CALCULATION OF ION FOCUSING WITH A PLASMA ELECTROSTATIC LENS IN A MAGNETIC FIELD FORMED BY COUNTER RING CURRENTS

V.I. Butenko
National Science Center "Kharkov Institute of Physics and Technology", Academicheskaya St., 1, Kharkov, 61108, Ukraine (E-mail: butenko@kipt.kharkov.ua)

PACS numbers: 41.85.–

1 INTRODUCTION

The problems of intense ion beam focusing are important for the nuclear physics, physics of high energies, physics and engineering of accelerators, beam technologies. The essential feature of intense ion beams is that they should be charge compensated during the focusing to prevent their destruction. In this case, the application of plasma-optic focusing systems which development is initiated by A.I. Morozov and co-workers [1], and recently successfully developed by A.A. Goncharov group [2–4] is expedient.

In the plasma electrostatic lens of Morozov type the magnetic surfaces are the equipotentials of the electrical field. It is supposed, that the current across a magnetic field is absent, and intensity and spatial distribution of electrical field in a plasma are completely determined by the magnetic field geometry and boundary condition. The last one is given as a continuous function $\Phi(R, z)$, where Φ is the potential (that is set from the outside), and R is the cylindrical surface radius. In practice the electrical potentials are entered in plasma by a discrete manner, using "basic" ring electrodes. The experimental researches [2–4] basically confirm the theoretical model [1], but some problems remain, in particular, the reasons of rather significant aberrations and methods of their elimination. In this work computer modeling of plasma-optic focusing devices is considered. Such simulations require the development of special-purpose computer codes aimed at modeling particular plasma-optic configurations. The numerical codes will make it possible to optimize efficiently the existing and planned experimental devices.

2 THE PROBLEM DEFINITION

In the previous work [5] a single-turn Morozov lens is considered. Its disadvantage is far-reaching magnetic surfaces. In experimental works [2–4] the configuration of a magnetic field with counter inclusion of solenoids is offered, that allows to locate basic electrodes near to the central plane of a lens. In the given work the lens is simulated by three coils with opposite currents.

The magnetic field of a ring current J (at radius of a coil a_c and coordinate l on an axis z) is described by azimuth component of vector potential:

$$A_\varphi = \frac{4J}{ck}\sqrt{\frac{a_c}{r}}\left[\left(1-\frac{k^2}{2}\right)\mathbf{K}(k)-\mathbf{E}(k)\right],$$

$$k^2 = \frac{4a_c r}{(a_c+r)^2+(z-l)^2}.$$

where c is the light velocity, **K** and **E** are the complete elliptic integrals of the 1-st and 2-nd kind.

The equation of magnetic surfaces has the form: $rA_\varphi = const$.

The topography of magnetic surfaces for a various ratio of currents in central and lateral coils (J_c and J_s, respectively) was calculated. Further we used topography of force lines $J_c = 1.5\ J_s$. The central coil is located at $z = 0$, lateral at $z = 5.7$ cm.

In the central part of a lens (-2.8 cm $< z < 2.8$ cm) the potentials of basic ring electrodes are applied to magnetic surfaces, and the magnetic surfaces at the left and to the right of the central area are considered as grounded.

The topography of equipotential surfaces at $J_c = 1.5\ J_s$. are presented in Figs. 2, 3, 6 and others.

3 BASIC EQUATIONS

Following [1], we shall enter the function of a magnetic flow:

$$\Psi(r, z) = rA_\varphi(r, z),$$

where A_φ is the azimuth component of the vector potential. In a Morozov lens the equipotentiality of magnetic surfaces is determined by the relation [1]:

$$\Phi(r, z) = F(\Psi),$$

where Φ is the potential of an electric field.

Let's express components of an electric and magnetic fields through Ψ:

$$B_r = -\frac{1}{r}\frac{\partial\Psi}{\partial z},$$

$$E_r = -\frac{\partial\Phi}{\partial r} = -\frac{dF}{d\Psi}\frac{\partial\Psi}{\partial r} = -r\frac{dF}{d\Psi}B_z,$$

$$E_z = -\frac{\partial\Phi}{\partial z} = -\frac{dF}{d\Psi}\frac{\partial\Psi}{\partial z} = r\frac{dF}{d\Psi}B_r.$$

The equations of motion in cylindrical system of coordinates:

$$\frac{dV_r}{dt} = \frac{e}{m}E_r + \frac{e}{mc}V_\varphi B_z + \frac{V_\varphi^2}{r},$$

$$\frac{dV_\varphi}{dt} = \frac{e}{mc}\left(V_z B_r - V_r B_z\right) - \frac{V_r V_\varphi}{r},$$

$$\frac{dV_z}{dt} = \frac{e}{m}E_z - \frac{e}{mc}V_\varphi B_r.$$

Substituting here the expressions for a component of electric and magnetic fields, we have:

$$\frac{dV_r}{dt} = \frac{e}{m} B_z \left(\frac{1}{c} V_\varphi - r \frac{dF}{d\Psi} \right) + \frac{V_\varphi^2}{r},$$

$$\frac{dV_\varphi}{dt} = \frac{e}{mc} \left(V_z B_r - V_r B_z \right) - \frac{V_r V_\varphi}{r},$$

$$\frac{dV_z}{dt} = \frac{e}{m} B_r \left(r \frac{dF}{d\Psi} - \frac{1}{c} V_\varphi \right).$$

If the initial azimuthal velocity of the beam $V_{\varphi 0} = 0$,

then $rV_\varphi + \frac{e}{mc}\Psi = \frac{e}{mc}\Psi_0$, (*)

where Ψ_0 is a function of a magnetic flow in the injection region.

4 RESULTS OF CALCULATIONS

Let's carry out calculations of the ion trajectories at parameters comparable to the Kiev lens [2–4]: energy of protons W = 20 keV, radius of an injected beam $r_0 = 3.5$ cm, beam is parallel, radii of current rings $r_a = 6.5$ cm, coordinate of protons injector z_0 = -30 cm, proton current of is 1 A. In the regions of a beam injection and its focus the magnetic field is practically equal to zero, thus in the focus $rV_\varphi = 0$, and moment aberrations are absent, see (*).

For focusing of ions the distribution $\Phi \propto r^2$ is usually considered. But in this case it does not carry to satisfactory results (see calculation of ion trajectories in *Fig. 1* at $\Phi = 0.65\ r^2$ and current density in the focus region in Fig. 2).

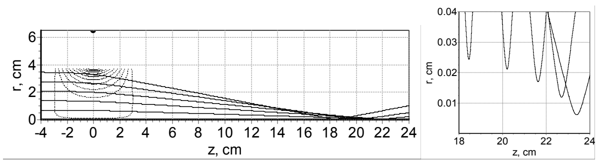

Fig. 1.

In a plane of the central coil the optimized distribution of potential on radius (in GS) is a polynomial:

$$\Phi = 0.75\ r^2 - 0.0121\ r^4,$$

where the factors are adjusted such that the focusing is best. In *Fig. 3* the proton trajectories for this case are submitted. In Fig. 4 the current density distribution of protons on radius is submitted in the focus region. From distribution of potential on radius in the plane of the central coil we shall pass to distribution of potential on length of a cylindrical surface with radius R, see Fig. 5. In experimental work [4] optimum focusing was received at distribution of the potential on a cylindrical surface proportional to distribution of a longitudinal magnetic field on an axis of lens. For such distribution we have found proton trajectories of (Fig. 6) and distribution of current density on radius (Fig. 7). Thus density of the current in focus $J_{max} = 1.2$ A/cm^2 is received at average beam radius 0.5 cm, that will be matched to the experimentally found results, but considerably concedes to the above-mentioned optimized calculated results.

Till now in calculations we accepted continuous distribution of potential on coordinates. As against it in experiments [2–4] the potentials in plasma are entered with the help of finite number (5 or 9) cylindrical electrodes.

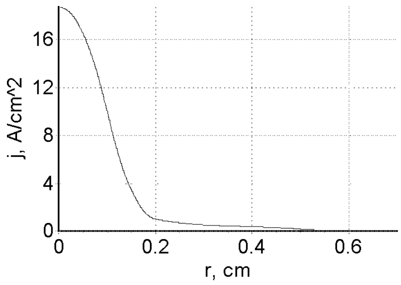

Fig. 2.

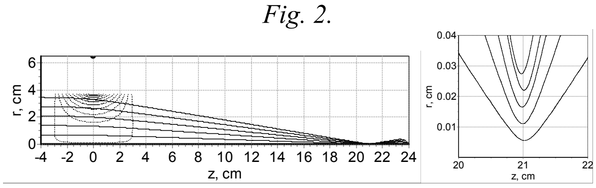

Fig. 3.

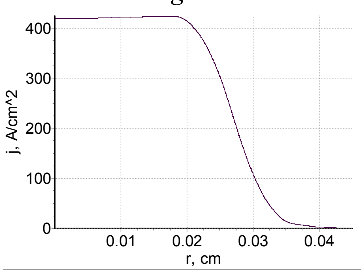

Fig. 4.

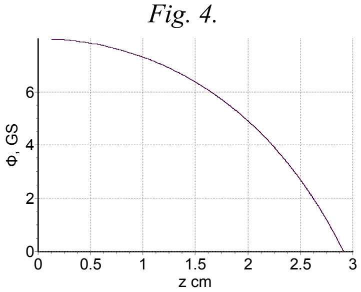

Fig. 5.

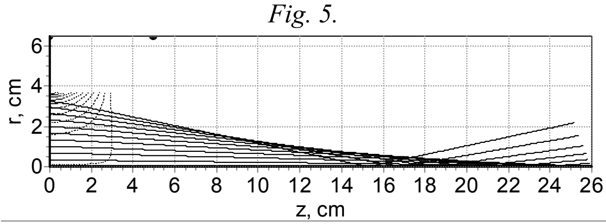

Fig. 6.

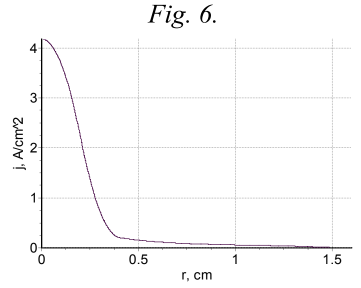

Fig. 7.

Let's consider a case of 9-electrode lens, that corresponds to the preset of 6 discrete values of potential on half of lens (6-th potential corresponds to zero potential on an axis). If these 6 values be set on the optimized curve submitted in Fig. 5, and be smooth by splines, the satisfactory concurrence of these curves will turn out.

However in experiment the electrodes of a finite length specifying the step distribution of potential were applied which in plasma smoothed out. Characteristics of this smoothing are not investigated yet experimentally. Formally, in calculations this smoothing was simulated by **B**-splines of the 3-rd order, and the degree of smoothing was defined by a ratio of effective electrode length that can be smaller than real one, and effective gap lenght between electrodes (this length can be larger than real one). For example, at a gap between electrodes of 5 mm the smoothed distribution was observed (Fig. 8). The proton trajectories, corresponding to this case, and their distribution on radius in the focal plate are presented in Fig. 9 and Fig. 10.

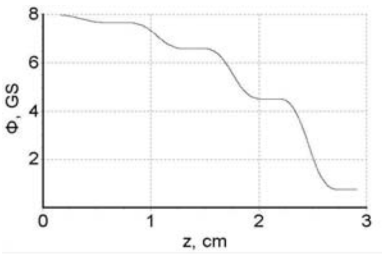

Fig. 8.

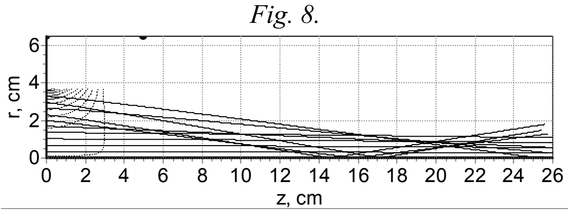

Fig. 9.

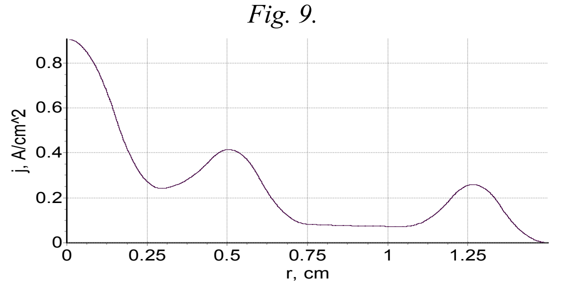

Fig. 10.

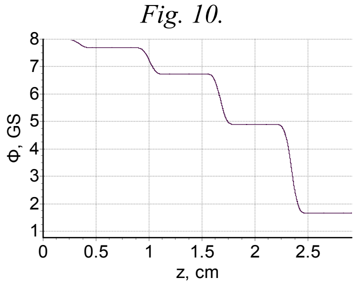

Fig. 11.

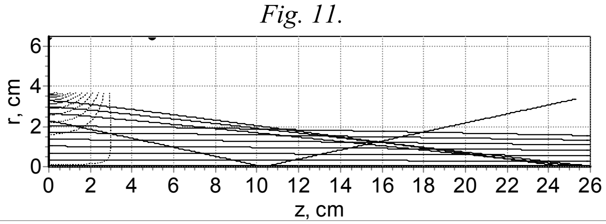

Fig. 12.

At the effective gap between electrodes of 2 mm (that is close to the experimental value) the smoothed distribution with more distinct stairs is received (Fig. 11). The proton trajectories which correspond to this case are shown in Fig. 12 and the current density in the focus region in Fig. 13. The current density (about 0.1 A/cm^2) and halfwidth of a focal spot (about 1 cm) under the order coincide with the experimental results [2–4]. In this case because of the step distribution of potential in plasma, bad beam focusing takes place. As the line of magnetic force in the marginal electrode region passes very closely and near to the axis and they settle down far apart, paraxial ions appear at the same potential, and therefore are not focused. Thus a divergence between experiment and calculations is significant, hence, in real plasma the potential smoothing is much stronger. The current density (~0.3 A/cm^2) and the focal spot half-length (~1 cm, see Fig. 13) corresponded to the experimental results by the order of values. Besides, in the paper [6] the ion focusing improvement was observed for the case while the magnetic field strength of the lens of such type decreased. Except of another causes, the smoothing of the potential step distribution can influence this effect in such case.

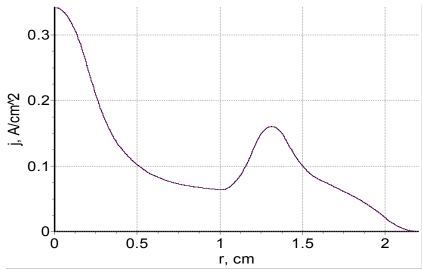

Fig. 13.

Taking into account these investigations, it is expedient to move the recommendations as follows. 1. Using the adequate computer model of the plasma lens for optimization of the electric field distribution. 2. Increasing the number and decreasing the thickness of the basic ring electrodes. 3. Testing the optimum electric field distribution in the plasma by a suitable precision experimental method.

The author would like to thank B.I. Ivanov for the proposition of the work subject, help in the work, and fruitful discussions.

REFERENCES

1. A.I.Morozov, S.V.Lebedev. // *Plasma Theory Problems*, V.8, P.247, Moscow, 1974.
2. A.A.Goncharov et.al. // *Plasma Physics Reports,* 1994, v. 20, p. 499.
3. A.A.Goncharov et.al. // *IEEE Trans. Plasma Sci*. 1997, v. 25, p. 709.
4. A.A.Goncharov et.al. // *Appl. Phys. Lett.* 1999, v. 75, p. 911.
5. V.I.Butenko, B.I.Ivanov. // *Problems of Atomic Science and Technology. Issue: Plasma Electronics and New Acceleration Methods* (2). 2000, No. 1, p. 164.
6. A.A.Goncharov et.al. // *Problems of Atomic Science and Technology. Issue: Plasma Physics* (6). 2000, No. 6, p. 124.