3000$) these effects are diminished.

Now consider the case when the target's diameter is near or less than $\lambda\gamma$. In this case the function F has strong oscillations under $u \approx 1$ and $F << 1$ under $u << 1$. The suppression of "radiation term" in the equation (9) is occurred. The additional distortion of summary electromagnetic energy flux through the small detector will be observed.

Fig. 2 shows the angular dependence of electromagnetic energy flux (9) in the "pre-wave zone" for various diameters of target. The detected summary spectral-angular electromagnetic energy "forward" flux is strongly distorted at the large angles when target diameter is near to the value $\lambda\gamma$.

5. CONCLUSIONS

We have considered the long-wavelength transition radiation from relativistic electron on a thin metallic disk in the "pre-wave zone". The general formula for spectral-angular density of electromagnetic energy flux through the small detector in the "forward" direction is obtained. We have analyzed the spectral-angular distribution of "forward" electromagnetic energy flux through the small detector.

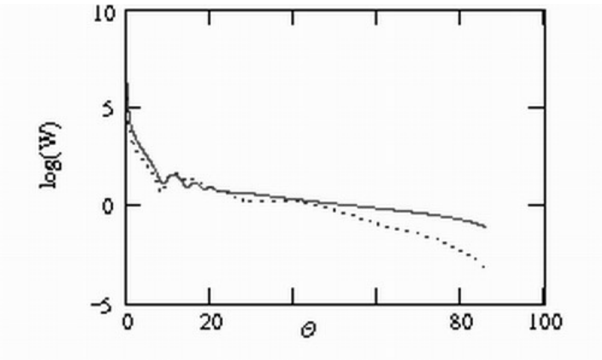

Fig. 2. *Angular distribution of spectral-angular density of electromagnetic energy flux in "forward" direction for electron with* $\gamma = 30$ *on the finite-diameter target. Dotted curve corresponds to the case when* $u = 0.5$*, solid curve -* $u = 2$*. The distance from the target* $y = 300$*,* $W(\Theta)=c\pi^2e^{-2}(dS/d\omega d\Omega)$ *and Θ is marked in a degree scale*

It is shown that within the limits of "pre-wave zone" the interference between radiation fields from various elementary radiators and interference between own electron field and whole radiation field are occurred. Both the spectral-angular density of whole electromagnetic energy flux and spectral-angular density of TR is depend on distance from the target.

We have analyzed the angular distribution of electromagnetic energy flux in the "pre-wave zone". The strong oscillation of intensity of electromagnetic energy flux is observed in the intermediate range of observation angles. For small angles (when detector is placed nearly from the electron trajectory) and for big angles (when detector is placed at the length much more than $\lambda\gamma$ from the electron trajectory) the own electron's field energy flux and the TR energy flux is dominated correspondingly.

For the case when the target diameter is equal or less than transversal size of "pre-wave zone" $\lambda\gamma$ the sufficient suppression of TR field contribution to the whole electromagnetic energy "forward" flux is observed.

A number of experiments concerning long-wavelength TR of relativistic electron bunches were performed in the last years [1-4,6,11,12]. For this experiments conditions (electrons with $\gamma \approx 10 \div 100$, long-wavelength TR with $\lambda \approx 1 \div 10$ $cm.$) the longitudinal ($l \approx 2\gamma^2\lambda$) and transversal ($l_\perp \approx \lambda\gamma$) size of "pre-wave zone" are the macroscopic values and can be compared with distance to the detecting setup and transversal target's size. So, the considered interference effects can play important role in such experiments.

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