SPECTRA OF CHERENKOV AND TRANSITION RADIATION EXCITED BY A TRAIN OF ELECTRON BUNCHES IN DIELECTRIC STRUCTURES OF RECTANGULAR CROSS-SECTION

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The peculiarities of radiation spectra of a single relativistic electron bunch or train of bunches in an infinite rectangular dielectric waveguide, in a semi-infinite dielectric waveguide, and in a dielectric waveguide of finite length were theoretically explored. Single-mode and multimode modes were surveyed. In the first case, the excited fields corresponding to Cherenkov and transition radiation were identified. In the second case, the substantial growth of field amplitude was obtained due to interference of many excited transversal modes. Limitation on maximum field amplitude and amount of contributing bunches caused by the fields removal with a group velocity was found. The features of excited spectra caused by a regular sequence of bunches were investigated. Selective excitation of transversal modes and corresponding field waveform were considered for the arbitrary rectangular geometry.

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1. INTRODUCTION

Wake field acceleration is considered to be a highly promising advanced accelerator concept. In particular, particle acceleration by a wake field excited in a rectangular dielectric waveguide by rectangular relativistic electron bunches of cross-section 10 μ×150 μ, time duration 3.5 fsec, energy 500 MeV, and charge 1…3 pC for each has been described [1]. Theoretical investigations of the dielectric wake fields in an infinite waveguide [2-4] have shown, that for planar geometry [2] (in contrast to cylindrical [3, 4]) equally-spaced radial modes of the wake field are excited. Since the decrease of amplitudes with the number of excited modes is slow, many modes contribute to the build up of the resulting wake field. For planar geometry this causes a localized wake field sequence of large amplitude pulses of alternating sign. Because of the non-equal spacing of modes in the cylindrical case, the regularity of the wake field pattern is rapidly broken. In a proposed experiment [1], as an approximation to 2D planar geometry a 3D rectangular geometry with one transverse direction elongated is chosen. Here we study this rectangular geometry complicated by 2-D matrix of eigen-frequencies.

In addition, we study the effect of the entrance boundary on the wake fields, i.e. we solve the semi-infinite dielectric structure problem. At the entrance of the rectangular dielectric waveguide (as well as in the cylindrical case [5]) transition radiation will be excited. Also, excited Vavilov-Cherenkov radiation moves away from the entrance with a group velocity that will result in a "quenching wave" [6] which cancels the Vavilov-Cherenkov radiation in the region between the entrance and a front moving with the group velocity. Thereby, the number of coherently radiating bunches will be restricted. In this work the exact solution of the problem is obtained for the case of a point electron bunch. The field consists of Vavilov-Cherenkov and transition radiations. For each of them, separate analytical expressions are given. By using Lagrange variables the topography of the excited wake field is numerically determined for a bunch of finite size. The process of wake field excitation by a sequence of bunches is explored.

For the dielectric waveguide of finite length the problem of maximum field amplitude at the waveguide entrance due to group velocity effect arises along with the maximum number of bunches contributing in build-up of this field.

Regular periodicity of bunch sequence leads to survival of selected resonant modes whose frequencies are multiplied to bunch repetition frequency. All mentioned phenomena are investigated analytically and numerically.

2. INFINITE DIELECTRIC WAVEGUIDE

Contrary to the planar geometry [2] in the rectangular case we have two-dimensional matrix of transverse eigen-functions and corresponding frequencies. There are three sets of equally-spaced radial modes: along diagonal and two sides of matrix. So, in general case the whole excited spectrum is not equidistant and field enhancement by amplitude peaking due to interference is not observed. However, at presence of regular sequence of bunches a lot of variants to excite various spectra appear.

3. SEMI-INFINITE DIELECTRIC WAVEGUIDE. POINT BUNCH

We consider a rectangular metal waveguide of width b and height d. The waveguide is filled with homogeneous dielectric of permittivity ε. In the longitudinal direction the waveguide extends over $0 \le z < \infty$. At the entrance $z = 0$ it is closed by a perfectly conducting metal wall.

We suppose that a monoenergetic point electron bunch is injected into the waveguide and propagates along the waveguide axis with constant velocity v_0. The charge and current densities of this bunch can be represented in the form:

$$\rho = -Ne\delta(x - x_L)\delta(y - y_L)\delta(t - t_L)/v_L, \quad j = e_z v_L \rho, \quad (1)$$

where $-e$ is charge of electron, N is the number of electrons in the bunch, $x_L(t_0, x_0, y_0, z)$, $y_L(t_0, x_0, y_0, z)$, $v_L(t_0, x_0, y_0, z)$ are the Lagrange velocity and spatial

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coordinates of bunch, accordingly, $t_L(t_0, x_0, y_0, z)$ is the Lagrange time of bunch, t_0 is injection time of the bunch into waveguide, x_0, y_0 are the coordinates of the origin of the bunch, e_z is the unit basis vector along an axis z.

In the uniform motion approximation the velocity and transverse coordinates of the bunch are stationary: $v_L = v_0$, $x_L = x_0$, $y_L = y_0$, and the Lagrangian time is $t_L = t_0 + z/v_0$.

Solving the wave equation taking into account the boundary conditions on the waveguide walls:

$$E_z(x=0) = E_z(x=b) = E_z(y=0) = E_z(x=d) = 0,$$

and on the waveguide entrance:

$$E_x(z=0) = E_y(z=0) = 0,$$

we obtain the expression for the Fourier-components of the longitudinal and transverse components of the electric field:

$$E_x^\omega = \sum_{k,l}\left\{\frac{8\pi kNe}{\varepsilon v_L b^2 d\left(k_{z_{k,l}}^2 - \frac{\omega^2}{v_L^2}\right)} e^{i\omega t_0}\sin\left(\frac{\pi k}{b}x_L\right)\sin\left(\frac{\pi l}{d}y_L\right)\times \cos\left(e^{ik_{z_{k,l}}z} - e^{i\omega z/v_L}\right)\sin\left(\frac{\pi k}{b}x\right)\sin\left(\frac{\pi l}{d}y\right)\right\}. \quad (2)$$

$$E_y^\omega = \sum_{k,l}\left\{\frac{8\pi lNe}{\varepsilon v_L d^2 b\left(k_{z_{k,l}}^2 - \frac{\omega^2}{v_L^2}\right)} e^{i\omega t_0}\sin\left(\frac{\pi k}{b}x_L\right)\sin\left(\frac{\pi l}{d}y_L\right)\times \left(e^{ik_{z_{k,l}}z} - e^{i\omega z/v_L}\right)\cos\left(\frac{\pi l}{d}y\right)\sin\left(\frac{\pi k}{b}x\right)\right\}. \quad (3)$$

$$E_z^\omega = \sum_{k,l}\left\{\frac{8iNe}{\varepsilon bd}e^{i\omega t_0}\sin\left(\frac{\pi k}{b}x_L\right)\sin\left(\frac{\pi l}{d}y_L\right)\frac{1}{\omega^2-\omega_0^2}\times\left(\omega e^{i\omega z/v_L} - \frac{\omega_0^2}{v_L k_{z_{k,l}}}e^{ik_{z_{k,l}}z}\right)\sin\left(\frac{\pi k}{b}x\right)\sin\left(\frac{\pi l}{d}y\right)\right\}, \quad (4)$$

where

$$k_{z_{k,l}}^2 = \varepsilon\omega^2/c^2 - (\pi k/b)^2 - (\pi l/d)^2 \quad (5)$$

is the longitudinal wave number of the electromagnetic eigenmodes in a dielectric waveguide.

We have derived the longitudinal component of the electric field as shown below.

Performing an inverse Fourier transform for E_z^ω, we obtain:

$$E_z = \frac{8iNe}{bd\varepsilon}\sum_{k,l}\sin\left(\frac{\pi k}{b}x_L\right)\sin\left(\frac{\pi l}{d}y_L\right)\times\sin\left(\frac{\pi k}{b}x\right)\sin\left(\frac{\pi l}{d}y\right)\{I_1 \quad I_2\}, \quad (6)$$

where

$$I_1 = \int_L d\omega\,\omega\frac{\exp[-i\omega t + i\omega(z/v_0 + t_0)]}{(\omega-\omega_0)(\omega+\omega_0)}, \quad (7)$$

$$I_2 = \int_L d\omega\frac{\omega_0^2\exp[-i\omega(t-t_0) + ik_{z_{k,l}}z]}{v_L k_{z_{k,l}}(\omega-\omega_0)(\omega+\omega_0)}. \quad (8)$$

Integral (7) describes the wake field of the charge moving in the infinite waveguide. It is easily derived by means of residue theory

$$I_1 = -2\pi i\cos\left[\omega_0\left(t - t_0 - \frac{z}{v_0}\right)\right]\theta\left(t - t_0 - \frac{z}{v_0}\right), \quad (9)$$

where integral (8) corresponds to the eigenmodes of a rectangular waveguide caused by the semi-finiteness of the system along *z* (entrance boundary). This term allows one to satisfy the boundary condition on the metal face plane of the waveguide. As will be shown further, term (8) consists of a “quenching wave” and the transition radiation. The analytical evaluation of such type of integrals has been explicitly given in [5]. It can be represented in as

$$I_2 = 2\pi i\left[U_2(r_2y, y) - U_2(r_1y, y)\right], \quad (10)$$

where $U_2(ry, y)$ is the Lommel function of two arguments, defined as

$$U_2(ry,y) = \begin{cases} -\sum_{m=1}^{\infty}(-1)^m r^{2m}J_{2m}(y) & \text{for } r \le 1, \\ -\cos\left(\frac{ry}{2} + \frac{y}{2r}\right) + \sum_{m=0}^{\infty}\frac{(-1)^m}{r^{2m}}J_{2m}(y) & \text{for } r > 1. \end{cases} \quad (11)$$

Introduced variables are defined as follows:

$$r_{1,2} = \sqrt{\frac{t - t_0 - z\sqrt{\varepsilon}/c}{t - t_0 + z\sqrt{\varepsilon}/c}}\sqrt{\frac{1 \mp c/v_L\sqrt{\varepsilon}}{1 \pm c/v_L\sqrt{\varepsilon}}},$$

$$y = \sqrt{c^2\left[(\pi k/b)^2 - (\pi l/d)^2\right]/\varepsilon}\sqrt{(t-t_0)^2 - (z\sqrt{\varepsilon}/c)^2}.$$

We denote

$$v_{ph} = c/\sqrt{\varepsilon}, \qquad v_{gr} = c^2/\varepsilon v_0.$$

At $t - t_0 - z/v_{ph} \ge 0$ the relations are fulfilled:

$$\begin{aligned} &0 < r_1 \quad 1, \\ &0 \le r_2 \le 1 \quad \text{for } t - t_0 - z/v_{gr} \le 0, \\ &r_2 > 1 \quad \text{for } t - t_0 - z/v_{gr} > 0. \end{aligned} \quad (12)$$

With the account Eqs. (11) and (12) for $t - t_0 - z/v_{ph} \ge 0$ and $t - t_0 - z/v_{gr} \le 0$ it can be written

$$U_2(r_2y, y) - U_2(r_1y, y) = \sum_{m=1}^{\infty}(-1)^m\left(r_1^{2m} - r_2^{2m}\right)J_{2m}(y) \quad (13)$$

Accordingly, for $t - t_0 - z/v_{gr} > 0$ we have

$$U_2(r_2y, y) - U_2(r_1y, y) = -\cos\left[\omega_0(t - t_0 - z/v_0)\right] + J_0(y) + \sum_{m=1}^{\infty}(-1)^m\left(r_1^{2m} + r_2^{-2m}\right)J_{2m}(y). \quad (14)$$

The first term on the right side of Eq.(14) describes an electromagnetic wave inside the region $0 < z < (t - t_0)v_{gr}$ which coincides with Vavilov-Cherenkov radiation (9) taken with opposite sign. This is the “quenching wave” that cancels the Vavilov-Cherenkov radiation in the corresponding region *z*. The remaining terms in Eq.(14) correspond to the part of transition radiation, which propagates with a velocity smaller than v_{gr}. Expression (13) corresponds to the faster part of transition radiation.

We now substitute (10) and (9) into (6), taking into account (13) and (14). It is convenient to represent the resulting field excited in a semi-infinite waveguide by a point charge moving with constant velocity as a superposition of spatially confined Vavilov-Cherenkov radiation E_z^{cher} and transition radiation E_z^{trans}.

$$E_z(t,x,y,z,t_0,x_0,y_0) = E_z^{cher} + E_z^{trans}, \qquad (15)$$

where

$$E_z^{cher}(t,x,y,z,t_0,x_0,y_0) = \frac{16\pi Ne}{bd\varepsilon}\sum_{k,l}\sin\left(\frac{\pi k}{b}x_0\right)\sin\left(\frac{\pi l}{d}y_0\right)\sin\left(\frac{\pi k}{b}x\right)\sin\left(\frac{\pi l}{d}y\right)\times$$
$$\times\cos\left[\omega_{kl}(t-t_0-z/v_0)\right]\times\left\{\Theta(t-t_0-z/v_0)-\Theta(t-t_0-z/v_{gr})\right\} \qquad (16)$$

$$E_z^{trans}(t,x,y,z,t_0,x_0,y_0) = \frac{16\pi Ne}{bd\varepsilon}\times$$
$$\times\sum_{k,l}\sin\left(\frac{\pi k}{b}x_0\right)\sin\left(\frac{\pi l}{d}y_0\right)\sin\left(\frac{\pi k}{b}x\right)\sin\left(\frac{\pi l}{d}y\right)\times$$
$$\times\left\{\left[\Theta(t-t_0-z/v_{ph})-\Theta(t-t_0-z/v_{gr})\right]\times\right.$$
$$\times\sum_{m=1}^{\infty}(-1)^m\left(r_1^{2m}-r_2^{2m}\right)J_{2m}(y)$$
$$\left.+\Theta(t-t_0-z/v_{gr})\left[J_0(y)+\sum_{m=1}^{\infty}(-1)^m\left(r_1^{2m}+r_2^{-2m}\right)J_{2m}(y)\right]\right\}.$$

$$\omega_{kl}^2 = [(\pi k/b)^2+(\pi l/d)^2]/(\varepsilon/c^2-1/v_0^2). \qquad (17)$$

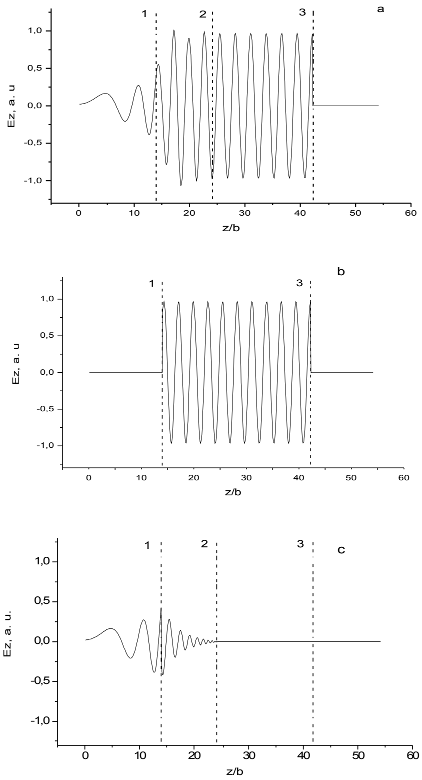

Fig. 1. Structure of the first mode of longitudinal component of wake field excited by a dot charge in a rectangular waveguide: a – total field E_z, b – field of the Vavilov-Cherenkov radiation E_z^{cher}, c – transition radiation E_z^{trans}. Numerals at curves: 1 – $z = z^{gr}$, 2 - $z = z^{ph}$, 3 – charge position. Time of observation $t = 260\,ns$, $t_0 = 0$, $\gamma = 9$, $\varepsilon = 3$.

The wake field (16) including the "quenching wave" is nonzero at $(t-t_0)v_{gr} \le z < (t-t_0)v_0$. In this region the envelope curve of Cherenkov radiation is constant (see Fig.1,b). The quantity v_{gr} is the group velocity of electromagnetic wave synchronous with the bunch. Evidently the plane $z^{gr} = (t-t_0)v_{gr}$ is the rear boundary of the wake field. This front moves behind the bunch with the group velocity v_{gr}.

The field of the transition radiation (17) exists in the region $0 \le z < (t-t_0)v_{ph}$. The quantity v_{ph} is the greatest velocity of propagation of the electromagnetic signal in the dielectric waveguide. It is the velocity, with which the fastest high-frequency part of the transition radiation – the so-called "precursor" – is propagating. The envelope curve of the transition radiation is maximum near the rear front (line 1 in Fig.1,c) and diminishes as one moves away from it. At the "precursor" (line 2 in Fig.1,c), it approaches zero. Near the entrance wall the envelope curve is small, but it is nonzero and decreases in time. The transition radiation (17) undergoes a sudden discontinuous change at the rear boundary of the wake field (see Fig.1,c). This is caused by the artificial split the continuous total field (15) into components, and the separated Vavilov-Cherenkov field (16) at the plane of rear front turns to zero abruptly too, as it shown in Fig.1,b.

At a fixed instant t the spatial structure of the spatial pattern of a single mode of the total field has the profile depicted in Fig.1,a. In front of the bunch (line 3) the field is absent. To the left of $z^{ph} = (t-t_0)v_{ph}$ (line 2) the field's envelope curve of the field starts to decrease, and at rear front $z = z^{gr}$ (line 1) it is equal to half of Vavilov-Cherenkov radiation amplitude. In the region $z << z^{gr}$ the field is small and decreases in time.

4. THE BUNCH OF FINITE SIZE

As a bunch of finite size we take the rectangular bunch of dimensions: width b_0, height d_0, and time duration t_b ($t_b = L_b/v_0$, where L_b – bunch length; the velocity v_0 is supposed constant and equal for all particles of the bunch) (see Fig.2). The charge density distribution of the bunch is considered homogeneous along both transverse and longitudinal coordinates. This bunch can be represented as a set of point bunches uniformly filling the volume of the finite sized bunch. Then the field excited by the bunch of finite size can be written as

$$E_z(t,x,y,r) = \sum_i E_z(t,x,y,r,t_{0i},x_{0i},y_{0i}),$$

where the elementary field $E_z(t,x,y,r,t_{0i},x_{0i},y_{0i})$ of a macroparticle with the number i is determined by the expressions (15)-(17).

Summation over the transverse coordinates can be changed by analytical integration of expressions (16), (17) over x_0, y_0.

In Fig.3 shown is the distribution of longitudinal wake field E_z along the waveguide axis excited in the semi-infinite dielectric waveguide by this relativistic electron bunch. The bunch has the following sizes: $b_0 = 10^{-3}$ cm, $d_0 = 0.15$ cm, $t_b = 3.5$ fs, charge of the bunch is 1pC, energy is 500 MeV. Waveguide sizes are $b = 18.8 \cdot 10^{-3}$ cm, $d = 0.15$ cm, $\varepsilon = 3$. The bunch moves along the waveguide axis. The time of observation is 32 ps.

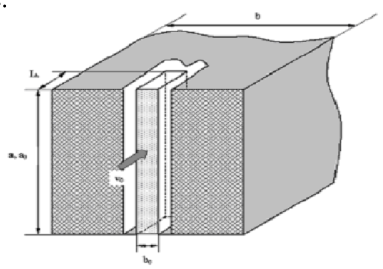

Fig. 2. Schematic of the model

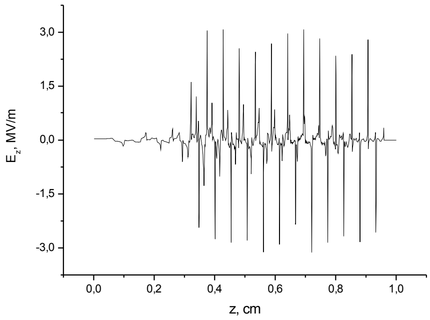

Fig. 3. Structure of longitudinal electric wake field excited in the semi-infinite waveguide axis by a single charged bunch. 50 modes in x, 50 modes in y, bunch charge is 1pC, energy is 500 MeV. The centre of a bunch is located on the waveguide axis.

The interference of many modes leads to the formation of the sequence of sharp peaks of wakefield with the amplitude considerably exceeding the amplitude of the principal mode. It is clearly seen that the region of maximum wakefield lies between the charged bunch and the rear front of the radiation. Behind the rear front the field is much less and has a different structure.

5. SEQUENCE OF BUNCHES. WAVGUIDE OF FINITE LENGTH

A proposed way to obtain an intense wake field is the use of a train of short bunches of moderate charge organized in such a way that these bunches should excite coherently. In this section we investigate the wake field excitation by a sequence of short relativistic bunches in the rectangular semi-infinite dielectric waveguide. We take a train of 15 bunches. It is seen in Fig.4 that the wake field grows approximately linearly with the number of bunches, and after the 13th bunch it decreases. Behind this last bunch (line 1), the field rapidly decreases to the back boundary of a wave packet behind which the field tends to zero.

The line 2 corresponds to the position of the rear front wake field excited by the leading bunch of the sequence. It determines the maximum number of bunches, from which fields can coherently sum up. The amplitude of the field in the fixed instant will attain the maximum value at the point

$$z^{gr} = v_{gr} t$$

and it will not increase with the growth of the number of bunches beyond a number

$$N^{\max} \approx \frac{t\left(v_0 - v_{gr}\right)}{\Delta z} + 1$$

where Δz is the distance between the neighboring bunches of the sequence. In our case $N^{\max} \approx 13$.

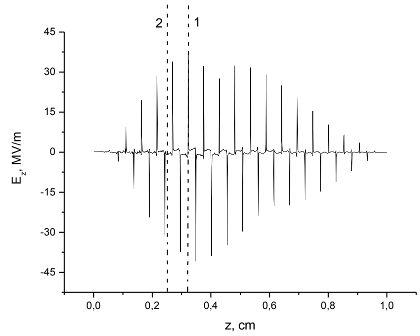

Fig. 4. Structure of longitudinal electric wake field excited on the semi-infinite waveguide axis by sequence of 15 charged bunches. 50 modes on x, 1 mode on y are taken into account. Others parameters are the same

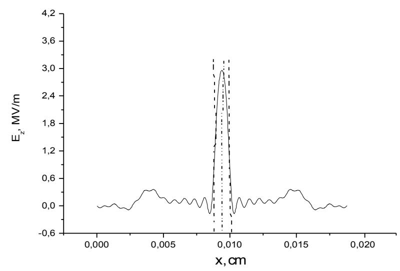

Fig.5. Distribution of longitudinal electric wake field on x, excited by a single charged bunch. The dashed line marks out a position of the bunch, the dot-dashed line marks out the axis of the waveguide. Others parameters are the same

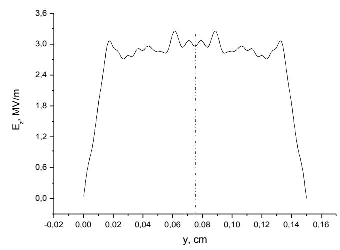

Fig.6. Distribution of longitudinal electric wakefield on y, excited by a single charged bunch. Others parameters are the same

The topography of the longitudinal component of the wake field is shown in Figs.5 and 6. In Fig.5 the profile of the longitudinal electric field excited by a single bunch over the width of dielectric waveguide is shown. Obviously, the features of wake field distribution excited by several bunches remain analogous. Here 50 modes are taken into account both in *x* and *y*. The field is computed at its maximum at the distance z = 0.64 cm. The field attains its maximum value on the

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waveguide axis and concentrates in the region of bunch location.

The distribution of longitudinal electric field over height of the waveguide is shown in Fig.6. The field amplitude is nearly homogeneous along the tall dimension of the waveguide.

6. WAKEFIELD IN THE PLANNED NSC KIPT EXPERIMENT

For a bunch of finite size or for a sequence of such bunches the expression for the wake field is obtained by integration over the transverse coordinates and the times when the electrons enter the structure. It should be noted that at using symmetric (respect to XZ and YZ planes) bunches injected along waveguide axis, only odd harmonics $k = 1,3,\ldots$, $l = 1,3,\ldots$ are excited.

To obtain high amplitude peaked wakefield in a rectangular waveguide we should select and excite only equally spaced modes among the many others in (17). In the planar geometry [2] all modes in the set of eigenmodes are equally spaced and the superposition problem for field peaking is solved automatically. As follows from (17) in the general case of arbitrary rectangular geometry, the diagonal frequencies ω_{kk} are always equally spaced. There is a good way to excite this partial series of eigenmode by using a sequence of bunches having repetition frequency equal to the frequency difference between these selected diagonal modes.

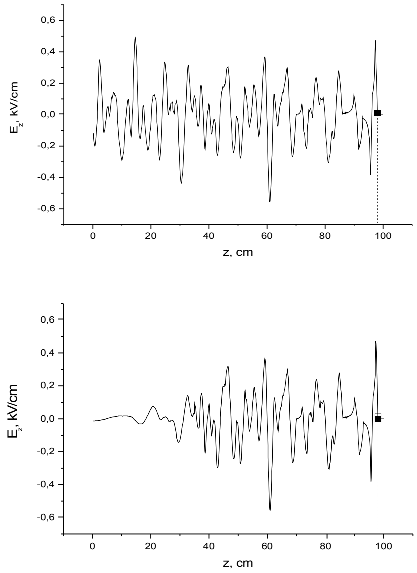

Fig. 7. Wakefield excited by a single bunch in the semi-infinite dielectric waveguide at the time point t=3.29 ns: upper - Cherenkov field, lower - full field. The label and dash line show bunch location

In Fig.7-Fig.8 the results of calculations for the following parameters of the NSC KIPT wake field experiment are presented: b=4.3 cm, d=8.6 cm, the charge of a single bunch Q_b=-0.32 nC, electron energy is $4\,\text{MeV}$; transverse sizes of a bunch- b_0=1.0 cm, d_0=1.0 cm, the bunch length is 1.71 cm; and $\varepsilon = 2.83$.

The single bunch excites many transverse harmonics (we consider 50 harmonics in x and 50 harmonics in y) . As all frequencies of the excited harmonics are not divisible by the lowest eigen-frequency as an integer, the longitudinal structure of the field has an irregular character even without taking into account the transition radiation. The presence of transition radiation especially complicates the field pattern as it contains a continuous spectrum of frequencies, extending from a cut-off frequency. The wake field trailing the bunch is partly cancelled by a disturbance moving with the group velocity, and, near the input plane of the system the amplitude of the field is close to zero.

If, in the semi-infinite waveguide, the sequence of bunches is injected with a repetition rate $f = 2886\,MHz$, which is equal to the lowest eigen-frequency of structure and consequently is equal to the frequency difference between diagonal elements of matrix $\{\omega_{kl}\}$, i.e. between frequencies ω_{kk}, the longitudinal structure of the field qualitatively changes. It becomes regular, with narrow peaks following the bunch, which have the same period as the bunch train: i.e., the sequence of bunches "cuts out" from the spectrum, excited by a single bunch, only frequencies, which are multiples of the bunch repetition rate. And, as we see from comparison of the curves in Fig.2, the transition field destroys only an insignificant amount of the regular structure of the field. "Cancellation" of the excited oscillations moving with the group velocity reduces the maximum number of bunches which can contribute to the growth of amplitude of the field [6,7]. For the length of this system $L = 100\,cm$ this bunch quantity is 17. Additional injection of bunches does not further increase the field amplitude with this given length of structure.

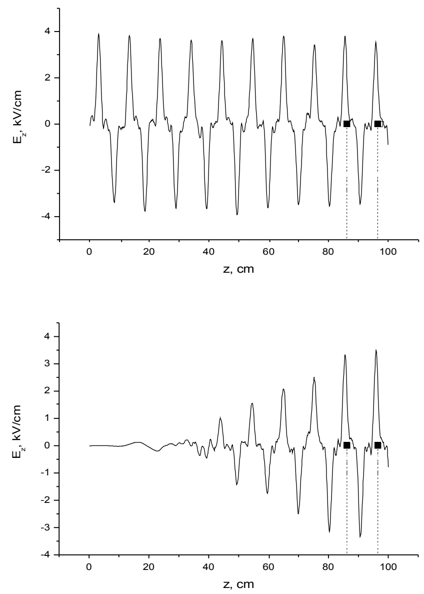

Fig. 8. Wakefield excited by a sequence of 17 bunches in the semi-infinite dielectric waveguide at the time t=8.43 ns: upper - Cherenkov field, lower - full field. Labels and dash lines show locations of the last 2 bunches of a sequence

7. "QUASI-MONOPOLAR" WAKE FIELD

A sequence of bunches, injected with a repetition rate equal to the lowest eigen-frequency ω_{11}, will excite the equally spaced harmonics with frequencies ω_{kk} and increase their amplitude due to coherent summation, while suppressing other non-equally spaced harmonics. For a certain relation between the transverse dimensions of the rectangular waveguide b and d, some non-diagonal elements of the matrix of eigen-frequencies ω_{kl} can also be divisible by the lowest excited eigen-frequency ω_{11} as an integer. These harmonics will survive and will be added to the harmonics with frequencies ω_{kk}.

In Fig.9 it is shown the longitudinal distribution of the wake field excited by the sequence of 5 bunches moving in a rectangular dielectric waveguide with dimensions $b = 4.88\,cm$; $d = 1.29b$. An exotic wake field with high amplitude peaks of one sign is excited in this case. The amplitude of the wake field of opposite sign is much lower.

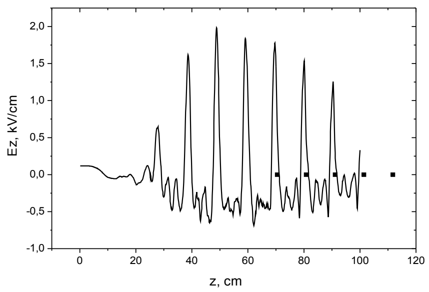

Fig. 9. Wakefield excited by a sequence of 5 bunches in a semi-infinite dielectric waveguide at t=3.78 ns. Labels show the locations of the last 4 bunches in the structure

8. SUMMARY

A 3-dimensional investigation of the excitation of Vavilov-Cherenkov and transition radiations in a semi-infinite dielectric waveguide of rectangular cross-section by a relativistic electron bunch or a train of identical bunches has been carried out.

Using Fourier transforms and the theory of functions of complex variables, the exact analytical solution of a problem of propagation of the electromagnetic signal excited by a moving point charged bunch in the semi-infinite rectangular dielectric waveguide is found. Analytical expressions for the fields of both Vavilov-Cherenkov and transition radiations are found. The longitudinal structure of the excited electric field looks like a periodic sequence of short peaks of alternative sign. It is the result of the interference from a great number of transverse modes. The topography of the wake field in cross-section conforms to the cross-sectional shape of the bunch.

The wake field growth is obtained by coherent addition of fields set up by an equally-spaced sequence of electron bunches. Phenomena originating at the entrance boundary, namely, excitation of transition radiation and the restriction upon the number of coherently exciting bunches are explored. The restriction arises from the disappearance of excited radiation in a region moving away from the entrance with the group velocity. The greatest amplitude of the longitudinal electric field is obtained when a new bunch crosses this frontier. The largest field that can be set up is determined by the parameters of the waveguide and the number of coherently exciting bunches. Taking the parameters of bunches in a proposed Brookhaven experiment [1], the wake field excited by 12 bunches in a rectangular dielectric waveguide, is equal to 40 MeV/m. To study the experiment adequately, it is necessary to optimize the waveguide sizes, take into account the vacuum channel needed for bunches to move through, and possibly to increase the number of coherently exciting bunches by using a resonator concept.

It is shown that the excited field is localized in the region occupied by the electron bunch. This might offer mitigation of limits imposed by dielectric breakdown; but to examine this possibility it is necessary to solve this problem again, taking into account the vacuum drift channel for electron bunch.

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СПЕКТРЫ ЧЕРЕНКОВСКОГО И ПЕРЕХОДНОГО ИЗЛУЧЕНИЯ, ВОЗБУЖДАЕМЫЕ ЦЕПОЧКОЙ ЭЛЕКТРОННЫХ СГУСТКОВ В ДИЭЛЕКТРИЧЕСКИХ СТРУКТУРАХ ПРЯМОУГОЛЬНОГО СЕЧЕНИЯ

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Исследовано возбуждение кильватерного поля электронными сгустками и их последовательностью в прямоугольных диэлектрических структурах: бесконечном волноводе, полубесконечном волноводе и волноводе конечной длины.

СПЕКТРИ ЧЕРЕНКОВСЬКОГО ТА ПЕРЕХІДНОГО ВИПРОМІНЮВАННЯ, ЗБУДЖЕНОГО ЛАНЦЮЖКОМ ЕЛЕКТРОННИХ ЗГУСТКІВ У ДІЕЛЕКТРИЧНИХ СТРУКТУРАХ ПРЯМОКУТНОГО ПЕРЕРІЗУ

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Досліджене збудження кільватерного поля електронними згустками та їх послідовністю в прямокутних діелектричних структурах: необмеженому хвилеводі, напівобмеженому хвилеводі та хвилеводі кінцевої довжини.