

# CHARACTERISTICS OF INHOMOGENEOUS ACCELERATING STRUCTURES

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We present the results of investigations of accelerating structures consisting of a segment of an inhomogeneous disk-loaded waveguide, input and output couplers. The sizes of the apertures inside the waveguide take random values in the interval  $1.2 < a < 1.3$  cm. The cavity radii and the parameters of the couplers are tuned depending on the iris apertures and the operating frequency. At the operating frequency (2797.2 MHz) the phase shift per cell is  $2\pi/3$ .

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## 1. INTRODUCTION

The transverse instability in linear accelerators is due to the dipole modes excited by the beam. There are different methods of beam break-up suppression: quality factor decreasing, reduction of the effective length of coherent interaction, using of external focusing. The so-called detuned structure is designed to be an approximately constant gradient with the dipole mode frequencies adjusted to have a Gaussian density distribution by varying the iris radius and the disk thickness. For evacuation of dipole modes the damped detuned structure has four rounded rectangular damping manifolds arranged at 90 degree intervals in azimuth around the accelerator cells. Another way to decrease the quality factor of dipole mode is making the radial cuttings on the disks. In the constant gradient and quasi-constant gradient structures the accelerating mode properties are essentially unchanged but the beam breakup passband is perturbed in frequency.

Existence of passbands in the structure is the consequence of its periodicity. It is possible to suppress the transverse instability by the use of non-periodic accelerating structures. From other hand, acceleration is possible only in the case when the synchronism condition between particles and electromagnetic field is fulfilled.

On the base of the method [1] the tuning method was developed for adjusting elements of disk-loaded structures, in which the sizes of apertures are arbitrary [2]. The radii of the cells are tuned in dependence on the sizes of the apertures so that at the operating frequency the phase shift per cell equals the transit angle of the relativistic particle. The consecutive tuning is used in the case of small coupling coefficients between cells, when only the adjacent coupling plays the important role. The cells are consecutively tuned in the special cavity stack. The negligence of the "remote" coupling produces errors during tuning of about  $\Delta\varphi \leq +0.5^\circ$  [3] which agree in value and sign with the obtained deviation of the operating frequency of about 150-200 kHz [2].

## 2. METHOD OF CONSECUTIVE TUNING

Under small coupling disk-loaded waveguides can be described with a definite accuracy as a chain of oscillators, every of which is coupled only with adjacent ones:

$$\left[ \omega_n^2 (1 + \Gamma_n) - \omega^2 \right] U_n = \omega_n^2 \Gamma_n^{(-)} U_{n-1} + \omega_n^2 \Gamma_n^{(+)} U_{n+1}, \quad (1)$$

where  $U_n$  is the amplitude of  $E_{010}$ -mode in the  $n$ -th cavity,  $\omega_n$  the eigenfrequency of  $E_{010}$ -mode in the  $n$ -th cavity.

In general, coefficients  $\Gamma_n$ ,  $\Gamma_n^{(-)}$  and  $\Gamma_n^{(+)}$  for the  $n$ -th cavity depend on the iris radii  $a_n$  and  $a_{n+1}$ , the dimensions of the  $n-1$ ,  $n$ ,  $n+1$  cavities, and the frequency [3]. In the most simple case of coupling through the small aperture in the infinitely thin wall coefficients  $\Gamma_n$ ,  $\Gamma_n^{(-)}$  and  $\Gamma_n^{(+)}$  have been obtained in the following form [4]:

$$\Gamma_n \rightarrow \alpha_{n,n-1}^{(0)} + \alpha_{n,n+1}^{(0)} = \frac{2}{3\pi J_1^2(\lambda_{01})} \cdot \frac{a_n^3}{b_n^2 d} + \frac{2}{3\pi J_1^2(\lambda_{01})} \cdot \frac{a_{n+1}^3}{b_n^2 d}, \quad (2.1)$$

$$\Gamma_n^{(-)} \rightarrow \beta_{n,n-1}^{(0)} = \frac{2}{3\pi J_1^2(\lambda_{01})} \cdot \frac{a_n^3}{b_n b_{n-1} d}, \quad (2.2)$$

$$\Gamma_n^{(+)} \rightarrow \beta_{n,n+1}^{(0)} = \frac{2}{3\pi J_1^2(\lambda_{01})} \cdot \frac{a_{n+1}^3}{b_n b_{n+1} d}, \quad (2.3)$$

where  $\lambda_{0l}$  is the first root of the zero order Bessel function  $J_0$ ,  $b$  and  $d$  are the radius and length of the cavity.

The coupling coefficients of two cavities, without any assumptions on the cavity dimensions, are given in [5]:

$$\Gamma_n \rightarrow \alpha_{n,n-1}^{(0)} \cdot \Lambda_{n,n-1} + \alpha_{n,n+1}^{(0)} \cdot \Lambda_{n,n+1}, \quad (3.1)$$

$$\Gamma_n^{(-)} \rightarrow \beta_{n,n-1}^{(0)} \cdot \tilde{\Lambda}_{n,n-1}, \quad (3.2)$$

$$\Gamma_n^{(+)} \rightarrow \beta_{n,n+1}^{(0)} \cdot \tilde{\Lambda}_{n,n+1}. \quad (3.3)$$

Coefficients  $\Lambda$  and  $\tilde{\Lambda}$  depend not only on the geometrical dimensions of the cells, but on the frequency as well. However, it was shown that the dependence of these coefficients on the iris radius and wall thickness is the most efficient. For the wall thickness  $t = 0.4$  cm the dependence of the coefficients  $\Lambda$  and  $\tilde{\Lambda}$  on  $a$  can be approximated by the following expressions:

$$\Lambda = 0.1633a^2 - 0.7078a + 1.5242, \quad (4.1)$$

$$\tilde{\Lambda} = -0.1966a^2 + 0.6541a - 0.0799. \quad (4.2)$$

We find the condition when the set of equations (1) at the operating frequency  $\omega = \omega_*$  have the solution of the following form:

$$U_n = U_{n,0} \exp(in\varphi), \quad (5)$$

where  $U_{n,0}$  is the real value. To obtain the solution of the set of equations (1) of such a form, the following condition is to be fulfilled:

$$\Gamma_n^{(-)} U_{n-1,0} = \Gamma_n^{(+)} U_{n+1,0}. \quad (6)$$

Under this condition the equations for the  $n$ -th and  $n-1$ -th cavities can be written in the form:

$$\left[ \omega_n^2 (1 + \Gamma_n) - \omega_*^2 \right] U_{n,0} = 2 \cos \varphi \cdot \omega_n^2 \Gamma_n^{(-)} U_{n-1,0} \quad (7.1)$$

$$\left[ \omega_{n-1}^2 (1 + \Gamma_{n-1}) - \omega_*^2 \right] U_{n-1,0} = 2 \cos \varphi \cdot \omega_{n-1}^2 \Gamma_{n-1}^{(+)} U_{n,0}, \quad (7.2)$$

hence

$$\left( \omega_n^2 (1 + \Gamma_n) - \omega_*^2 \right) \left( \omega_{n-1}^2 (1 + \Gamma_{n-1}) - \omega_*^2 \right) = 4 \cos^2 \varphi \cdot \omega_n^2 \omega_{n-1}^2 \Gamma_n^{(-)} \Gamma_{n-1}^{(+)} \quad (8)$$

So, if the values of  $b_{n-1}$ ,  $a_{n-1}$ ,  $a_n$ ,  $a_{n+1}$ ,  $d$  are already known, the value of  $b_n$  can be obtained from this equation.

We suppose that all cells have an equal length  $D = d + t$ , which is determined by the synchronism condition between the electromagnetic field and the particle. If  $b_n$  has been obtained, and  $a_n$ ,  $a_{n+1}$ ,  $a_{n+2}$  are known, then one can obtain  $b_{n+1}$  from equation (8), and so on. So, the whole accelerating structure can be consecutively tuned. At the first step two identical cells ( $a_1 = a_2 = a_3 = a_0$ ,  $b_1 = b_2 = b_0$ ) are to be tuned on the fixed phase shift per cell  $\varphi$ . Then, using disks with arbitrary values of  $a$ , all the radii of the cells  $b$  can be determined. Any segment of the structure tuned by such a way can be considered as an independent structure.

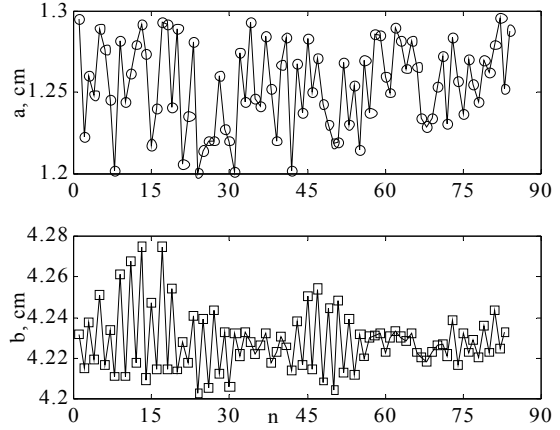
### 3. CHARACTERISTICS OF INHOMOGENEOUS STRUCTURES

Smooth variation of the iris radius is used in constant gradient and quasi-constant gradient structures. The method of consecutive tuning makes it possible to tune the structures not only with the smooth low of aperture variation, but with the abrupt one as well. Random distribution of the iris aperture can be considered as an example of this variation.

Consider the structure in which the value of the iris radius deviates from the constant value of  $a_0$  on some random quantity  $r_n$ :

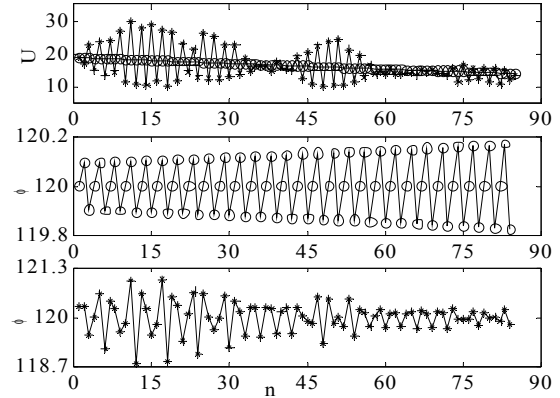
$$a_n = a_0 + r_n. \quad (9)$$

Lets the quantity  $r_n$  is uniformly distributed in the interval  $\pm 0.05$  cm. The iris and cell radii in the “random” constant impedance structure are shown in Fig. 1. The structure is 3.036 meters long with 85 cells (including the couplers). Frequency of the accelerating mode is  $f_* = 2797.2$  MHz. Phase shift per cell is  $\varphi = 2\pi/3$ .



**Fig. 1** Iris radii –  $a$  and cell radii –  $b$  in the “random” constant impedance structure

The iris radius in the constant impedance structure is  $a_0 = 1.25$  cm. In Fig. 2 we have shown the amplitude distribution and the phase shift per cell in the “random” constant impedance structure and in the usual constant impedance structure.



**Fig. 2** Field amplitude distribution  $U$  MV/m and phase shift per cell  $\varphi^\circ$  in the “random” structure (\*) and in the constant-impedance structure (o)

The energy gain, rate of acceleration and the power attenuation in the constant-impedance and in the “random” constant-impedance structure are shown in Table 1.

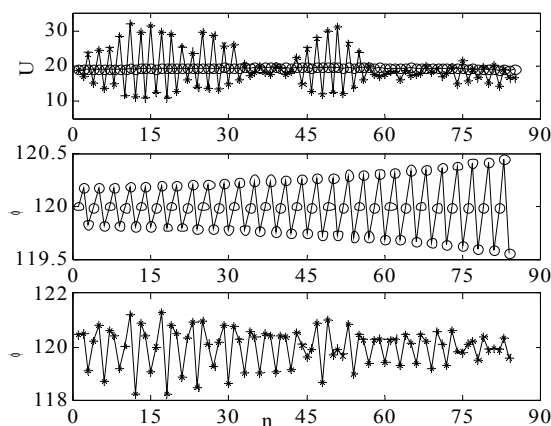
The length of the input (output) coupler is equal to that of the structure cell. The frequencies of the couplers and the coupling values are determined from the condition of reflected wave absence at the operating frequency.

**Table 1.** Characteristics of constant-impedance and “random” constant-impedance sections

Input power 13 MW	Constant-impedance	“random” structure
Energy gain, MeV	36.76	36.96
Rate of accel., MeV/m	12.11	12.17

Attenuation, nep.	0.69	0.79
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Amplitude distribution and phase shift per cell in the constant gradient and "random" constant gradient structures are shown in Fig. 3. In Table 2 we have shown characteristics of these sections.



**Fig. 3** Field amplitude distribution  $U$  MV/m and phase shift per cell  $\phi^\circ$  in the "random" structure (\*) and in the constant gradient structure (o)

**Table 2.** Characteristics of constant-gradient and "random" constant-gradient sections

Input power 13 MW	Constant-gradient	"random" structure
Energy gain, MeV	43.23	42.92
Rate of accel., MeV/m	14.23	14.13
Attenuation, nep.	1.107	1.23

The values of the energy gain and the rate of acceleration in Tables 1, 2 are higher than one can expect in reality. It is caused by the fact that the influence of higher modes on the field distribution in the cell is not considered in the using model. However, these values can be used for the comparison of different structures.

The characteristics of "random" structures are comparable with the characteristics of constant impedance and constant gradient ones. Despite the fact that there would be some worry about field breakdown, we assume that the "random" structure can be used for the suppression of beam blow-up instability.

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## ХАРАКТЕРИСТИКИ НЕОДНОРОДНЫХ УСКОРЯЮЩИХ СТРУКТУР

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Мы представляем результаты исследования ускоряющих структур, состоящих из отрезка неоднородного диафрагмированного волновода, входного и выходного трансформаторов типа волны. Размеры радиусов отверстий связи внутри волновода принимают случайные значения в интервале  $1.2 < a < 1.3$  см. Радиусы ячеек и параметры трансформаторов типа волны настраиваются в зависимости от размеров пролетных отверстий и значения рабочей частоты. На рабочей частоте (2797.2 МГц) набег фазы поля на ячейке составляет  $2\pi/3$ .

## ХАРАКТЕРИСТИКИ НЕОДНОРІДНИХ ПРИСКОРЮЮЧИХ СТРУКТУР

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Ми приводимо результати дослідження прискорюючих структур, які складаються з відрізка неоднорідного хвилеводу, вхідного та вихідного трансформаторів типу хвилі. Розміри отворів зв'язку у середині хвилеводу приймають випадкові значення в інтервалі  $1.2 < a < 1.3$  см. Радіуси ячійок і параметри трансформаторів типу хвилі налагоджуються в залежності від розмірів отворів зв'язку та робочої частоти. На робочій частоті (2797.2 МГц) набіг фази поля на комірці складає  $2\pi/3$ .