COUPLING STATES IN MOVING QUASI-NEUTRAL MEDIUM

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The possibility of existence of spherically symmetrical quasi-neutral collective coupling states (CCS) in collisionless plasma is discussed. Correlation phenomena in quasi-neutral systems with hot electron component moving with average velocity close to ion one may occur in greatest degree and guide to the advantage of CCS forming. CCS consists of large numbers of electrons and ions, and represent regions with the potential differing from average (zero) plasma potential. The concept of CCS can be considered as non-linear BGK-waves. This definition is just applied in great degree for equilibrium configurations with non-linear electrostatic field. The simplest spherically symmetrical configuration has been considered without spin and polarisation in the frame of kinetic description for monochromatic on energy electron and ion components and distribution functions on full momentum in forms of simple power functions of different powers for electrons and ions. Asymptotic analytical and full numerical solutions were found for CCS with captured and passing particles turning into final density neutral plasma created chaotically moving electrons and ions in rest.

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1. INTRODUCTION

In moving plasma medium formed for example during neutralization processes of high-current ion beams by electrons spontaneous growth of the phase volume of ion beam and fast thermalization of electron component are observed [1, 2]. The main paradox of the problem of neutralization is in the following: initially, there are two cold flows of electrons and ions, finally, there is one mixed electron-ion flow of larger entropy [3]. Therefore, it can be supposed that the growth of entropy is the consequence of neutralization processes. Estimations of different processes via pair Coulomb collisions do not allow to explain observed phenomena.

Below we shall consider an intermediate stage of less entropy than the neutralized stationary state. It permits to allow fast thermalization of the electron-ion flow. The main concept of this stage is collective coupling state (CCS). The moving regions of potential significantly differing from the average (zero) potential of mixed electron-ion flow are the manifestation of CCS. The correlation phenomena appearing in moving quasi-neutral medium arise from the CCS Coulomb (collective) energy negativity, i.e. the possibility of existence of CCS is energetically profitable. Such regions consist of large number of electrons and ions and the cross section of neutral (in the sense that the potential decreases as faster as 1/r) CCS interaction may be large enough to lead to fast thermalization of the system (the cross section of CCS interactions is proportional to $(Z^2/M)^2$, where Z is the full ion charge, M – the mass of CCS). The CCS concept can be called also as non-linear BGK-waves [4, 5]. This term is just applicable to steady-state states with nonlinear electrostatic fields.

2. SPHERICALLY-SYMMETRIC CCS

Here we try to construct the simplest spherically-symmetric CCS in the ion frame. We shall find neutral CCS (the electrostatic potential drops faster than 1/r) transferred to the neutral plasma consists of chaotic moving electrons and ions at rest in the limiting case of large radius.

The Lagrangian of a non-relativistic particle in spherical coordinates r, θ, φ is given by $\Lambda = (1/2m)(p_r^2 + p_\theta^2 + p_\varphi^2) - e\varphi + (e/mc)(p_rA_r + p_\theta A_\theta + p_\varphi A_\varphi)$, where $p_r = mr'$, $p_\theta = mr\theta'$, $p_\varphi = mr\varphi' \sin\theta$ are mechanical momenta. Since we are interested in spherically symmetric CCS, the conditions for the components of the vector-potential are $A_r = A_\varphi = A_\theta = 0$. There are known three constants of motion of a particle in the field of central forces: H – Hamiltonian, N – full momentum and L – the projection of the full momentum on the polar axis. Transformation from mechanical momenta to H, N, L is given by

$$D = \frac{mN}{r^3 \sin\theta} \frac{1}{\sqrt{(2m(H \pm e\varphi) - \frac{N^2}{r^2})(\frac{N^2}{r^2} - \frac{L^2}{r^2}\sin^2\theta)}},$$

where the plus and minus signs represent ions and electrons, respectively.

The distribution functions of the particles should not depend on L in the case of spherically symmetric states and we define these as follows

$$f_e = f_e^0 \delta(H - H_e) F_e(N), f_i = f_i^0 \delta(H - H_i) F_i(N). \quad (1)$$

To simplify calculations and evaluate restriction to the behavior of the particle distributions, we take the N-distribution in the following form:

$$F_e(N) = N^\alpha, F_i(N) = N^\beta. \quad (2)$$

The constants H_e and H_i are defined from the conditions: the full energies $H_i = 0$ (ions at rest) and $H_e = W$ provided $\varphi = 0$. Using equations (1), (2) one obtains the charge densities of electrons and ions:

$$\rho_e = \pi m_e f_e^o \frac{1}{r^2\sqrt{m_e W}} \int_0^{N_m} \frac{NF_e(N)dN}{\sqrt{2(1-\Phi) - N^2 / m_e W r^2}},$$
$$\rho_i = \pi m_i f_i^o \frac{1}{r^2\sqrt{m_e W}} \int_0^{N_m} \frac{NF_i(N)dN}{\sqrt{2(m_e/m_i)\Phi - N^2 / m_e W r^2}}, \quad (3)$$

where dimensionless electrostatic potential $\Phi = e\varphi/W$ and the integration is performed on the areas of nonnegative values of the radical functions. Using equations (2) and (3) and defining the quantities E and I:

$$E = f_e^0 \frac{4\pi e m_e}{W}(2m_e W)^{(1+\alpha)/2} 2^\alpha B[\frac{\alpha+2}{2}, \frac{\alpha+2}{2}],$$
$$I = f_i^0 \frac{4\pi e m_i}{W}(2m_i W)^{(1+\beta)/2} 2^\beta B[\frac{\beta+2}{2}, \frac{\beta+2}{2}], \qquad (4)$$

where B is beta-function - Euler integral of 1-st kind, the charge densities can be written as

$$\rho_e = \frac{W}{4\pi e} E r^\alpha (1-\Phi)^{(1+\alpha)/2},$$
$$\rho_i = \frac{W}{4\pi e} I r^\beta \Phi^{(1+\beta)/2}. \qquad (5)$$

Defining dimensionless radius and parameter G by

$$x = rI^{1/(\beta+2)}, G = EI^{(\alpha+2)/(\beta+2)}, \qquad (6)$$

the equation for self-consistent potential can be written as

$$\frac{1}{x^2}\frac{d}{dx}x^2\frac{d\Phi}{dx} = -Gx^\alpha(1-\Phi)^{(1+\alpha)/2} + x^\beta \Phi^{(1+\beta)/2}. \qquad (7)$$

This equation describes spherically symmetric BGK-waves and can be compared with the Thomas-Fermi equation that is obtained under somewhat different conditions: distributed ion charge instead of point charge and non-Fermi statistics of energy distribution.

The boundary conditions are defined as

$$\Phi(\infty) \to 0, \Phi'(\infty) \to 0,$$

i.e. $\rho_e(\infty) \to \rho_i(\infty) \to$ *const.* The potential of CCS has to be dropped faster then 1/x when $x \to \infty$, i.e. we shall seek neutral CCS. The charge densities have to be integrated and the electrostatic potential has to be limited at the center of CCS.

In the following we restrict our consideration to the case of uniform electron distribution on N ($\alpha = 0$) which satisfies the condition $\rho_e(\infty) \to$ *const.* Assuming $\alpha=0$ equation (7) becomes

$$\frac{1}{x^2}\frac{d}{dx}x^2\frac{d\Phi}{dx} = -G\sqrt{1-\Phi} + x^\beta \Phi^{(1+\beta)/2}. \qquad (8)$$

3. QUALITATEVELY ANALYSIS AND ASYMPTOTIC SOLUTIONS FOR CCS

Under given boundary conditions the neutral CCS can exist in the following intervals of the exponent β of ion distribution function:

- $\infty > \beta > 1$, here φ drops asymptotically faster than 1/x and slower then $1/x^2$;
- $-1 > \beta > -2$, here φ drops asymptotically faster than $1/x^4$.

If $\beta < 0$, the ion and electron densities at the center of CCS diverge proportionally x^β and $x^{\beta+1}$, respectively. The condition of limited value of the potential at the center of CCS is satisfied under the condition of $\beta > -2$.

The asymptotic solution of equation (8) satisfying the given boundary conditions has the following form:

$$\Phi = G^{2/(1+\beta)} t^{2\beta/(1+\beta)} - \frac{G^{4/(1+\beta)} t^{4\beta/(1+\beta)}}{1+\beta}$$
$$- \frac{(\beta-2)G^{6/(1+\beta)} t^{6\beta/(1+\beta)}}{2(1+\beta)^2} + \frac{G^{(3-\beta)/(1+\beta)} t^{6\beta/(1+\beta)+2/(1+\beta)}}{(1+\beta)^3} + \cdots, \qquad (9)$$

where t = 1/x.

Equation (8) was solved numerically to satisfy the asymptotic (9). There is only one value of the parameter G for each values $\Phi(0)$ and β satisfying the boundary conditions.

If $\beta > 1$ the central part of CCS is formed by passing particles coming from infinity ad going to infinity and has negative (electron) charge. In the case if $-2 < \beta < -1$ the central part of CCS is formed as passing as captured electrons and captured ions, and has positive (ion) charge.

It will be noted that another equilibrium state can be described be equation (8). This state is not transferred to neutral plasma. Let us change the boundary conditions as follows:

$\Phi(\infty) \to 1$ and $\Phi'(\infty) \to 0$ faster than 1/x.

It is seen that such states can exist only if $\beta < 0$, but electron and ion densities diverge as x^β when $x \to 0$. As distinct from the above considered case, here ions move and electrons at rest when $x \to \infty$. Neutral CCS can exist only if $\beta < -1/2$. While the densities diverge at the center of CCS the potential is limited and electric field is zero if $-1 < \beta < -1/2$.

CONCLUSIONS

It is shown the possibility of existence of spherically symmetrical quasi-neutral collective coupling states (CCS) in collision-less plasma. The concept of CCS was discussed. The simplest spherically symmetrical configuration has been considered without spin and polarisation in the frame of kinetic description for monochromatic on energy electron and ion components and distribution functions on full momentum in forms of simple power functions of different powers for electrons and ions. Asymptotic analytical and full numerical solutions were found for CCS.

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СВЯЗАННЫЕ СОСТОЯНИЯ В ДВИЖУЩЕЙСЯ КВАЗИНЕЙТРАЛЬНОЙ СРЕДЕ

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Обсуждается возможность существования сферически симметричных квазинейтральных коллективных связанных состояний (КСС) в бесстолкновительной плазме. В квазинейтральных системах, образованных горячей электронной компонентой, движущейся со средней скоростью близкой к скорости ионов, существенно выражены корреляционные явления, обуславливающие формирование КСС. КСС образуются большим числом электронов и ионов и представляют собой области с потенциалом, отличающимся от среднего (нулевого) потенциала плазмы. КСС можно рассматривать также как нелинейные БГК-волны. Последнее определение как раз в большей степени используется для описания равновесных состояний с нелинейными электростатическими полями. В качестве примера в рамках кинетического описания, в котором электроны и ионы монохроматичны по энергии, а распределения по моментам выбраны в виде простых степенных функций с различными показателями для электронов и ионов, рассмотрены простейшие сферически симметричные конфигурации, не обладающие спином и поляризацией. Получены аналитические асимптотические и полные численные решения, описывающие КСС с захваченными и проходящими частицами, переходящими в нейтральную плазму конечной плотности, образованную хаотически движущимися электронами и покоящимися ионами.

ЗВ'ЯЗАНІ СТАНИ В КВАЗІНЕЙТРАЛЬНОМУ СЕРЕДОВИЩІ, ЩО РУХАЄТЬСЯ

А.В. Агафонов

Обговорюється можливість існування сферично симетричних квазінейтральних колективних зв'язаних станів (КЗС) у беззіштовхувальній плазмі. У квазінейтральних системах, утворених гарячим електронним компонентом, що рухається із середньою швидкістю близької до швидкості іонів, істотно виражені кореляційні явища, що обумовлюють формування КЗС. КЗС утворюються великим числом електронів і іонів і являють собою області з потенціалом, що відрізняється від середнього (нульового) потенціалу плазми. КЗС можна розглядати також як нелінійні БГК-хвилі. Останнє визначення саме в більшому ступені використовується для опису рівноважних станів з нелінійними електростатичними полями. Як приклад у рамках кінетичного опису, у якому електрони й іони є монохроматичними по енергії, а розподіли по моментах обрані у виді простих степеневих функцій з різними показниками для електронів і іонів, розглянуті найпростіші сферично симетричні конфігурації, що не мають спин і поляризацію. Отримано аналітичні асимптотичні і повні чисельні рішення, що описують КЗС із захопленими і минаючими частками, що переходять у нейтральну плазму кінцевої густини, утворену електронами, що хаотично рухаються, і непорушними іонами.