PARAMETRIC EXCITATION OF SURFACE WAVES IN PLASMA-METAL STRUCTURES WITH PERPENDICULAR MAGNETIC FIELD

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This paper deals with the parametric excitation of potential surface waves (SWs) propagating in a planar plasma-metal waveguide structure with a magnetic field perpendicular to the plasma-metal boundary. An external, spatially uniform, alternating electric field at the second harmonic of the excited wave is used as the source of parametric excitation. It is considered two cases, when the pump field is an eigen perturbation of the system, and when it is a non-eigen one.
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1. INTRODUCTION

At present, plasma-metal waveguides are widely used in plasma and semiconductor electronics, gas discharges, and various plasma technologies [1-4]. In practice many types of waveguide structures operate with a magnetic field oriented perpendicular to plasma-metal boundary [5-7]. Such waveguides are typical of RF and microwave discharge devices, magnetrons, Penning sources, magnetic discharge pumps, Hall detectors, divertor- and limiter-equipped fusion systems, devices for the plasma processing of metal surfaces, and so on.

The linear theory of potential SWs at a plasma-metal boundary with a such magnetic field configuration has been developed fairly well [6, 7], and some nonlinear mechanisms for the self-interaction of these SWs have also been investigated [8, 9]. However, the construction of a nonlinear theory of SWs requires a detailed study of the mechanisms for their excitation. In the waveguide structures in question, SWs are difficult to excite by charged particles because of the presence of an external magnetic field perpendicular to the plasma-metal boundary. Our objective here is to investigate the efficiency of parametric excitation [10] of these waves.

2. PUMP FIELD THRESHOLD AMPLITUDE

We analyze the parametric excitation of a high-frequency SW propagating along a plane plasma-metal boundary in the y direction. A nonisothermal plasma ($T_e >> T_i$, where T_e and T_i are the electron and ion plasma temperatures, respectively) occupies the half-space $x > 0$ and is bounded at $x = 0$ by a perfectly conducting metal surface. A steady magnetic field $\vec{H}_0$ is directed along the x axis, which is perpendicular to the plasma-metal boundary. The properties of SWs in an inhomogeneous plasma are strongly influenced by the spatial distribution of plasma density in the boundary layer. In plasmas with large and small density inhomogeneities, the properties of SWs are determined by the integral parameters of the plasma in the region where the wave field is localized. [3]. In those cases, the plasma-metal boundary can be assumed to be sharp and the plasma density can be treated as uniform and set equal to its mean value in the localization region of the SW. Below, the efficiency of the parametric excitation of SWs will be considered under the assumptions that the plasma-metal boundary is sharp and the plasma is homogeneous.

In [6], it has been shown that, in the waveguide structure under consideration, high-frequency potential SWs can be excited at frequencies higher than the electron cyclotron frequency, $\omega > \omega_{ce}$. In what follows, we consider the parametric excitation of such waves at the boundary between a weakly collisional dense plasma ($\omega^2 << \omega_{pe}^2$, where ω_{pe} is the electron plasma frequency) and a metal. In this case, the wavenumber k and frequency ω of the excited SW are related by [6]

$$k^2 = \omega^2(\omega^2 - \omega_{ce}^2)/(V_{Te}^2\omega_{pe}^2)\,, \tag{1}$$

where V_{Te} is the electron thermal velocity. Analysis of relation (1) shows that the phase velocity of an SW is much higher than the electron thermal velocity.

To note, an important property of the waves under study is that they are reciprocal [6]. This means that there exist two oppositely propagating waves with the same frequency ω and the same (in absolute value) wavenumbers $k_1 = k(\omega)$ and $k_2 = -k(\omega)$. This property makes possible parametric excitation of the waves in question due to decay instability [12]. That this method is efficient is evidenced by the fact that the self-interaction of SWs [8] is accompanied by the excitation of purely surface perturbations both at the static and second harmonics. Consequently, SWs excited at a plasma-metal boundary are not subject to the nonlinear damping associated with the excitation of volume modes [3], which can result in a loss of energy.

We assume that the source of parametric excitation is an external, spatially uniform electric field oscillating at a frequency ω_0 and directed along the external magnetic field:

$$E = E_0 \cos(\omega_0 t)\,. \tag{2}$$

In this case, the spatiotemporal synchronization condition [10] takes the form

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$$\omega_0 = \omega + \omega, 0 = k_1(\omega) + k_2(\omega) . \quad (3)$$

We can see that the interaction between SWs with the frequency $\omega = \omega_0 / 2$ and the pump wave is the most efficient.

The parametric excitation of SWs will be investigated in a weakly nonlinear approximation [3, 10], which is valid for sufficiently small SW amplitudes and in which the small nonlinearity parameters are $\mu_{1,2} = e|A_{1,2}|/(m_e V_{Te}^2) << 1$, where $A_{1,2}$ are the amplitudes of the excited SWs and e, m_e are the absolute value of charge and mass of an electron, respectively. In this approximation, Poisson's equation and the nonlinear quasi-hydrodynamic equations for electron motions in the SW field can be written as:

$$\begin{aligned} &\Delta\varphi = 4\pi e n_e , \\ &\frac{\partial n_e}{\partial t} + div(n_e \vec{V}_e) = 0, \\ &\frac{\partial \vec{V}_e}{\partial t} + (\vec{V}_e \nabla)\vec{V}_e = \frac{e}{m_e}\left\{\nabla\varphi - \frac{[\vec{V}_e, \vec{H}_0]}{c}\right\} - V_{Te}^2 \frac{\nabla n_e}{n_e} - \nu \vec{V}_e , \end{aligned} \quad (4)$$

where φ is the wave potential, $\vec{V}_e$ and n_e are the hydrodynamic velocity and density of the plasma electrons, and ν is the effective frequency of their collisions.

In the weakly nonlinear approximation, we can substitute linear expressions [6] into the nonlinear terms in quasi-hydrodynamic equations (4) to obtain a set of nonlinear equations describing the dependence of the amplitudes of SWs on time in their interaction with the pump field. It should be kept in mind that, in a weakly collisional plasma, the damping of SWs can have a strong impact on their excitation. That is why, in analogy with [10], we introduce additional terms that take into account a weak nonlinear damping of the excited waves. Thus, the spatiotemporal dynamics of the excitation of SWs by an external, spatially uniform, alternating electric field at the second harmonic of the frequency of the excited wave can be described by the following set of nonlinear equations:

$$\left.\begin{aligned} &\frac{\partial A_1}{\partial t} \pm V_g \frac{\partial A_1}{\partial y} + \nu A_1 = -i\alpha A_0 A_2^* , \\ &\frac{\partial A_2}{\partial t} \mp V_g \frac{\partial A_2}{\partial y} + \nu A_2 = -i\alpha A_0 A_1^* , \end{aligned}\right\} \quad (5)$$

where the coefficient $\alpha = e\omega F/(4 m_e V_{Te}^2)$ characterizes the interaction of the excited SWs with the pump field; the parameter

$$F = \omega_{pe}^2(\omega^2 - \omega_{ce}^2)/[\omega^2(2\omega^2 - \omega_{ce}^2)]$$

accounts for the influence of the magnetic field and plasma density on the efficiency of the wave excitation; $V_g = V_{Te}\,\omega_{pe}(\omega^2 - \omega_{ce}^2)^{1/2}/(2\omega^2 - \omega_{ce}^2)$ is the group velocity of the SWs, and $A_0 = r_{de} E_0$ with $r_{de} = V_{Te}/\omega_{pe}$ being the electron Debye radius. The upper (lower) sign in Eqs. (5) corresponds to the propagation of the first wave in the positive (negative) direction along the y-axis and the propagation of the second wave in the negative (positive) direction. In what follows, we consider the temporal dynamics of the SW amplitudes in the case in which the second terms on the left-hand sides of Eqs. (5) can be neglected.

Analysis of Eqs. (5) yields the following time dependence of the amplitudes of the excited waves:

$$\begin{aligned} &|A_j(t)|^2 = |A_j(0)|^2 + (|A_1(0)|^2 + |A_2(0)|^2 \times \\ &sh^2\left(\alpha|A_0|t\right)\exp(-2\nu t), \quad j = 1, 2. \end{aligned} \quad (6)$$

For $\alpha|A_0|t > 2$, this expression becomes

$$|A_j(t)| = 1/2\sqrt{|A_1(0)|^2 + |A_2(0)|^2}\,\exp(\alpha|A_0| - \nu)t , \quad (7)$$

in which case the phases approach the steady-state value $\pi/4$.

Expression (7) implies that SWs can be excited under the condition $\gamma = \alpha|A_0| - \nu > 0$. This condition determines the threshold amplitude of the pump field $|A_0|_{cr}$, above which the SWs can be excited parametrically:

$$|A_0|_{cr} = 4\nu\, m_e V_{Te}^2/(eF\omega) . \quad (8)$$

3. NON-RESONANT INTERACTION

First, let us investigate the saturation of the decay instability in the case of non-resonant interaction, i.e. when ω_0 is non-eigen frequency. To do that, it is necessary to consider interactions between harmonics of the wave, according to the nonlinear quasi-hydrodynamic equations. It gives the following nonlinear equations for the amplitudes of the excited SWs (to third order in the field amplitude):

$$\left.\begin{aligned} &\frac{\partial A_1}{\partial t} + \nu A_1 = -i\alpha A_0 A_2^* - i(\beta_1|A_1|^2 + \beta_2|A_2|^2)A_1 , \\ &\frac{\partial A_2}{\partial t} + \nu A_2 = -i\alpha A_0 A_1^* - i(\beta_1|A_2|^2 + \beta_2|A_1|^2)A_2 , \end{aligned}\right\} \quad (9)$$

where $\beta_1 = 10 e^2/(m_e^2 V_{Te}^4)F\omega$ is the coefficient of self-interaction of SWs of the given type, obtained in [8], $\beta_2 = -7e^2\omega^2/(18 m_e^2 V_{Te}^4\,\omega_{pe}^2)F\omega$ is the coefficient of interaction of the both excited waves.

Accounting for the self-interaction of each SW and the interaction between them violates spatiotemporal synchronization condition (3). For the dense plasma under consideration, we have $|\beta_2| << |\beta_1|$, hence, the nonlinear frequency shifts of both the first ($\Delta\omega_{NL1} = \beta_1|A_1|^2 + \beta_2|A_2|^2$) and the second ($\Delta\omega_{NL2} = \beta_1|A_2|^2 + \beta_2|A_1|^2$) excited SWs are governed primarily by their self-interaction. Thus, the frequency mismatch between the excited SWs and the pump field increases with time. As a result, SWs saturate at the same amplitude, which is independent of the initial conditions:

$$\frac{|A_1|_{st}}{|A_0|} = \frac{|A_2|_{st}}{|A_0|} = \left[\frac{\alpha^2 A_0^2 - \nu^2}{A_0^4(\beta_1 + \beta_2)^2}\right]^{1/4} . \quad (10)$$

As time elapses, the phases of the SWs approach the value

$$\arg A_{jst} = -0{,}5 \arccos\left[-\sqrt{1-\nu^2/(\alpha^2 A_0^2)} \right]. \quad (11)$$

Numerical solution of equations (9) shows that the time required for the phases to reach this value decreases as the pump field amplitude and plasma density increase and the electron temperature decreases. Hence, the development of parametric instability leads to the excitation of two oppositely propagating SWs with the same frequency and amplitude. The superposition of these waves produces a standing SW.

4. RESONANT INTERACTION

Now let us consider the resonance interaction when pump field is an electric field of Langmuir wave. In this case the system (5) should be expanded with one dynamical equation for the amplitude of Langmuir wave:

$$\partial A_0 / \partial t = -i\alpha_0 A_1 A_2 , \quad (12)$$

where α_0 is the resonant interaction constant. In many cases the amplitude A_0 can be assumed as constant. This imposes the following restriction to energy of SWs:

$$W_{1,2} / W_0 \propto (A_{1,2} / A_0)^2 (d / L) << 1 , \quad (13)$$

where d is the SW penetration depth into the plasma, L is the characteristic length of the system. For the semi-bounded plasma: $d << L$. Thus, even in the case of comparable amplitudes $A_{1,2}$ with the amplitude of pump field, A_0 can be considered as constant. In this case the resonant interaction is also described by (9).

5. CONCLUSIONS

The analysis carried out has shown that, in both cases of resonant and non-resonant interaction at a fixed amplitude of the pump field, a strengthening of the external magnetic field, as well as a reduction in the plasma density, leads to an increase in the threshold pump field amplitude and to a decrease in both the linear growth rate and the saturation amplitude of the excited SWs.

An increase in the plasma electron temperature also leads to an increase in the threshold pump field amplitude and a decrease in the linear growth rate.

Thus, the parametric excitation of the SWs under study is found to be most efficient for waveguide structures with a sufficiently dense plasma in weak magnetic fields.

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ПАРАМЕТРИЧЕСКОЕ ВОЗБУЖДЕНИЕ ПОВЕРХНОСТНЫХ ВОЛН В ПЛАЗМЕННО-МЕТАЛЛИЧЕСКИХ СТРУКТУРАХ С ПЕРПЕНДИКУЛЯРНЫМ МАГНИТНЫМ ПОЛЕМ

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В работе рассмотрено параметрическое возбуждение потенциальных поверхностных волн, распространяющихся в планарной волноводной структуре “плазма-металл” с перпендикулярным к границе магнитным полем. В качестве источника параметрического возбуждения используется внешнее однородное в пространстве и переменное во времени электрическое поле на второй гармонике возбуждаемых волн. Рассмотрены случаи, когда поле накачки является как собственным, так и несобственным возмущением системы.

ПАРАМЕТРИЧНЕ ЗБУДЖЕННЯ ПОВЕРХНЕВИХ ХВИЛЬ У ПЛАЗМОВО-МЕТАЛЕВИХ СТРУКТУРАХ ІЗ ПЕРПЕНДИКУЛЯРНИМ МАГНІТНИМ ПОЛЕМ

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В роботі розглянуто параметричне збудження потенціальних поверхневих хвиль, що розповсюджуються у планарній хвилеводній структурі “плазма-метал” з перпендикулярним до межі магнітним полем. Як джерело параметричного збудження використовується зовнішнє однорідне у просторі та змінне у часі електричне поле на другій гармоніці хвиль, що збуджуються. Розглянуто випадки, коли поле накачки є як власним, так і невласним збудженням системи.