NON-LINEAR EVOLUTION OF VORTICES IN HIGH-CURRENT ELECTROSTATIC PLASMA LENS

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The spatial structure and nonlinear dynamics of vortices in plasma lens for high-current ion beam focusing have been investigated theoretically.
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1. INTRODUCTION

It is known from numerical simulations and experiments that vortices are long-lived structures in vacuum. However, the acceleration of evolution of vortices in electron plasma was observed in laboratory experiments. Same dynamics of vortices should take place in near wall turbulence of nuclear fusion installations, where the crossed configuration of electrical and magnetic fields also is realized.

The charged plasma lens, intended for focusing of high-current ion beams, has the same crossed configuration of fields [1]. It is important to know the properties of vortices at the nonlinear stage of their evolution. It has been shown theoretically in this paper, that after reaching the quasi-stationary state the electrons in a field of a vortex rotate around its axis with the higher velocity in comparison with the velocity of azimuthal drift of electrons in the fields of the lens. Slow and quick vortices are contacting combinations of two vortices rotated in the opposite directions.

The instability development in the initially homogeneous plasma causes that the vortices are born pairs. Namely, if the vortex-bunch of electrons is generated, the vortex-hole of electrons occurs near it. It has been shown, that at small inhomogeneous electron density in the real experimental lens the preference is realized in the behaviour of vortices. Namely, the vortex - bunch goes to the region of a higher electron density ne, and vortex – hole goes to the region of lower ne.

2. JOINT DEVELOPMENT OF TWO INSTABILITIES

In [2] the dispersion law of oscillations, possible in the plasma lens is presented. The obtained dispersion law describes the joint development of two instabilities. Namely, in a limiting case l0|\omega_e|<<k_*V_b, basically, the instability of the ion stream relative to electrons develops. Here V_b is the ion beam velocity, k_* is the longitudinal wave vector, l0 is the azimuthal angular number, \omega_e is the angular velocity of the electron drift in the crossed fields. Thus, the growth rate of the instability development in the case l0|\omega_e|<<k_*V_b increases with the growth of k_*. In the limiting case l0|\omega_e|>>k_*V_b the instability of electrons, drifting relative to ions in the crossed fields in the cylindrical system with a radial gradient of the magnetic field develops. Slow vortices have the highest growth rate. Joint development of two instabilities under conditions typical for experiments l0\omega_e<<k_*V_b, results to that the growth rate of the slow vortical perturbation is more for the most homogeneous perturbation in the longitudinal direction, as the finite dimensions of the lens allow.

At instability development the vortices are born as follows. The non-uniform electric field \mathbf{E}(r, t) (=\nabla \varphi(r, t)), arising as a result of the instability development, leads to the nonuniform electron dynamics with a velocity perturbation \delta\mathbf{V}(r, t)=\langle e/m_o \omega_\perp \rangle (\mathbf{e}, \nabla \varphi). As a result of a nonuniform \delta\mathbf{V}(r, t) the electron bunching is performed which results in the nonuniform distribution of the electron density perturbation \delta n_e=n_e(\mathbf{k}\cdot \nabla)/(\omega_\perp l_0\omega_\perp). Last automatically results in the vortical movement of the electrons with a vorticity \alpha=e_\perp \mathbf{r} \times \mathbf{V}=(\omega_\perp^2/n_e)\delta n_e/n_e. 

3. SPATIAL STRUCTURE OF VORTICES

Let us describe the structure of a quick vortex in the rest frame, rotating with the angular velocity \omega_\perp=V_{ph}/r_0. Let us consider a chain on \theta of vortices - bunches and vortexes - holes of electrons. Neglecting nonstationary and nonlinear - on \phi - terms, we derive the following equation

\mathbf{V}_*=-(e/m_0\omega_{bi})\mathbf{e}, \mathbf{E}_*+\langle e/m_0\omega_\perp \rangle (\mathbf{e}, \nabla \varphi), \tag{1}

describing the quasi-stationary dynamics of electrons in the fields of the lens and the vortical perturbation. From (1) we obtain the expression for radial and azimuthal electron velocities

\mathbf{V}_*=-(e/m_0\omega_{bi})\mathbf{e}, \mathbf{E}_*+\langle e/m_0\omega_\perp \rangle (\mathbf{e}, \nabla \varphi), \tag{2}

\mathbf{V}_* can be presented as a sum of the phase velocity of perturbation, V_{ph}, and velocity of azimuth electron oscillations, \delta\mathbf{V}_*, in the field of perturbation, \mathbf{V}_*V_{ph}+\delta\mathbf{V}_*. Because \mathbf{V}_*=rd\theta/dt, we present d\theta/dt as \mathbf{d}/dt+rd\theta/dt, where \omega_{bi}=(\Delta n_e/n_e)(\omega_\perp^2/2\omega_{bi})r_{in}, r is the radius of the vortical perturbation location. Then from (2) we obtain

\mathbf{d}/dt=(\omega_\perp^2/2\Delta n_e/n_e)(1/\omega_{bi}(r)-1/\omega_{bi}(r))+(e/m_0\omega_{bi})\mathbf{e}, \mathbf{d}/dt=-\langle e/m_0\omega_\perp \rangle (\mathbf{e}, \nabla \varphi)\mathbf{e}, \tag{3}

At small diversions of \theta from \theta_0, decomposing \omega_{bi}(r) on \theta=\theta_0+\xi, and integrating (3), we derive

(\delta r)^2 - 2\omega_{bi}(r)\varphi(r)\varphi(r)\mathbf{d}/dt = \mathbf{d}/dt, \tag{4}

The vortex boundary separates the trapped electrons, forming the vortex and moving on closed
trajectories and untrapped electrons, moving outside the boundary of the vortex and oscillating in its field. For vortex boundary we derive the following expression from the condition \(\delta \tau = \delta \tau_{cl} \).

\[
\delta \tau = \frac{2(\phi_{\text{cl}} \omega_{0}(r_{s})/\pi e^2 A_{\text{cl}})}{\pi e^{2} A_{\text{cl}}(\delta \omega_{0\text{cl}}/r_{s})} \) \]  \( \delta \tau_{cl} \) \] (5)

Here \(\delta \tau_{cl}\) is the radial width of the vortex - bunch of electrons. From (5) the radial size of the vortex - hole of electrons follows

\[
\delta r_{v} = \frac{2(\phi_{\text{cl}} \omega_{0}(r_{s})/\pi e^2 A_{\text{cl}})}{\pi e^{2} A_{\text{cl}}(\delta \omega_{0\text{cl}}/r_{s})} \] \] (6)

From the equation of electron motion and Poisson equation it is possible to derive approximately the expression for the vortex size \(\alpha = e\omega_{0}A_{\text{cl}}\), which is characteristic of the vortex motion of electrons

\[
\alpha \approx -2e\omega_{0}/m \equiv -2e\omega_{0}/n_{0} \phi_{\text{cl}} \]

From here it follows that up to certain amplitude of vortices the structure of electron trajectories in the field of the chain on \(s\) of quick vortices in the system of rest, rotated with \(\omega_{0} \equiv V_{ph}/r_{s}\), is similar to the structure, presented in [2].

For large amplitudes of quick vortices in the region of the electron bunches the contraflows are formed. The vortex - hole rotates in the rest frame, rotating with a frequency \(\omega_{0}\), the vortex - bunch rotates in the rest frame, rotating in the opposite direction of rotation of unperturbed plasma at \(\omega_{n} \equiv \omega_{0}/n_{0}\). It is seen that the size of the vortex is inversely proportional to \(\omega\) and is proportional to \(\phi_{cl}\). That is the size of the vortex essentially depends on \(\phi_{cl}\) with \(\omega_{n}\) already at small perturbations of electron density the sizes of the vortex, \(\delta r_{v}\), can reach \(\delta r_{v} = 2R, \ R \) is the plasma lens radius

(3) can be integrated without decomposition \(\omega_{0}(r)\) on \(\delta \tau = \tau_{cl}\). For this purpose we approximate \(\omega_{0}(r) = \omega_{0}(1 + \mu r^{2}/R^{2})\). Then, integrating (3), we derive

\[
2(\phi_{\text{cl}} \omega_{0}(r_{s})/\pi e^2 A_{\text{cl}})\] \[\] (7)

From the condition \(\phi_{\text{cl}} = \phi_{cl}(\mu)\) and (7) we obtain the expression, determining the boundary of the vortex - hole of electrons,

\[
[r^{2} + (\delta r_{cl})(t)]^2 - [\omega_{0}(r_{s})/\omega_{0}(r)]^2 [r^{2} - (\delta r_{cl})^2] = - \mu^{2} \omega_{0}(r_{s})/2 \] \[\] (8)

From (8) and \(\phi_{\text{cl}} = \phi_{cl}(\mu)\) we derive the expression, determining the radial width of the vortex - hole of electrons,

\[
\phi_{\text{cl}} = \frac{4R^{2} \omega_{0}(r_{s})/\pi e^{2} A_{\text{cl}}}{\delta r_{cl}} + \frac{2(\phi_{\text{cl}} \omega_{0}(r_{s})/\pi e^2 A_{\text{cl}})}{\pi e^{2} A_{\text{cl}}(\delta \omega_{0\text{cl}}/r_{s})} \] \[\] (9)

Let us consider the vortex with the small phase velocity \(V_{ph}\) in comparison with the drift electron velocity, \(V_{ph} < < V_{n0}\). The spatial structure of the electron trajectories in its field for small amplitudes of the vortex looks like that shown in Fig.1. It is determined by that in all lens \(\theta\) has an identical sign, \(\alpha = 0\). In other words, the radial electric field, created by the vortex is less, than the electric field of the lens, \(E_{n} < E_{\text{n0}}\). Then in all lens the azimuthal electron velocities have an identical sign and there are not contraflows of electrons. The slow vortex of a small amplitude does not have a separatrix. For the description of the electron trajectories we use (2). Using in them \(V_{ph} = d\phi/d\tau\) and excluding \(\theta\), we obtain for boundary of the vortex \(r(\theta)\)

\[
\delta \tau = \frac{2(\phi_{\text{cl}} \omega_{0}(r_{s})/\pi e^2 A_{\text{cl}})}{\pi e^{2} A_{\text{cl}}(\delta \omega_{0\text{cl}}/r_{s})} \] \[\] (10)

In the case of small amplitudes (10) becomes

\[
\delta \tau = \frac{2(\phi_{\text{cl}} \omega_{0}(r_{s})/\pi e^2 A_{\text{cl}})}{\pi e^{2} A_{\text{cl}}(\delta \omega_{0\text{cl}}/r_{s})} \] \[\] (11)

From (10) we derive the radial size of the slow vortex

\[
\delta r_{cl} = \frac{2(\phi_{\text{cl}} \omega_{0}(r_{s})/\pi e^2 A_{\text{cl}})}{\pi e^{2} A_{\text{cl}}(\delta \omega_{0\text{cl}}/r_{s})} \] \[\] (12)

In the case of small amplitudes (12) becomes

\[
\delta r_{cl} = \frac{2(\phi_{\text{cl}} \omega_{0}(r_{s})/\pi e^2 A_{\text{cl}})}{\pi e^{2} A_{\text{cl}}(\delta \omega_{0\text{cl}}/r_{s})} \] \[\] (13)

For the description of the slow vortex structure one can also use the equation

\[
d\omega_{0}/d\tau = \omega_{0}(V_{ph})/\pi e^{2} A_{\text{cl}} \]

(14)

We obtain approximately from (14) the equation, describing the slow vortex of the small amplitude \(\delta r = [\omega_{0}(V_{ph})/\pi e^{2} A_{\text{cl}}] \)

\[
d\delta r/d\tau = \omega_{0}(V_{ph})/\pi e^{2} A_{\text{cl}} \]

(15)

for the condition of the electron trapping

\[
\omega_{0}(V_{ph})/\pi e^{2} A_{\text{cl}} \]

(16)

Fig.1.

4. SATURATION OF EXCITED HOMOGENEOUS SLOW VORTICAL TURBULENCE

For quick vortices the cause of the instability is the gradient of the velocity \(\omega_{0}\), therefore for development of instability the nonadiabatic dynamics of electrons is necessary. For slow vortices the reason of the instability is the interaction of the drifting electron stream with ions, therefore amplitude of the saturation of the slow vortex is determined from the condition of the ion trapping

\[
V_{n0} = V_{ph} \]

(15)

or from the condition of the electron trapping

\[
V_{n0} = (V_{ph}) \]

(16)

and is determined by smaller of them. For the plasma lens, close to the optimum plasma lens, the saturation is determined by electron trapping. For the plasma lens, far from the optimum plasma lens, the saturation is determined by ion trapping. The slow homogeneous turbulence is not separated into single vortices.

5. NONLINEAR DYNAMICS OF VORTICES

The development of instability in initially homogeneous plasma lens causes that the vortices are born pairs: if the vortex - bunch of electrons is generated, the vortex - hole of electrons occurs near it.

Let us consider how the nonhomogeneity of electron density effects on the behaviour of vortices. Finiteness of time of the vortices symmetrization and also the reflection of resonant electrons from vortices - bunches result that the vortices are asymmetrical. Namely, on opposite on θ parties of vortices the small bunches and holes are formed. It results in formation of polarization azimuth electric fields $E_\theta$, directed along $e_\theta$. The formation of fields $E_\theta$ causes the radial drift and spatial separation of vortices (see fig.2). In other words, the property of preference of motion of the vortex - hole on the peripherals of the plasma column and the vortex - bunch to its axis is realized. The polarization electric fields in the vortex - hole and the vortex - bunch have opposite signs. Namely, the vortex - hole goes to the region of a lower electron density, and the vortex - bunch goes to the region of higher electron density).

![Fig. 2. The opposite radial shift of the vortex - bunch of the electron density and the vortex - hole](image)

The resonant electrons are reflected from the vortex-bunch. Thus the distribution of the electron density being asymmetrical on azimuth is formed. It results in the radial motion of the vortex - bunch of electrons and leads to simultaneous formation of spiral distribution of the electron density. In the case of the azimuthally symmetrical vortex its velocity of radial drift is equal to

$$V_r = \left( \frac{\rho^2}{20n_0} \right) \left( \frac{\delta n_e}{\delta x} \right) \left( \frac{dn_e}{dr} \right) |_{r=v} \quad (17)$$

$R_v$ is the radius of the vortex. The width of the spiral is equal to the radial width of the vortex in the case of its high radial velocity. In the case of a low radial velocity of the vortex the width of the spiral is less, than the radial width of the vortex.

When two vortices - bunches of electrons begin to concern each other, the electrons of each vortex, taking place near to its boundary, are reflected from the next vortex. Thus the asymmetry is formed on the azimuth distribution of the electron density in the neighbourhood of each vortex. It leads to occurrence of a relative velocity of vortices.

$$V_{rel} = \left[ \frac{\rho^2}{20n_0} \right] R_v \quad (18)$$

The similar behaviour of electrons was observed in experiments in the only electron plasma, in the charged plasma of the lens [1,3] and in the plasma, placed in crossed radial electrical and longitudinal magnetic fields.

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REFERENCES