DICKE SUPERRADIANCE ON LANDAU LEVELS

P.I. Fomin, A.P. Fomina

Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine
e-mail: pfomin@bitp.kiev.ua

It is shown that in the inverted system of nonrelativistic electrons in rarefied magnetized plasma, when electron density on high Landau levels exceed some critical value defined by its transversal energy, magnetic field and temperature, the nonequilibrium phase transition occures with domain ordering of mutual orientations of interacting rotated dipoles. The intensity of cyclotron radiation of each domain in ordered phase becomes proportional to square of electron number in it.

PACS: 41.60.-m; 42.50.Fx

1. INTRODUCTION

The phenomenon of superradiance (SR) was considered first in the famous work by Dicke [1] on the example of two-level model. Now rather significant literature (see for example, reviews [2,3,4]) is devoted to it, but, as it is marked by many authors, a theme is far from being exhausted, many interesting and physically important questions and situations remain not investigated.

In the present work the question of possbility and conditions of SR formation in the inverted system of electrons on high Landau levels (1) with $n >> 1$ are investigated

$$E_{\perp} = n\hbar\omega_H, n = 0,1,2,... \tag{1}$$

$$\omega_H = eH/mc. \tag{2}$$

As it is known, for generation of the induced coherent radiation in systems like masers the equidistancy of levels is an obstacle because of specific competition of radiation and absorption processes in this case. In a case of SR we deal not with induced, but with spontaneous radiation and here, as we shall see, equidistancy appears of advantage. This is, first, because the SR regime is usually realized in open finite systems without mirrors when radiation leaves active volume of generation quickly enough, practically having no time to get in absorption regime [3]. Second, in this case all inverted electrons, as a rule, occupying not one level, but some significant interval of high levels $\Delta n (n >> \Delta n >> 1)$, because of equidistancy, radiate the same mode on frequency (2), and also, as we shall see, in the same rate, do not depend on initial energy.

The phenomenon of SR arises when in "coherence domains", with the sizes R_0 smaller than a wave length λ all N_0 radiating dipoles gradually, during radiation, are aligned in one direction by a dipole - dipole interaction between them in "a near zone" $R_0 << \lambda$, - so that in result the total dipole of the domain D appears in N_0 times larger than an elementary dipole d. Therefore the intensity of collective dipole radiation becomes proportional to N_0^2 - as opposite to N_0 in the case of radiation of uncorrelated dipoles.

Transition in such a correlated polarized state is similar to phase transition in magnetics or ferroelectrics, and for its description it is convenient to use the Weiss method of mean self-consistent field [6]. Let's note, that the considered phase transition is nonequilibrium and has all features of the self-organizing phenomenon in dissipative systems.

2. CYCLOTRON SUPERRADIATION ON LANDAU LEVELS

Levels (1) at high n correspond to quantum states which wave functions are located near the classical Larmor orbits with radiuses $r_L = V_{\perp}/\omega_H$.

In this connection we can proceed to (quasi-) classical description of such states and transitions between them. In a classical limit in coordinate system, in which longitudinal movement is excluded, these orbits are given as

$$r_{\perp}(t) = r_L\{\cos(\omega t + \alpha), \sin(\omega t + \alpha), 0\}, \tag{3}$$

where in braces the cartesian components of the vector $r_{\perp}$ are written out. In other words, quantum states are set by narrow enough wave packages at "Landau orbits" which represent the certain superpositions of the Landau wave functions [5].

Being inverted at the beginning on high levels, electrons start to fall downwards on stear steps (1), radiating quants with frequency (2). Differential (on corners) and integral intensities of dipole radiation of one electron in a classical limit are described by the known formulas [7]

$$dI/d\Omega = [\ddot{d} \times \vec{n}_k]/4\pi c^3, \vec{n}_k \equiv \vec{k}/k, \tag{4}$$

$$I = \frac{2e^2\omega_H^2 V_{\perp}^2}{3c^3} = \frac{4e^2\omega_H^2}{3mc^3}E_{\perp}, \tag{5}$$

From (5) the following evolution the law of electron energy follows

$$\frac{dE_{\perp}(t)}{dt} = -I = -\frac{E_{\perp}}{\tau}, \tau = \frac{3mc^3}{4e^2\omega_H^2}, \tag{6}$$

$$E_{\perp}(t) = E_{\perp}(0)\exp(-t/\tau). \tag{7}$$

We see that time of radiation τ does not depend on $E_{\perp}$, i.e. electrons with various (within the limits of dispersion $\Delta E_{\perp}(0) << E_{\perp}(0)$) initial energies will fall downwards at the same rate. It is easy to see, that at the same rate the dispersion of energy will decrease also

$$\Delta E_{\perp}(t) = \Delta E_{\perp}(0)\exp(-t/\tau), \tag{8}$$

so that the relation is true

$$\Delta E_{\perp}(t)/E_{\perp}(t) = Const << 1. \tag{9}$$

This allows judging about evolution of the whole collective of electrons by evolution of their mean energy and other mean quantities. Therefore, for simplicity, we shall understand further under the symbols $E_{\perp}, V_{\perp}, d_0 = er_L$ and etc., their average values over ensemble of inverted electrons.

It is possible to divide the total volume V, occupied by N inverted electrons on subvolumes V_{coh}, or "coherence areas", which sizes R_0 are smaller in comparison with radiated wave length λ but are greater in comparison with radiuses of Larmor orbits, describing the sizes of elementary dipoles

$$r_L << R_0 << \lambda. \tag{10}$$

Thus, a big enough number of N_0 radiating dipoles will be in volume V_{coh}

$$n_e V = N >> N_0 = n_e V_{coh} >> 1 \tag{11}$$

Let us consider a total dipole of such subsystem

$$\vec{D}(t) = \sum_{j=1}^{N_0} \vec{d}_j(t) = e\sum_{j=1}^{N_0} \vec{r}_{j\perp}(t). \tag{12}$$

Similarly to (4) for one electron, collective dipole radiation of our subsystem will be described by the formula

$$dI = \frac{[\ddot{\vec{D}} \times \vec{n}_k]^2}{c^3}\frac{d\Omega}{4\pi} = \frac{\omega_H^4}{c^3}\left[\vec{D}^2 - (\vec{D}\cdot\vec{n}_k)^2\right]\frac{d\Omega}{4\pi}. \tag{13}$$

By substituting here (12) and (3) and averaging over the period $T = 2\pi/\omega_H$, we have

$$< dI > = \frac{e^2\omega_H^2 V_{\perp}^2}{c^3}[N_0 + \sum_{i\neq j}^{N_0(N_0-1)} \cos(\alpha_i - \alpha_j)]\frac{1+n_z^2}{2}\frac{d\Omega}{4\pi}. \tag{14}$$

The multiplier $(1+n_z^2)/2$ reflects the anisotropy properties of dipole radiation of nonrelativistic electrons. The second member in square brackets describes the correlation effects connected with mutual aligning of dipoles. If correlations are not present, i.e. contributions of all $\cos(\alpha_i - \alpha_j)$ are mutually compensated and in the sum give zero, then in (14) works the first member only that corresponds to the total radiation of N_0 independent elementary dipoles.

In the case of full correlation, when all $\cos(\alpha_i - \alpha_j) = 1$, the formula (14) gives

$$< dI >_{corr} = N_0^2 \frac{e^2\omega_H^2 V_{\perp}^2}{c^3}\frac{1+n_z^2}{2}\frac{d\Omega}{4\pi}, \tag{15}$$

i.e. in comparison with radiation of N_0 noncorrelative dipoles the intensity grows in N_0 times. This is just SR [1].

Radiation time of such correlated dipoles radiating coherently will decrease in N_0 times in comparison with time (6)

$$\tau_{coh} = \tau / N_0 \tag{16}$$

Really, it is possible to expect only partial positive correlation of phases, i.e. partial aligning of all dipoles, when the average over ensemble value of phase difference *cos* is positive. Having replaced in (14) all $\cos(\alpha_i - \alpha_j)$ by their average value, we receive

$$< dI > = N_0^2 \frac{e^2\omega_H^2 V_{\perp}^2}{c^3}[N_0 + N_0(N_0-1) < \cos\alpha >]\frac{1+n_z^2}{2}\frac{d\Omega}{4\pi} \tag{17}$$

In this case the intensity of coherent radiation is proportional to $N_0^2 < \cos\alpha >$.

We proceed now to consideration of the mechanism of spontaneous aligning of the dipoles giving rise to SR regime.

3. POLARIZATION PHASE TRANSITION IN THE "COHERENCE DOMAINS"

To solve the problem of the phase transition we apply here the Weiss method of self-consistent mean field [6] approved in the theory of spontaneous magnetization. Let us consider the potential energy of trial dipole $d_0(r_0,t)$ with the electric field $E(r_0,t)$, created at the point r_0 by the other $(N_0 - 1)$ dipoles

$$U(d_0) = -d_0(r_0,t)\cdot E(r_0,t), \tag{18}$$

where

$$\vec{E}(\vec{r}_0,t) = \sum_{j}^{N_0-1} \frac{3n_j(n_j\cdot d_j(t)) - d_j(t)}{\left|\vec{r}_0 - \vec{r}_j\right|^3},$$

$$\vec{n}_j = (\vec{r}_0 - \vec{r}_j)/\left|\vec{r}_0 - \vec{r}_j\right|. \tag{19}$$

All dipoles rotate under the law (3) and radiate and their electric field is not static, but it also rotates with frequency ω_H. Therefore, the use of expression (19) for a field $E(t)$ demands explanation. The matter is that the conditions (10), determining the coherence volume, mean that various dipoles of the domain are in the so-called "near zone" $(R_0 << \lambda)$ relativly to one another, where the main member in decomposition of retarded potentials and fields over degrees of small parameters (r_L/R_0) and (r_L/λ) appears just in expression (19) (see on this subject, for example, [7]).

Averaging (18) over the rotation period, we notice, that points r_0 and r_j characterize not instant positions of rotating electrons, but positions of the motionless rotation centres and consequently do not depend on time. Having substituted (19) to (18) and averaging over the period, we receive

$$< U(\vec{r}_0) > = -\frac{d_0^2}{2}\sum_{j=1}^{N_0-1}\frac{1-3(n_{jz})^2}{\left|\vec{r}_0-\vec{r}_j\right|^3}\cos(\alpha_0-\alpha_j), \quad (20)$$

where $d_0 = er_L, n_{jz} = (z_0 - z_j)/\left|r_0 - r_j\right|$. It is possible to replace the sum (20) approximately by the integral over coordinates in limits of "coherence volume" with an obvious measure $n_e dV_j$, representing an average number of dipoles in the element of volume $dV_j \equiv (dr_j)$ in a vicinity of a point r_j. But before writing out this integral, we shall notice, that because the position of our trial dipole should not be allocated, it is necessary to average (20) over this parameter, i.e. to enter additional integration $(dr_0)/V_{coh}$. Besides, likewise an average field method, we replace $\cos(\alpha_0 - \alpha_j)$ in (20) by its value averaged over ensemble. After all these averagings we get the expression

$$< U > = -\frac{d_0^2 n_e}{2} < \cos\alpha > \int\frac{(d\vec{r}_0)}{V_{coh}}\int d\vec{r}_j\frac{1-3(n_{jz})^2}{\left|\vec{r}_0-\vec{r}_j\right|^3}. \quad (21)$$

By the replacement of variables $\{r_0, r_j\} \to \{r = r_0 - r_j, R = 1/2(r_0 + r_j)\}$ and integrating over dR, we get

$$< U > = -\frac{d_0^2 n_e}{2} < \cos\alpha > \int_{V_{coh}}\frac{r^2 - 3z^2}{r^5}(d\vec{r}). \quad (22)$$

Aligning of dipoles over directions is energetically favourable by virtue of increasing, thus, of the negative contribution to potential energy < *U* >. Therefore, the correlations necessary for that will occur only in that part of coherence volume where the area of integration over relative coordinates satisfies the condition

$$r^2 - 3z^2 > 0, \quad (23)$$

i.e. in the area like a flattened out circular cylinder. We will name this part of coherence area a "coherence domain" or otherwise a "domain of self-polarization". In the similar next domain the direction of an average vector of polarization should be close to opposite to minimize in system as a whole the positive energy of polarization electric field. It is known that by virtue of the similar reason the macroscopical volumes of magnetics and ferroelectrics are broken into domains also. Domains are divided by transition regions ("domain walls") within which limits the turn of polarization vector occur.

We return now to an estimation of integral in (22) with restriction (23). It is convenient, obviously, to calculate it in cylindrical coordinates, in which

$$r^2 = \rho^2 + z^2, r^2 - 3z^2 = \rho^2 - 2z^2 > 0. \quad (24)$$

We limit the integration over ρ by (ρ_1, ρ_2), so it is easy to show that

$$< U > = -\frac{2\pi}{3\sqrt{3}}\ln\left(\frac{\rho_2}{\rho_1}\right)^2 d_0^2 n_e < \cos\alpha >. \quad (25)$$

We will consider now a question about the minimal and maximal limits (ρ_1, ρ_2) over relative coordinate $\rho = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ in a plane between two dipoles in the coherence domain. We remind that the characteristic sizes of initial "coherence volume" were determined by conditions (11): $r_L << R_0 << \lambda$. The inequality (24) means that in relative coordinates "the coherence domain" has the form of flattened out circular cylinder with radius $2R_0$. Hence, the maximal value of ρ is $\rho_2 = 2R_0$, the minimal value should be taken about double Larmor radius $\rho_1 \sim 2r_L$, because at smaller distances between centres of dipoles the interaction between pair of electrons does not carry dipole character any more and it is impossible to use dipole formulas. Thus, we can write $\ln(\rho_2/\rho_1) \approx \ln(R_0/r_L)$. As the relation R_0/r_L enters under a mark of the logarithm the result is poorly sensitive to exact value of this relation. Thus, taking into account a condition $R_0 << \lambda$ we can substitute here R_0 by the quantity of the order of $\lambda/10$. Taking into account, that $r_L = V_\perp/\omega_H$ and $\lambda\omega_H = 2\pi c$, we can write

$$\ln\left(\frac{\rho_2}{\rho_1}\right)^2 \approx \ln\left(\frac{\lambda\omega_H}{10V_\perp}\right)^2 \approx \ln\left(\frac{mc^2}{5E_\perp}\right). \quad (26)$$

With the account of (26) the expression (25) takes the following form

$$< U > = -\frac{2\pi}{3\sqrt{3}}\ln\left(\frac{mc^2}{5E_\perp}\right)^2 d_0^2 n_e < \cos\alpha >. \quad (27)$$

To find the $< \cos\alpha >$, we address now to Weiss method [6]. For this purpose at the beginning it is necessary to consider the response of our system of rotating dipoles in plane (*x,y*)

$$d_j = d_0\{\cos(\omega t + \alpha_j), \sin(\omega t + \alpha_j), 0\} \quad (28)$$

to the external homogeneous electric field

$$E_e = E_e\{\cos(\omega t + \alpha_0), \sin(\omega t + \alpha_0), 0\} \quad (29)$$

rotating synchronously with dipoles. The potential energy of a dipole in this field, averaged on the period of rotation, obviously, is

$$\overline{U} = -\overline{d_j(t)\cdot E_e(t)} = -d_0 E_e \cos(\alpha_j - \alpha_0), \quad (30)$$

thus it is seen, that the aligning of dipoles along the field with radiation of released energy is energetically favourable. The thermal fluctuations resist this tendency. They are realized in rarefied magnetized plasma mainly in the form of plasma fluctuations and Alfven waves. The distribution over the phase differences is given by Bolzman formula

$$\rho(\alpha) = C\exp\left(\frac{-U(\alpha)}{kT}\right) = C\exp\left(\frac{d_0 E_e}{kT}\cos\alpha\right) \quad (31)$$

with normalizing factor

$$C^{-1} = \int_0^{2\pi} d\alpha\, \exp\left(\frac{d_0 E_e}{kT}\cos\alpha\right) = 2I_0\left(\frac{d_0 E_e}{kT}\right), \quad (32)$$

where $I_0(x)$ is a modified Bessel function of a zero order [8]. Average value of $< \cos\alpha >$ is determined by the integral

$$< \cos\alpha > = \int_0^{2\pi}\cos\alpha\,\rho(\alpha)d\alpha = \frac{d}{dx}\ln I_0(x) = \frac{I_1(x)}{I_0(x)}, \quad (33)$$

where

$$x = d_0 E_e / kT \quad (34)$$

So, the external field induces the nonzero correlator of a phase difference (33), i.e., otherwise, polarizes the system. A polarization measure is an average dipole moment of unit volume

$$P = n_e d_0 < \cos\alpha > . \quad (35)$$

Polarization generates an additional internal electric field

$$E_p = \nu \cdot P , \quad (36)$$

where ν is some dimensionless parameter which we shall define later. The field E_p, in turn, strengthens the polarization. This feedback effect will be taken into account, if in (34) we replace E_e by the sum $E_e + E_p = E_e + \nu P$ that corresponds to ideology of self-consistent mean field by Weiss. As a result, we receive the nonlinear equation for polarization P

$$P = n_e d_0 < \cos\alpha >= n_e d_0 F\left(\frac{d_0 (E_e + \nu P)}{kT} \right), \quad (37)$$

$$F(x) = I_1(x) / I_0(x) . \quad (38)$$

Excluding an external field, we receive "the equation of the self- consistency"

$$P = n_e d_0 \cdot F\left(\frac{d_0 \nu}{kT} P \right) . \quad (39)$$

We consider now a condition of existence of its nontrivial solution. Having entered a variable $z = \frac{d_0 \nu P}{kT}$, we rewrite the equation (39) in the following form

$$z = \frac{{d_0}^2 \cdot \nu n_e}{kT} \cdot F(z) . \quad (40)$$

Function *F (z)* has asymptics [8]

$$F(z) = 1 - \frac{1}{2z} - \frac{1}{8z^2} - ..., \quad (z >> 1) \quad (41)$$

$$F(z) = \frac{z}{2}\left[1 - \frac{z}{8} + ... \right], \quad (z << 1) . \quad (42)$$

From (40) and (41) it follows that at large z the solution exists and corresponds to the maximal polarization

$$z = \frac{{d_0}^2 \cdot \nu n_e}{kT} >> 1 , \quad (43)$$

$$P_{\max} = n_e d_0 = \frac{kT}{\nu d_0} \cdot z >> \frac{kT}{\nu d_0} . \quad (44)$$

To determine threshold value of electron density, higher of which there is a nontrivial solution of the equation (39), it is necessary to consider the asymptotic (42). Being limited by the first member, we receive from (40)

$$z = \frac{{d_0}^2 \cdot \nu n_e}{2kT} \cdot z, \quad (z << 1) . \quad (45)$$

It follows from here, that the density n_e has a critical (threshold) value, if it satisfies the condition

$$\left(\frac{\nu}{2} \frac{{d_0}^2 \cdot n_e}{kT} \right)_c = 1 , \quad (46)$$

at $n_e \geq n_{ec}$ a nontrivial domain self-polarization appears.

To define the factor ν, it is necessary to compare the expression for potential energy following from (35), (36) and (30)

$$< U >= -\nu n_e d_0^2 < \cos\alpha > , \quad (47)$$

with the expression (27) received earlier. Demanding equality between them, we receive

$$\nu = \frac{2\pi}{3\sqrt{3}} \ln\left(\frac{mc^2}{5E_\perp} \right), \quad (E_\perp << mc^2 / 5) . \quad (48)$$

It is useful to express d_0^2 by energy $E_\perp$ and magnetic field H

$$d_0^2 = (e r_L)^2 = \frac{2mc^2 E_\perp}{H^2} . \quad (49)$$

As a result, the criterion of occurrence of domain self-polarization of the inverted electron system on high Landau levels, leading to SR, takes the form

$$\frac{2\pi}{3\sqrt{3}} \ln\left(\frac{mc^2}{5E_\perp} \right) \frac{mc^2}{H^2} \frac{n_e E_\perp}{kT} \geq 1 . \quad (50)$$

ACKNOWLEDGMENT

The authors would like to thank V.N. Mal`nev for helpful discussion.

REFERENCES

1. T.L. Dicke R.H. Coherence in Spontaneous Radiation Processes // *Phys. Rev.* 1954, v. 93, № 1-2, p. 99-110.
2. V.V. Zheleznyakov, V.V. Kocharovskiy, and Vl.V. Kocharovskiy. Polarization waves and superradiance in active media // *Uspekhi Fizicheskikh Nauk.* 1989, v. 159, № 2, p. 193-260 (in Russian).
3. A.V. Andreev. Optical superradiation: new ideas and new experiments // *Uspekhi Fizicheskikh Nauk.* 1990, v. 160, № 12, p. 1-46 (in Russian).
4. L.I. Men`shikov. Superradiance and related phenomena // *Uspekhi Fizicheskikh Nauk.* 1999, v. 169, № 2, p. 113-154 (in Russian).
5. L.D. Landau, E.M. Lifshitz. *Quantum mechanics*, M.: “Nauka”, 1974, 752 p.
6. J.S. Smart. *Effective field theoris of magnetism.* 1966, W. Saunders Company, Philadelphia-London.
7. L.D. Landau, E.M. Lifshitz. *Field theory.* M.: ”Nauka”, 1973, 504 p.
8. *Handbook of Mathematical Functions*, Eds.M. Abramovitz and I. Stegun, Nat. Bureau of Stand. 1964 (in Russian: Справочник по спецфункциям, 1979, M.: “Наука”).