THE CAUSAL APPROACH TO SCALAR QED VIA DUFFIN-KEMMER-PETIAU EQUATION

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In this work we consider the scalar QED *via* Duffin-Kemmer-Petiau equation in the framework of Bogoliubov-Epstein-Glaser causal perturbation theory. We calculate the lowest order distributions for Compton scattering, vacuum polarization, the self energy and, by using a Ward identity, the vertex correction. The causal method provides a mathematically well defined and noneffective theory which determines, in a natural way, the propagator and the vertex of the usual effective theory.

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1. INTRODUCTION

The usual way to approach scalar quantum electrodynamics (SQED) is by performing the electromagnetic minimal coupling in the free Lagrangian of Klein-Gordon (KG) scalar field theory [1]. An alternative way is to start from the free Duffin-Kemmer-Petiau (DKP) Lagrangian instead of the KG one [2].

It is known that in the free field case the DKP and KG theories are equivalent, both in classical and quantum pictures [2,3,4]. However, there are still no general proofs of equivalence between these theories when interactions and decays of unstable particles are taken into account [5,6] (in this context see also references [7,8], which relate different results by using both DKP and KG formalisms with strong interactions). Some progress in this direction has been made recently. For instance, it was shown that this equivalence holds at the classical level when minimal interaction with electromagnetic [4,9] and gravitational [10] fields are present. Also, strict proofs of this equivalence were given for the quantized scalar field interacting with classical and quantized electromagnetic, Yang-Mills and external gravitational fields [6,11].

Perhaps the most evident advantages in working with DKP theory are the formal similarity with spinor QED, the fact that do not appear derivative couplings between DKP and the gauge field and that this theory allows an unified treatment of the scalar and vector fields. Despite this, the theory shares some difficulties with SQED based on KG one (SQED-KG), which are usually surpassed by dealing with an effective theory [2,6].

In this work we shall consider SQED-DKP theory in the framework of Bogoliubov-Epstein-Glaser causal perturbative method [12], which gives a mathematical rigorous treatment of ultraviolet divergences in quantum field theory. Our goals are to obtain a non-effective and mathematically well defined theory for SQED-DKP, at the same time recovering the results of the effective theory already obtained in the usual perturbative approach. In addition, this work must be viewed as an initial step in the attempts to rigorously establish the renormalizability of the theory.

2. THE DUFFIN-KEMMER-PETIAU THEORY

The free DKP theory is given by the Lagrangian [2,4]

$$L = \frac{i}{2}\bar{\psi}\beta^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi \ , \tag{1}$$

where ψ is a multicomponent wave function, $\bar{\psi} = \psi^{+}\eta^{0}$, and $\eta^{0} = 2(\beta^{0})^{2} - 1$. β^{μ} are a set of matrices ($\mu = 0,1,2,3$) satisfying the algebraic relations

$$\beta^{\mu}\beta^{\nu}\beta^{\rho} + \beta^{\rho}\beta^{\nu}\beta^{\mu} = \beta^{\mu}g^{\nu\rho} + \beta^{\rho}g^{\mu\nu} \ .$$

It is known that the above algebra has only three irreducible representations, whose degrees are 1, 5 and 10 [13]. The first one is trivial, having no physical content. The second and the third correspond, respectively, to scalar and vectorial representations. In this work we shall restrict us to the scalar case.

The standard procedure of canonical quantization for the free lagrangian (1) gets [2]

$$\left[\psi^{-}_{a}(x), \overline{\psi^{+}}_{b}(y)\right] = \frac{1}{i}S^{+}_{ab}(x-y)$$

and

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$$\left[\overline{\psi}^{-}{}_a(x),\psi^{+}_b(y)\right] = -\frac{1}{i}S^{-}_{ba}(y-x),$$

where ψ^{-} and ψ^{+} contains only annihilation and creation operators, respectively, and

$$S^{\pm}_{ab} \stackrel{def}{=} \frac{1}{m}\left[i\partial\left(i\partial + m\right)\right]_{ab}\Delta^{\pm}(x), \tag{2}$$

where $\Delta^{\pm}(x)$ are the positive (negative) frequency parts of the Pauli-Jordan distribution $\Delta(x) = \Delta^{+}(x) + \Delta^{-}(x)$. This later distribution has causal support [12] and can be written as $\Delta(x) = \Delta^{ret}(x) - \Delta^{adv}(x)$, where $\Delta^{ret}(x)$ and $\Delta^{adv}(x)$ has, respectively, retarded and advanced supports with respect to the point x. Analogously we define

$$S(x) \stackrel{def}{=} S^{+}(x) + S^{-}(x) = S^{ret}(x) - S^{adv}(x) \tag{3}$$

where again $S^{ret(adv)}(x)$ have retarded (advanced) support with respect to x and it is defined by equation (2) just replacing $\Delta^{\pm}$ by $\Delta^{ret(adv)}$. The above splitting of $S(x)$ into retarded and advanced parts is not the unique possible, as we will see in the next sections.

The interaction with the electromagnetic field is introduced by the minimal substitution $\partial_\mu \to \partial_\mu - ieA_\mu$ in the Lagrangian (1), which becomes

$$L = L_F + L_I,$$

where $L_F =$ is the free lagrangian (1) and

$$L_I = e:\overline{\psi}\beta^{\mu}\psi:A_\mu \quad . \tag{4}$$

is the interaction lagrangian. In this work we use $e > 0$.

3. THE BOGOLIUBOV-EPSTEIN-GLASER APPROACH

In the Bogoliubov-Epstein-Glaser causal method [12] the S-matrix is introduced as an operator-valued functional given by the perturbative series

$$S(g) = 1 + \sum_{n=1}^{\infty}\frac{1}{n!}\int d^4x_1\cdots d^4x_n T_n(x_1,\cdots,x_n)\times \\ g(x_1)\cdots g(x_n) \tag{5}$$

where $g(x)$ is a c-number test function supposed to belong to the Schwartz space, $g(x)\in S^4(R^4)$. The symmetric n-point functions $T_n(X)\,(X \stackrel{def}{=} \{x_1,\cdots,x_n\})$ are distributions written in terms of *free* fields and are the basic building blocks to be inductively constructed from the knowledge of $T_1(x)$ by means of the requirements of causality

$$S(g_1+g_2) = S(g_1)S(g_2), \text{ if supp } g_1 > \text{supp } g_2$$

and translational invariance. The above causality condition implies that

$$T_n(x_1,\cdots,x_n) = T_m(x_1,\cdots,x_m)T_{n-m}(x_{m+1},\cdots,x_n),$$

if

$$\{x_1,\cdots,x_m\} > \{x_{m+1},\cdots,x_n\}; \tag{6}$$

Based on general arguments such as correspondence [3], we have that $T_1 = iL_I^{(1)}$, where $L_I^{(1)}$ is the term of first order in the coupling constant in the interaction Lagrangian.

The inductive procedure works as follows. From the assumption that all $T_m(X)$, with $m \le n-1$, are known and satisfy (6), we can construct the distributions

$$A'_n(x_1,\cdots,x_n) = \sum_{P_2}\widetilde{T}_{n_1}(X)T_{n-n_1}(Y,x_n), \tag{7}$$

$$R'_n(x_1,\cdots,x_n) = \sum_{P_2}T_{n-n_1}(Y,x_n)\widetilde{T}_{n_1}(X), \tag{8}$$

where P_2 stands for all partitions $P_2:\{x_1,\cdots,x_{n-1}\} = X\cup Y, X\neq\emptyset$ into disjoint subsets with $|X| = n_1$ $|Y| = \le n-2$. In these expressions $\widetilde{T}_n(X)$ refers to the n-point distributions corresponding to a series for the S^{-1}-matrix analogous to (5), which can be obtained from the formal inversion of $S(g)$. If in the above expressions the sums are extended in order to include the empty set $X = \emptyset$ we get

$$\begin{aligned} A_n(x_1,\cdots,x_n) &= \sum_{P_2^0}\widetilde{T}_{n_1}(X)T_{n-n_1}(Y,x_n) \\ &= A'_n(x_1,\cdots,x_n) + T_n(x_1,\cdots,x_n) \end{aligned} \tag{9}$$

$$\begin{aligned} R_n(x_1,\cdots,x_n) &= \sum_{P_2^0}T_{n-n_1}(Y,x_n)\widetilde{T}_{n_1}(X) \\ &= R'_n(x_1,\cdots,x_n) + T_n(x_1,\cdots,x_n) \end{aligned} \tag{10}$$

where now P_2^0 stands for all partitions $P_2^0:\{x_1,\cdots, x_{n-1}\} = X\cup Y$. A glance at Eqs. (9) and (10) shows that A_n and R_n are not known because they contain the unknown T_n. However, the distribution defined by

$$D_n(x_1,\cdots,x_n) \stackrel{def}{=} R_n - A_n = R'_n - A'_n \tag{11}$$

is known. It can be proved that the supports of R_n and A_n are retarded and advanced with respect to x_n, respectively. Then, D_n has a causal support with respect to this point, i.e.,

$$\operatorname{supp} D_n(X, x_n) \subseteq \Gamma_n^+(x_n) \cup \Gamma_n^-(x_n),$$

After splitting D_n into its advanced and retarded parts we can obtain T_n from the relations (9) or (10).

The above splitting is the nontrivial step of the inductive procedure. In dealing with this problem we need only to consider the numerical distribution d associated with D_n (i.e., we neglect the operator *field* distributions that appear as factors in the causal distribution D_n). Then, we write $d = r - a$, where r and a are respectively the associated retarded and advanced numerical distributions. To solve the splitting problem we must determine the singular order ω of a distribution. We have to consider two distinct cases: *i*) $\omega < 0$ - in this case the solution of the splitting problem is unique and the r and a distributions can be found by multiplication of d by step functions; *ii*) $\omega \geq 0$ - now the solution can be no longer obtained by multiplying d by step functions and, after a careful mathematical treatment, it may be shown that the retarded distribution (which suffices to determine T_n) is given, in momentum space, by the *central splitting solution*

$$\hat{r}(p) = \frac{i}{2\pi}\int_{-\infty}^{+\infty} dt \frac{\hat{d}(tp)}{(t - i0)^{\omega+1}(1 - t + i0)}, \tag{12}$$

where the symbol ^ denotes the Fourier transform. However, in contrast with the case $\omega < 0$, this expression does not give a unique solution for the splitting problem. By assuming that the splitting procedure cannot raise the singular order of d, it can be shown that the general solution in momentum space is given by

$$\tilde{r}(p) = \hat{r}(p) + \sum_{|a|=0}^{\omega} C_a p^a, \tag{13}$$

where the C_a are constant coefficients which are not fixed by the causal structure - we need additional physical conditions in order to determine them.

We now apply the inductive steps above to construct the two point distributions for SQED-DKP theory. Looking for (4) we see that the one-point distribution for the DKP field interacting with electromagnetic field is given by $T_1(x) = ie :\overline{\psi}(x)\beta^{\mu}\psi(x): A_{\mu}(x)$, where e is the *physical* charge. To go from $n = 1$ to $n = 2$ we construct the distribution $D_2(x_1, x_2)$, which in this case is given by

$$\begin{aligned} D_2(x_1,x_2) &= [T_1(x_1), T_1(x_2)] \\ &= e^2\{:\overline{\psi}(x_2)\beta^{\nu}\psi(x_2)::\overline{\psi}(x_1):\beta^{\mu}\psi(x_1): \\ &- :\overline{\psi}(x_1)\beta^{\mu}\psi(x_1)::\overline{\psi}(x_2)\beta^{\nu}\psi(x_2):\}A_{\mu}(x_1)A_{\nu}(x_2) \end{aligned} \tag{14}$$

We can verify explicitly that this distribution is causal from the fact that $[T_1(x_1), T_1(x_2)] = 0$ if $(x_1 - x_2)^2 < 0$.

4. APPLICATIONS

We now apply the formalism sketched above to determine some lowest order processes in SQED, namely the Compton scattering, vacuum polarization, self-energy of the scalar particle and the vertex correction at the limit of zero transferred momentum.

4.1. COMPTON SCATTERING

By using Wick's theorem [12] we normally expand (14) and keep only those terms contributing to Compton scattering, given by

$$\begin{aligned} D_2^I(x_1,x_2) &= ie^2\beta^{\mu}_{ab}\beta^{\nu}_{cd} :\overline{\psi}_a(x_1)\psi_d(x_2):\times \\ &:A_{\mu}(x_1)A_{\nu}(x_2): S_{bc}(x_1 - x_2) \end{aligned} \tag{15}$$

$$\begin{aligned} D^{II}(x_1,x_2) &= ie^2\beta^{\mu}_{ab}\beta^{\nu}_{cd} :\psi_b(x_1)\overline{\psi}_c(x_2):\times \\ &:A_{\mu}(x_1)A_{\nu}(x_2):\{-S_{da}(x_2 - x_1)\} \end{aligned} \tag{16}$$

In both these terms, the numerical distribution we have to split is $S(x)$, which is causal. Thus, a splitting solution is trivially obtained from (3). But it can be shown that $S(x)$ has singular order $\omega = 0$, which implies that this splitting is not unique. The general solution for the numerical retarded distribution in configuration space is $\tilde{r}(x) = S^{ret}(x) + C\delta(x)$, where C is an arbitrary constant. The general result for the numerical distribution $t^I(x_1, x_2)$ is

$$t^I(x_1,x_2) = -S^F(x_1 - x_2) + C\delta(x_1 - x_2), \tag{17}$$

where we have defined $-S^F(x) \overset{def}{=} -\frac{1}{m} i\partial(i\partial + m)\Delta^F(x)$ ($\Delta^F(x)$ is the usual scalar Feynman propagator). In a similar way we find $t^{II}(x_1,x_2) = -S^F(x_2 - x_1) + C'\delta(x_2 - x_1)$. The condition of charge conjugation invariance of the theory requires that $C = C'$. The gauge invariance requirement yields $C = \frac{I}{m}$, where I is the 5 × 5 identity matrix. Turning this result into (17) we obtain

$$t^I(x_1,x_2) = -T^C(x_1 - x_2) = t^{II}(x_2,x_1), \tag{18}$$

where we have defined $T^C(x) \equiv S^F(x) - \frac{1}{m}\delta(x)$. It is straightforward to see that this distribution is the Green function for DKP equation, i. e., $(i\partial - m)T^C(x) = \delta(x)$. So, in this sense the causal approach gives, in a natural way, the correct *effective propagator* for the DKP scalar particle, which agrees with the results of [2,6].

4.2. VACUUM POLARIZATION

Considering now those terms contributing to vacuum polarization we have

$$D_2^{Vac}(x_1,x_2) = \{P^{\mu\nu}(y) - P^{\nu\mu}(-y)\} : A_\mu(x_1)A_\nu(x_2): \quad (19)$$

where $y \overset{def}{=} x_1 - x_2$ and $P^{\mu\nu}(y) \overset{def}{=} -e^2 Tr\{\beta^\mu \times S^+(y)\beta^\nu S^-(-y)\}$. It turns out that the expression into curl brackets in the above expression is the numerical distribution we have to split, which have singular order $\omega = 2$. Using the central splitting formula (12) to determine its retarded part $\hat{r}(k)$ we can determine the corresponding numerical distribution $\hat{t}(k)$. The final result for the complete two-point distribution for the vacuum polarization in configuration space is given by

$$T_2^{Vac}(x_1,x_2) = -i : A_\mu(x_1)\Pi^{\mu\nu}(x_1 - x_2)A_\nu(x_2):,$$

where

$$\hat{\Pi}^{\mu\nu}(k) = \frac{1}{2\pi^4}\left(\frac{k^\mu k^\nu}{k^2} - g^{\mu\nu}\right)\hat{\Pi}(k);$$

$$\hat{\Pi}(k) = \frac{e^2}{12}k^4\int_{4m^2}^{\infty} ds\frac{1}{s(k^2 - s + i0)}\left(1 - \frac{4m^2}{s}\right)^{3/2}.$$

However, from the fact that the singular order of the distribution we had to split was $\omega = 2$, the above result is not unique. The most general solution is given by $\tilde{\Pi}(k) = \hat{\Pi}(k) + C_0 + C_\mu k^\mu + C_2 k^2$, where the normalization constants C_0, C_μ and C_2 are not determined by causality, but from the requirements of parity invariance, zero mass for the gauge field, and the identification of e with the physical charge. These conditions imply that $C_0 = C_\mu = C_2 = 0$.

4.3. SELF-ENERGY AND VERTEX CORRECTION

Now, the terms in (14) contributing to the scalar self-energy are

$$D_I^{self}(x_1,x_2) = -e^2 :\overline{\psi}(x_1)\beta^\mu[S^-(x_1 - x_2)D_0^+(x_2 - x_1)$$

$$+ S^+(x_1 - x_2)D_0^+(x_1 - x_2)]\beta_\mu\psi(x_2):; \quad (20)$$

$$D_{II}^{self}(x_1,x_2) = e^2 :\overline{\psi}(x_2)\beta^\mu[S^+(x_2 - x_1)D_0^+(x_2 - x_1)$$

$$+ S^-(x_2 - x_1)D_0^+(x_1 - x_2)]\beta_\mu\psi(x_1):. \quad (21)$$

The numerical distributions we have to split are the terms into brackets, which have singular order $\omega = 1$. The final general solution, involving two arbitrary constants, is given by

$$T_I^{self}(x_1,x_2) = ie^2 :\overline{\psi}(x_1)\Sigma(x_1 - x_2)\psi(x_2):$$

$$+ ie^2 :\overline{\psi}(x_2)\Sigma(x_2 - x_1)\psi(x_1): \quad (22)$$

where

$$\hat{\Sigma}(p) = \frac{ie^2}{4(2\pi)^4}\left\{\left[\log\left|1 - b^2\right| - i\pi\theta\left(p^2 - m^2\right)\right]\times\right.$$

$$\left.\left[m\left(1 - \frac{1}{b^2}\right) + \frac{p}{2}\left(1 - \frac{1}{b^4}\right)\right] - \frac{p}{2b^2} + C_0 + C_1 p\right\}. \quad (23)$$

By considering the complete propagator for the scalar particle and by requiring its normalized mass to be m, we find that these constants satisfy the condition

$$C_0 = m\left(\frac{1}{2} - C_1\right),$$

which can be used to eliminate one of them, say C_0. The remaining constant C_1 is related to another one, which appears in the splitting of the vertex causal distribution, by the following Ward identity

$$\hat{\Lambda}^\mu(p,p) = \frac{1}{(2\pi)^2}\frac{\partial}{\partial p_\mu}\hat{\Sigma}(p), \quad (24)$$

where Λ^μ is the vertex numerical distribution. Substituting the explicit form of the self-energy (23) into this identity we obtain

$$\hat{\Lambda}^\mu(p,p) = \frac{ie^2}{4(2\pi)^6}\left\{\left[\log\left|1 - b^2\right| - i\pi\theta\left(p^2 - m^2\right)\right]\times\right.$$

$$\left[\frac{2p^\mu}{mb^4}\left(1 + \frac{p}{mb^2}\right) + \frac{\beta^\mu}{2}\left(1 - \frac{1}{b^4}\right)\right]$$

$$+ \frac{2p^\mu}{m^2}\left[P.V.\frac{1}{b^2 - 1} - i\pi\delta(b^2 - 1)\right]\left[m\left(1 - \frac{1}{b^2}\right) + \right.$$

$$\left.\left.\frac{p}{2}\left(1 - \frac{1}{b^4}\right)\right] + \frac{p^\mu p}{m^2 b^4} - \frac{\beta^\mu}{2b^2} + C_1\beta^\mu\right\} \quad (25)$$

This result is well defined at $p = 0$, but it is singular on mass shell $p^2 = m^2$ due to the logarithmic term. The above form of the vertex function suffices to study the physical meaning of the constant C_1, which is done in connection to charge normalization. The physical charge is defined in the scattering of a scalar particle by an external electromagnetic field at low energies. Thus we must consider the contributions to S matrix (in the limit of zero transferred momentum) from the terms containing C_1 in both the self-energy and vertex distributions. Because of the above mentioned mass shell singularity, we must be care in taking the adiabatic limit. Making so, we can prove that all these contributions cancel themselves and conclude that this constant has no physical meaning. Nevertheless it can be specified by requiring the vertex function to satisfy the central splitting condition, i.e., $\hat{\Lambda}^{\mu}(0,0)$. Using this condition into (25) we obtain $C_1 = \frac{1}{4}$.

5. CONCLUSIONS AND FINAL REMARKS

In this work we considered SQED-DKP theory in the framework of Bogoliubov-Epstein-Glaser causal method. The starting point was the identification of the one point distribution $T_1(x)$, which was determined by the interaction Lagrangian (4) written in terms of free fields. The causal method thus dictated completely the form of the interaction, giving us a non-effective and mathematically well-defined theory. In calculating the two-point distribution for Compton scattering, and by using the requirements of charge conjugation and gauge invariance, we recovered in a natural way the scalar propagator of the DKP usual effective theory. At one loop level we calculate the vacuum polarization, the scalar self-energy and the vertex correction in the limit of zero transferred momentum. We determined the physically meaningful normalization constants by using physical requirements, as charge and mass normalization, parity and gauge invariance. All our results agree with those obtained in the context of the effective SQED-DKP theory as well as in the context of SQED-KG, what corroborates the belief about the equivalence of KG and DKP theories. A complete analysis at one loop level, as well as the study of the renormalizability of the theory, is in course. As future perspectives we can quote the use of the causal approach to study DKP field interacting with external fields.

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