FEATURES OF ANGULAR DISTRIBUTIONS OF 1 GEV ELECTRONS SCATTERED BY THIN SILICON MONOCRYSTAL

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The result of theoretical and experimental investigations of angular distribution structure of 1 GeV electrons scattered by silicon crystal of 10 μm thickness are presented. The electron beam was falling on the crystal under different angles (from zero to the critical channeling angle) in respect to the crystal axis <111>. The analysis of the experimental data was carried out with the help of computer simulation of electron beam passage through the crystal on the basis of binary collision model. It is shown, that the existence of several maxima in the angular distributions of scattered electrons is stipulated by contributions of different fractions of electron beam in crystal, namely: the channeled and the above-barrier. The combined technique (simulation-experiment) of investigation of elastic scattering makes it possible to obtain important quantitative information about relativistic electron beam dynamics in aligned crystals.

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1. INTRODUCTION

When a beam of relativistic electrons passes through a crystal at a small angle to one of crystallographic axes there takes place a significant orientation effect in electron scattering exhibited as characteristic annular angular distributions of the particles outgoing from the crystal ("doughnut scattering") [1,2]. In this case the magnitude of a root-mean-square scattering angle of electrons can exceed essentially (in several times) the corresponding parameter at electron scattering in an amorphous target of the same thickness [2], and the less is the target thickness, the more is this difference. This shows the existence of strong correlation in electron scattering, when it sequentially collides with lattice atoms located along the given crystallographic axis.

Another important manifestation of these correlations is the coherent effect in Bremsstrahlung of relativistic electrons, when radiation intensity in crystal in a low-energy part of spectrum can ten times exceed radiation intensity in an amorphous target (see [3] and ref. there). As it is in the case of elastic scattering, at Bremsstrahlung in crystal the strong dependence of radiation intensity on the angle ψ_0 of electron arrival to a crystallographic axis is observed. The radiation intensity grows fast while the angle ψ_0 is decreasing. The radiation intensity reaches maximum value at $\psi_0 < \psi_c$, where ψ_c is the critical angle of an axial channeling [4].

The last circumstance, apparently, was the reason why during a long period of time high intensity of radiation of relativistic electrons, incident along a crystallographic axis, was associated with a radiation of channeled electrons, i.e. electrons involved in a finite motion in a field of atomic strings located along the given axis (see, for example, [5]). However, the analysis of the stability of 1 GeV electron motion in an axial channeling regime has shown, that the incoherent scattering on thermal oscillations of lattice atoms and on electronic subsystem of crystal results in a fast dechanneling of electrons, i.e. transition to a regime of above-barrier (infinite) motion.

The radiation of an above-barrier electron ($\psi_0 > \psi_c$) has also a coherent character, high intensity (though smaller, than for channeled electron) and maximum in a low-energy part of a spectrum [3]. While solving many problems, in particular, the problem of optimization of a gamma-radiation source, it is necessary to know the contribution of each of the indicated mechanisms. It is worth to point out, that the position of a maximum in radiation spectrum of above-barrier electrons coincides with the position of a maximum of channeled electrons radiation [3]. For this reason it is impossible to establish the relative contribution of these two mechanisms to radiation of an electron beam, passing through a crystal, immediately from the analysis of a radiation spectrum. Theoretical calculations of a share of channeled particles, as a function of depth of an electron beam penetration into a crystal, that were based on the use of the kinetic equation method (see., for example, [5]), gave overstated results contradicting the value of radiation intensity observed at the experiments.

To define a relative share of channeled electrons from a relativistic electron beam passing through aligned crystal the special experimental techniques that use the effect of redistribution of a channeled electron stream to the impact parameters to an atomic string have been developed in NSC KIPT [6,7]. The matter is that on average a channeled electron goes in a crystal close to atomic strings during a longer period of time, than above-barrier does. Thus, it has a higher probability of processes stipulated by small impact parameters. Δ-electron emission [6], electronuclear reactions [7] etc. belong to such processes. Each of these techniques has its advantages and drawbacks. The technique [6] is the most simple for realization, however, it shows only an integral effect on a crystal thickness. The technique [7], that uses layer-by-layer etching of the crystal, irradiated by an electron beam, and then the measurement of induced activity of etched layers, enables to trace the dependence of an output of electronuclear reactions (thus, probability of close impacts) with the depth of electron beam penetration into a crystal. The technological difficulties connected with layer-by-layer etching and the measurement of layer activity, however, have an effect for the measurement accuracy. Furthermore both these techniques use secondary electrodynamic processes for revealing a share of channeled electrons. This also adds errors to measurements.

The purpose of the present work is to research the features of elastic scattering of relativistic electrons on crystal atomic strings, and to study a possibility to determine a share of particles of an electron beam moving in crystal in an axial channeling regime, by means of analyzing angular distributions of particles scattered by a crystal.

The advantage of the given method is that the elastic scattering of electrons, instead of secondary electrodynamic processes is analyzed. The experiment is complicated, because to observe a thin structure of angular distributions it is necessary to have an electron beam with an initial divergence that is considerably smaller than the critical angle of channeling, and a rather thin monocrystal, so that the channeled electrons would make a noticeable part of a beam on withdrawal from crystal.

2. EXPERIMENT

These conditions were realized on the Kharkov linear accelerator of electrons (LAE-2 GeV). The collimated electron beam 0.3 x 0.3 *mm* sizes with energy $\varepsilon = 1$ *GeV* and divergence 0.01 *mrad* fell on silicon monocrystal of thickness $T = 10\,\mu m$, located in goniometer at the distance 11.5 *m* from the collimator. The orientation of monocrystal by the axis <111> in relation to the axis of an electron beam was carried out by secondary electron emission effect [6].

Electrons scattered by crystal were registered by a glass plate, which was located at the distance of 14.9 *m* from monocrystal, and the doze of glass irradiation by electrons was selected so that the darkening of glass was in linear dependence on a doze. Then the irradiated glass plates were scanned photometrically by a microphotometer IFO-451. The photometric measurement was conducted through the center of angular distribution of electrons on glass, with the breadth of the window of the microphotometer $\Delta x = 2$ m*m*. The finite breadth of the window of photometer, naturally, results in an integration of data in a strip of capture Δx and this determines the angular resolution of the given installation at measurement of angular distributions of scattered electrons.

The characteristic "print" of the angular distribution of electrons scattered by monocrystal obtained on a glass plate located across the beam (plane (x, y)) is indicated on Fig. 1a. The electron beam fell on crystal at the angle $\psi_0 = \psi_c$ to crystallographic axis <111> (axis z). Under the conditions of the given experiment the value of the critical angle of axial channeling makes $\psi_c = 0.41$ *mrad.* As the figure shows the angular distribution of electrons has a complex structure: the azimuthal inhomogeneous annular distribution round the direction of the crystallographic axis with the apex angle $\psi = \psi_0$ and with additional azimuthally homogeneous "spot" in the center.

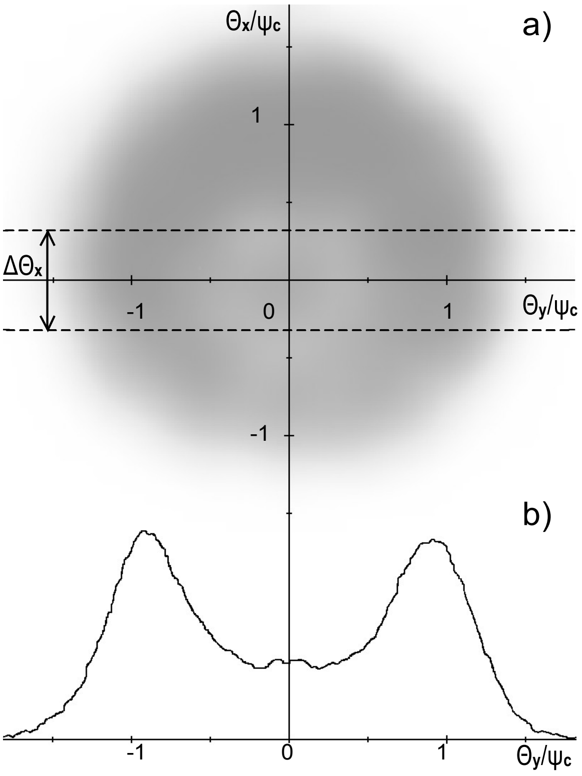


Fig. 1

On Fig. 1b the angular distribution of electrons (solid curve), obtained as a result of a photometric measurement of glass darkening along the axis Oy through the center of a ring is demonstrated. In our case the capture breadth of photometer in angular units made $\Delta\theta_x$ = 0.13 *mrad* (see Fig. 1a). This angular distribution contains two clearly expressed maxima at $\Delta\theta_y = \psi_0$, which are stipulated by scattering of above-barrier electrons. Besides, for $\theta_y = 0$ the local maximum is observed. This maximum cannot be explained by scattering of above-barrier particles, and consequently, it is connected with scattering of channeled electrons.

At $\psi_0 = \psi_c$ practically all particles of the beam, when entering the crystal, are in a regime of above-barrier motion. Channeled electrons can appear when a beam is passing through a crystal as a result of above-barrier electrons scattering on thermal fluctuations of lattice atoms, on an electronic subsystem of crystal, and on various kinds of crystalline structure defects.

When a crystal misalignment angle diminishes from ψ_c to zero, there grows the number of electrons, captured into a channeling regime, when entering the crystal. Therefore in the conducted experiment the angular distributions of scattered electrons with various values of misalignment angles were investigated $\psi_0 = 0$; 0.05; 0.21; 0.31; 0.41 *mrad* or in terms of ψ_c accordingly $\psi_0 =$ 0; 0.125; 0.5; 0.75; 1.0 ψ_c. However, it is necessary, to bear in mind that when an angle diminishes, the characteristic ring of angular distribution of above-barrier electrons narrows and all three maxima merge to one, and this does not allow to divide visually the contributions of channeled and above-barrier fractions of a beam to the formation of angular distribution of particles scattered by crystal.

3. COMPUTER SIMULATION

Generally the dynamics of relativistic particle beam in aligned crystal is rather complicated, as the various fractions of a beam are involved in various regimes of motion: finite and infinite, regular and chaotic with transitions between them. The analytical description of beam dynamics can be conducted only in some limit cases. Thus, for example, the theory of multiple scattering of relativistic charged particles on atomic strings of crystal, based on the employment of a continuous string approximation, describes the coherent azimuth scattering of above-barrier electrons ("doughnut scattering effect") [2, 3]. However, this theory does not describe transitions of particles between various fractions of an electron beam in crystal, since the continuous string approximation does not take into account the contribution of incoherent scattering. It is possible to take incoherent scattering into account by analytical methods only in case of rather large incident angles $\psi_0 >> \psi_c$ [8]. At the same time, as it was already mentioned, orientation effects in scattering and radiation of a relativistic electron beam, passing through a crystal, are most brightly exhibited in the range of angles $\psi_0 < \psi_c$. Therefore for the quantitative description of these effects computer simulation of passing of an electron beam through aligned crystal appears to be the most adequate.

With the purpose of theoretical analysis of angular distributions, obtained in the experiment, a computer simulation on the basis of Monte-Carlo method with utilization of a model of binary collisions of a relativistic electron with atoms of a crystalline lattice was conducted. Such approach allows taking into account both coherent (correlated) scattering of fast electrons on atomic strings of crystal located along a crystallographic axis, and incoherent scattering of electrons on thermal fluctuations of atom positions of atoms in a lattice and electronic subsystem of crystal. A rather small thickness of a crystal ($T = 10$ μm) allows gathering a sufficient statistic of events ($N = 10000$) during the acceptable period of time.

Belonging of a particle of a beam to a fraction of channeled or above-barrier electrons was controlled along all the trajectory of a particle in crystal according to the sign of transverse motion energy

$$\varepsilon_\perp = \varepsilon v_\perp^2/2 + U(\rho) \ , \qquad (1)$$

where ε is the energy of an electron, $v_\perp$ is the component of particle velocity in the plane (x, y) in terms of light velocity, $U(\rho)$ is the field of atomic string of crystal, averaged along the axis z. For the channeled electron $\varepsilon_\perp \leq 0$, for the above-barrier $\varepsilon_\perp > 0$. Thus, there was carried out a monitoring of a number of channeled electrons, as a function of penetration depth of electrons into crystal.

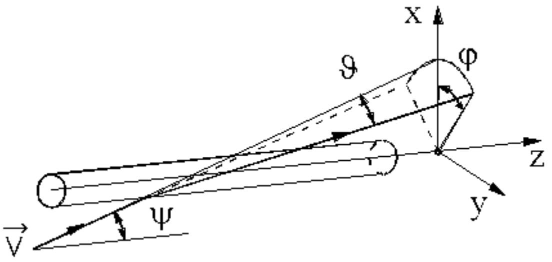


Fig. 2

In a continuous string approximation the transverse motion energy of an electron $\varepsilon_\perp$ is an integral of motion [3]. The scattering of electrons in this case is possible only at an azimuth angle φ (see Fig. 2), the polar angle ψ, defining the transverse motion energy of a particle before and after scattering on a string $\varepsilon_\perp = \varepsilon \psi^2/2$ does not vary. However, thermal oscillations of atoms in lattice knots, other imperfections of a crystalline structure result in changing of $\varepsilon_\perp$ and in the possibility of transition of particles from one fraction into another. Thus, the coherent scattering on an atomic string ensures "fast" azimuth scattering of electrons on φ, and the incoherent scattering on imperfections of a crystalline lattice is responsible for a rather "slow" scattering on the angle ψ.

The angle of electron departure from crystal in relation to crystallographic axis ψ_s is determined by the energy value $\varepsilon_\perp$ in the point of departure, and also by the distance to the nearest atomic string ρ_s:

$$\psi_S = \sqrt{2(\varepsilon_\perp - U(\rho_S))/\varepsilon} \ . \qquad (2)$$

The channeled electron ($\varepsilon_\perp < 0$) makes a finite motion in a plane, orthogonal to the axis of the crystal. It is obvious, that the angular distribution of this fraction of beam particles should be concentrated within the limits of a cone apex angle $\psi_c = \sqrt{2U_m/\varepsilon}$ (where U_m is the depth of a potential hole $U(\rho)$) and with the maximum along the crystallographic axis.

The above-barrier electron ($\varepsilon_\perp > 0$) makes infinite motion in the plane (x, y), being scattered sequentially on various atomic strings of a crystal. The distribution of above-barrier electrons in the angle ψ should have a minimum in the direction of the crystallographic axis.

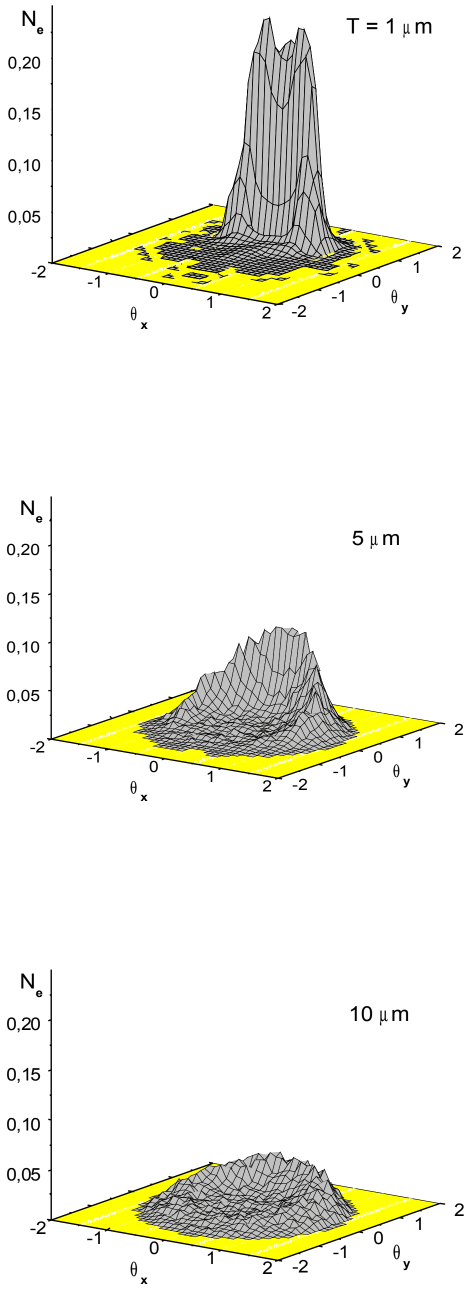


Fig. 3

In Fig. 3 the results of computer simulation of a *1 GeV* electron beam passing through a silicon crystal when $\psi_0 = \psi_c$ are presented. This figure visually demonstrates the evolution of angular distribution of scattered electrons when the thickness of crystal is growing from *1 μm* to *10 μm*. It is necessary to mark, that practically all the electrons, entering the crystal, with $\psi_0 = \psi_c$ initially are above-barrier. Fig. 3 shows that the coherent azimuth scattering of above-barrier electrons on crystal atomic strings is much stronger than the incoherent, so that a ring of azimuth scattering “has time” to get already "closed" at a thickness of about *10 μm*, having a little blurring under the influence of isotropic incoherent scattering.

With the growth of thickness it is possible to notice the emerging of a small local maximum in the angular distribution of electrons in the crystallographic axis direction, i.e. when $\theta_x = \theta_y = 0$. The analysis of the motion parameters of the electron departing along the crystallographic axis and forming this maximum shows that the majority of them are channeled, i.e. have $\varepsilon_\perp \le 0$.

4. ANALYSIS OF EXPERIMENTAL DATA

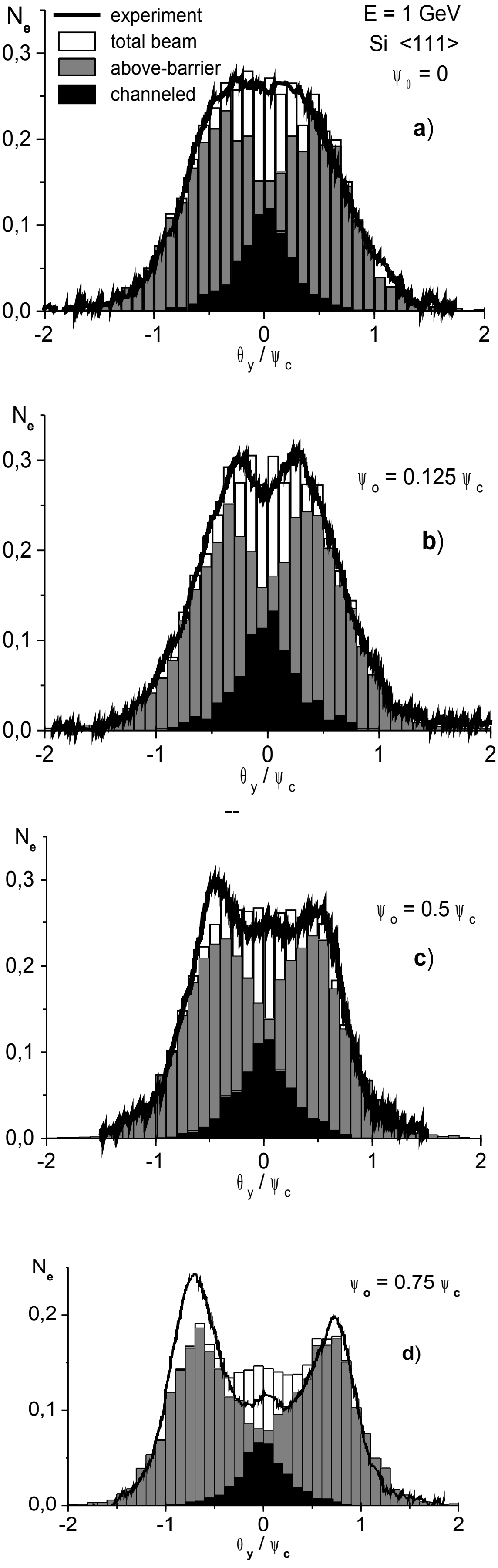
The results of measurements of angular distributions of electrons with energy $\varepsilon = 1\ GeV$, scattered by silicon monocrystal of thickness $T = 10\ \mu m$ with various values of an incident angle of an electron beam to the crystallographic axis <111> are represented in Fig. 4 by solid curves. These curves are obtained by a photometric measurement of a print of two-dimensional angular distribution of scattered electrons along the axis Oy through the center of the ring with photometer capture breadth $\Delta\theta_x = \pm\, 0.32\ \psi_c$ (see Fig. 1a). In Fig. 4 the results of computer simulation of the process of elastic electron scattering for the same conditions are represented as histograms. The black histogram corresponds to the angular distribution of the channeled electrons, the grey to the above-barrier electrons, and the white to their sum, i.e. the angular distribution of the whole electron beam, that has past through the crystal.

The comparison of the experimental data on angular distributions of scattered electrons of various values of the crystal misalignment angle ψ_0 with the results of the computer simulation as a whole shows not only qualitative, but also good quantitative agreement. The noticeable divergence at $\psi_0 = 0.75\ \psi_c$, apparently, is connected to inaccuracy of azimuth orientation of a glass plate at its photometric measurement, what is proved by the obvious asymmetry of the measured angular distribution in relation to the axis x. Unfortunately, the realization of an updating measurement now is impossible because of a stop of the accelerator LAE - 2 GeV.

When an electron beam is falling along the crystallographic axis ($\psi_0 = 0$, see Fig. 4a) the significant widening of the angular distribution of electrons is observed not only in comparison with an initial divergence of the beam $\Delta\psi_0 = 0.025\ \psi_c$, but also in comparison with electron scattering in an amorphous target of the same thickness $\sqrt{\overline{\theta^2_{am}}} = 0.34\psi_c$. This effect is explained by coherent scattering of electrons by atoms of crystal atomic string and it can be described as the influence of an average field of an atomic string $U(\rho)$ on an electron beam

$$\begin{aligned}\varepsilon_\perp &= \varepsilon\psi_0^2/2 + U(\rho_0) = \\ &= \varepsilon\psi^2(t)/2 + U(\rho(t)) = \varepsilon\psi_s^2/2 + U(\rho_s)\end{aligned}, \qquad (3)$$

where ρ_0 is the distance to the nearest atomic string at the electron entrance into the crystal. If the incident angle



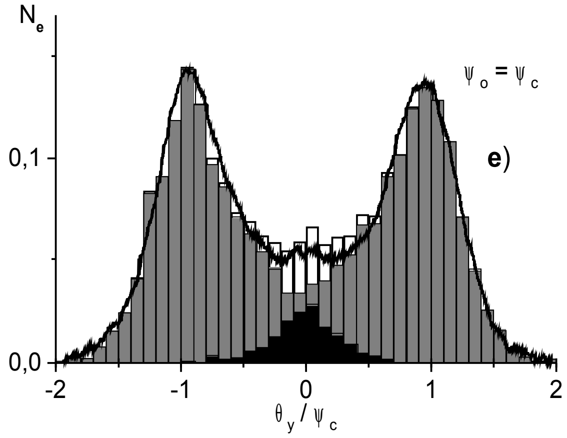


Fig. 4

$\psi_0 = 0$, then according to (3), the angle between the direction of particle motion and the string axis changes under the influence of the field $U(\rho)$ as

$$\psi(t) = \sqrt{2\left(U(\rho_0) - U(\rho(t))\right)/\varepsilon}\ , \qquad (4)$$

where $\rho(t)$ is the trajectory of electron motion in the plane (x, y). Depending on the point of departure from crystal ρ_s the particle will have an angle $\psi_s = \sqrt{2\left(U(\rho_0) - U(\rho_s)\right)/\varepsilon}$. Averaging of values of particle departure angle on an electron beam results in the angular distribution with a breadth $\Delta\theta \sim \psi_c$ even at zero divergence of an incident electron beam. The thickness of crystal in this case should exceed only a quarter of the average value of the oscillation period of the channeled electron that is about one micron for the conditions of the discussed experiment.

The dependence of the mean-square scattering angle of 1 GeV electron beam from a target thickness is represented in Fig. 5. The solid curve corresponds to the case of $\psi_0 = 0$; the dashed curve corresponds to the random orientation of the crystal in relation to the electron beam direction that is equivalent to the scattering in an amorphous target. Both these curves are obtained by computer simulation of electron beam scattering with initial angular divergence $\Delta\psi_0 = 0.2\ \psi_c$.

Fig. 5 shows the sharp increase of electron multiple scattering angle $<\theta>$ in aligned crystal in comparison with a scattering in amorphous medium at the first microns from beam entering the target. This is the manifestation of coherent mechanism of electron scattering on crystal string atoms with the maximum value of scattering angle $<\theta> = \psi_c$ (see Eq. (2)). Further increase of $<\theta>$ with target thickness T increasing is stipulated by incoherent scattering and it occurs in the same manner as in amorphous medium (see Fig. 5).

It should be noted that in the case $\psi_0 \neq 0$ the ultimate angle $<\theta>$ stipulated by coherent scattering mechanism is equal to $2\psi_0$, when the characteristic angular distribution ring becomes closed. In this case the electron beam as a whole is turned by crystal to the angle ψ_0 with the angular distribution width $<\theta> \approx \psi_0$.

Availability of particles with angles $\theta_y > \psi_c$ in angular distribution in Fig. 4a is stipulated by the contribution of the incoherent scattering, which, as it was marked above, results in dechanneling of electrons, i.e. transition to an above-barrier condition. The results of the simulation show, that at $\psi_0 = 0$ about 90 % of the electrons are captured to an axial channeling regime at the entrance into the crystal. However, after passing *10 μm* through the crystal only 10 % of beam electrons stay in the channel.

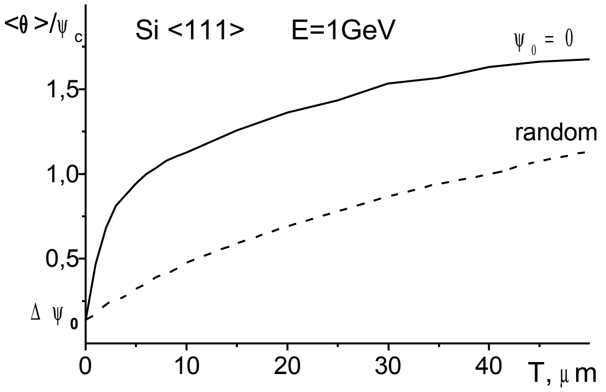


Fig. 5

Of course, there is an opposite process, called rechanneling, when above-barrier electron is captured to the channeling regime of motion. At $\psi_0 = \psi_c$ all the electrons are above-barrier at entering the crystal ($\varepsilon_\perp > 0$). However, as computer simulation shows, about 2% of beam particles become channeled after passing 10 μm in the crystal in this case.

It is impossible to make a conclusion about a quantitative ratio between fractions of an electron beam from the analysis of Fig. 4 immediately, since the area of a photometric measurement covered not the whole angular distribution of scattered particles (see Fig. 2). However, the selected method of photometric measurement contains the important information about the structure of angular distributions of scattered electrons and allows to observe the contribution of channeled electrons to this angular distribution at $\psi_0 \leq \psi_c$.

When the gradual disalignment of the crystal as related to the electron beam takes place, the initial circular distribution gradually passes to the annular with some darkening in the centre of the ring. In Fig. 4 (a-e) this transition corresponds to gradual division of one broad maximum of angular distribution (see Fig. 4a) into three separate maxima (see Fig. 4e). The computer simulation of an electron beam passing through a crystal allows to distinguish between the channeled and the above-barrier electrons and to give the unambiguous interpretation of a complex structure of the angular distributions, observed in the experiment: two extreme maxima correspond to the intersection of the photometric measurement strip with the ring of angular distribution of above-barrier electrons; the emerging of the central maximum is stipulated by the angular distribution of channeled particles.

5. CONCLUSION

The executed experiment demonstrates a possibility of direct observation of the effect of relativistic electron channeling in a thin monocrystal by means of angular distributions of an electron beam that has past through a crystal. Quantitative analysis of the angular distributions, based on the computer simulation of the process of electron beam passing through a crystal, allows getting important information about dynamics of an electron beam moving in a crystal along a crystallographic axis.

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